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ABSTRACT

Primary school mathematics teachers should, at the most basic level, have mastery of the content knowledge that they are required to teach. In this paper we test empirically whether this is the case by analyzing the South African SACMEQ 2007 mathematics teacher test data which tested 401 grade 6 mathematics teachers from a nationally representative sample of primary schools. Findings indicate that 79% of grade 6 mathematics teachers showed content knowledge levels below the grade 6/7 band, and that the few remaining teachers with higher-level content knowledge are highly inequitably distributed.

Keywords: mathematics teacher knowledge, SACMEQ, South Africa, mathematics
JEL codes: I20, I21, I28

1. Introduction

An extensive body of assessment data points to poor performance in mathematics across all levels of the schooling system in South Africa. This data ranges from classroom observation and localized small-scale studies (Schollar, 2008; Ensor et al, 2009) to nationally representative assessments of mathematics achievement, such as SACMEQ, TIMSS and the National School Effectiveness Study (NSES)[1]. An analysis of this literature reveals a variety of frameworks and methodologies, but across these, there is unanimous agreement on the very low and highly unequal performance of South African students.

Over the last decade, ongoing low student performance has led to increasing interest in understanding how teacher characteristics, pedagogical practices and content knowledge may figure within these patterns of poor academic development (Taylor & Vinjevold, 1999; Carnoy, Chisholm, & Chilisa, 2012; Taylor & Taylor, 2013). A common finding across these studies relates to the presence of large numbers of South African mathematics teachers who lack fundamental understandings of mathematics.

Our focus in this paper is on one specific element of teacher quality: teacher content knowledge. Using the South African SACMEQ 2007 data we analyze the test responses of 401 grade 6 mathematics teachers drawn from a nationally representative sample of South African primary schools. While not designed with reference to the South African mathematics curriculum specifically, this test consisted of 42 multiple-choice items drawn from several content domains. Mapping these questions to the current South African curriculum revealed that some items could be linked to curriculum specifications below the grade 6 level, some at the grade 6/7 level, and some above this level.

Our research questions center around understanding what South African grade 6 mathematics teachers know relative to the South African school curriculum. Our interest in this research question is driven by concerns for conceptualizing mathematics teacher development and for policy implications that might guide this conceptualization. We thus begin the paper by overviewing previous studies that have looked at mathematics teacher content knowledge in South Africa in recent years. We include in this overview findings from previous analyses of the SACMEQ 2007 data. We note in particular the ways in which teacher content knowledge is conceptualized in these studies and contrast these conceptualizations with key international conceptualizations. Gaps in the categories used in prior analyses of the SACMEQ 2007 teacher performance dataset provide a rationale for the present re-analysis. Thereafter we present our findings and discussion along with the policy implications for in-service mathematics teacher development.

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[1] SACMEQ stands for the Southern and East African Consortium for Monitoring Educational Quality, TIMSS stands for the Trends in International Mathematics and Science Study. The most comprehensive reports for these studies are Moloi & Chetty (2011) for SACMEQ, Reddy et al. (2012) for TIMSS (2012) and Taylor, Van der Berg & Mabogoane (2013) for the NSES.

2. The extant literature on mathematics teacher content knowledge in South Africa

It is impossible to survey the literature on mathematics teacher knowledge in South Africa without taking cognizance of the country's political history. After 46 years of legislated racial exclusivity, South Africa emerged from apartheid and embraced democracy in 1994. In trying to grapple with a past gripped by severe educational inequity, policy makers systematically dismantled apartheid-era educational policies and replaced them with non-racial policies aimed at rectifying the wrongs of the past. At the time of the transition there were 18 education departments, 32 universities and technikons and 105 colleges of education, which varied widely in terms of quality (Sayed, 2004, p. 248). Teacher education colleges were subsequently integrated into higher-education institutions, but graduates of the former segregated institutions still populate many South African schools alongside post-apartheid teacher education graduates. While the variable quality of teacher education under apartheid has been widely written about, concerns have continued to be expressed post-apartheid about the highly variable quality of in-service teacher training (Council on Higher Education, 2010). The central concern we overview in this section remains that many of these teachers lack the content knowledge and pedagogical skill required to provide access to the disciplinary ideas of mathematics.

Following the transition to democracy, the President's Education Initiative commissioned research on a range of topics including teacher content knowledge. Taylor & Vinjevold (1999, p. 230), summarizing the 54 studies that made up this initiative, concluded that:

> "The most definite point of convergence across the [President's Education Initiative] studies is the conclusion that teachers' poor conceptual knowledge of the subjects they are teaching is a fundamental constraint on the quality of teaching and learning activities, and consequently on the quality of learning outcomes."

In the wake of these findings, curriculum reform moved away from the sparse specifications that had characterized the first post-apartheid curriculum (Curriculum 2005) towards a more overt specification, but concerns about teacher content knowledge and knowledge about the sequencing and pacing of mathematical ideas in the middle years continued to be expressed (Reeves & Muller, 2005).

A key point to note in the early post-apartheid period is that concerns about teachers' mathematical knowledge were predominantly inferred from studies of classroom practice (e.g. Taylor & Vinjevold, 1999). With learner performance continuing to be low, and seemingly not 'address-able' through waves of curriculum reform, attention turned more directly to measuring teacher knowledge, with classroom-based studies continuing alongside this turn. We overview the findings of the more recent classroom-based studies, before turning our attention to research that has focused on measuring teachers' mathematical knowledge more directly.

A range of smaller scale studies have analyzed data based on classroom teaching to make inferences about the nature of primary teachers' disciplinary knowledge of mathematics and their subject knowledge for teaching. Sorto & Sapire (2012) and Ally & Christiansen (2013) both note the prevalence of highly procedural orientations to mathematics teaching among different sets of grade 6 teachers in South Africa, pointing to a disposition to view mathematics as driven towards the production of answers.

A second strand of inferences about teachers' mathematical knowledge points to teaching analysed for mathematical coherence. Early primary years' data points to lack of awareness of 'givens' and 'unknowns' within teachers' explanations (Venkat, 2013). More broadly, focus on explanations points to absence of mathematical criteria for judging whether steps and results are appropriate (Hoadley, 2006) and a range of disruptions to connections (Venkat & Adler, 2012; Adler & Venkat, 2014). These findings echo the attention to the 'conveying' of ideas that Van der Berg et al (2011) have noted as a potential reason for differences in content knowledge in South Africa not playing through in any direct way to learner performance. This point is made widely in the international literature on assessment of teachers' mathematical knowledge, with content knowledge seen as necessary but not sufficient for high quality teaching (e.g. Prestage & Perks, 2001).

While acknowledging the limitations of content knowledge per se, research findings relating to measuring South African teachers' mathematical content knowledge suggest that a focus on content knowledge remains critical. The National School Effectiveness Study (NSES) included a short teacher test based on grade 6 related curriculum items given to participating grade 4 and 5 teachers from all but one of the South African provinces (Taylor, Van der Berg, & Mabogoane, 2013). The items focused on estimation/rounding, fraction addition, pattern continuation, time, and perimeter of a composite shape. Analysis of grade 4 and 5 teachers' performance on these tasks indicated significant gaps on grade 6 related items, with the highest performance seen on the estimation/rounding task with 64% of grade 4 and 68% of grade 5 teachers getting this item correct. About half the teachers from both grades were able to answer the fraction addition, time, and pattern continuation items correctly, with this facility dropping to below a third of teachers in both grades getting the perimeter item correct. Taylor (2011) pointed out through their linked analysis of learner and teacher performance, that only those teachers getting all five items correct showed positive impact on learner performance.

This study provides evidence of gaps at the level of simply being able to 'do the mathematics' at the grade level of teaching for many teachers across the topics that were tested, negating possibilities for problems to be shifted to the level of 'conveying mathematics'. Given that the NSES mathematics teacher test consisted of only five items, one cannot point out relative areas of strength or weakness in relation to mathematics. However, given that the NSES and other tests overviewed here point to gaps related, or very close to, content that these teachers were expected to teach, further insight into mathematical content strands that may be classified as particularly problematic, or worth focusing on as priorities within teacher development, would seem to be useful. This provided one of the motivations for our re-analysis of the SACMEQ teacher test data.

Following on from the NSES study of 2007/8/9, Carnoy *et al.* (2011, p. 89) found that grade 6 mathematics teachers in the North West province of South Africa achieved an average score of 40% on a more extended test consisting primarily of grade 6 level items. In their earlier pilot study they also concluded that:

> "The relatively low level of mathematics knowledge that teachers have in all but the highest student [socioeconomic status] schools is somewhat troubling. It raises some doubts about the preparation of the teacher force" (Carnoy & Chisholm, 2008, p. 33).

This study, once again, confirms gaps at grade 6 level but does not offer guidance on the grade level knowledge that can be assumed. Knowing what large numbers of teachers cannot do does not provide useful information for policy to input on the content level at which teacher development could usefully begin. The SACMEQ 2007 dataset, with items based on a range of content areas and levels, provided a dataset where we could analyze performance in relation to a spectrum of curricular grades.

3. Studies analyzing the SACMEQ 2007 mathematics teacher test

Prior to the SACMEQ 2007 study, published research projects looking at mathematics teacher content knowledge were largely localized studies focusing on only a few schools or at most a single district (Van der Sandt & Nieuwoudt, 2003; Taylor & Moyana, 2005). Where the SACMEQ 2007 study differed was that it tested grade 6 mathematics teachers drawn from a nationally representative sample of primary schools in South Africa. While the primary aim of the SACMEQ study was to test grade 6 students' numeracy and literacy proficiency, one element of the survey included a mathematics teacher test administered to the mathematics teacher(s) teaching the grade 6 students being tested, and a language teacher test administered to the language teacher(s) teaching the grade 6 students being tested. In the case of South Africa there were 401 grade 6 mathematics teachers who wrote the mathematics teacher test.

To date, three studies have analyzed the SACMEQ mathematics teacher data in a comprehensive way, namely the report by Spaull (2013b), the quantitative analysis by Shepherd (2013) and the more in-depth analysis by Taylor & Taylor (2013). Two other studies have provided broader surface-level analyses of the SACMEQ 2007 teacher test data, namely that of Hungi *et al.* (2011) and Moloi & Chetty (2011). It is worth noting here that there is disagreement in the literature between Hungi *et al.* (2011) and Moloi & Chetty (2011) as to the findings from the SACMEQ 2007 mathematics teacher test results for South Africa. While Moloi & Chetty (2011, p. 60) find that "teachers reached high competency levels in both the Reading and Mathematics tests", Hungi *et al.* (2011, p. 13) report that only 32% of South African grade 6 mathematics teachers have "desirable levels" of mathematics knowledge. Hungi *et al* (2011, p.13) further note that almost all other countries that took part in the study reported higher proportions of teachers with desirable levels of content knowledge, for example: Kenya (90% of teachers), Zimbabwe (76% of teachers) and Swaziland (55% of teachers). The difference between these two studies relates to what constitutes “desirable” or “acceptable”, with Moloi & Chetty reporting high achievement of teachers relative to SACMEQ scales *calibrated for student achievement levels*. We return to this issue in our analysis of the SACMEQ data below.

Both Moloi & Chetty (2011) and Hungi *et al.* (2011) use Rasch total scores when discussing teacher test scores. Rasch analyses calculate difficulty values for each item and then use these difficulty values in conjunction with student and teacher responses to each question to create a composite score with a particular mean (500) and standard deviation (100) using data from all participating countries. Given that the teacher test was different to the student test, and that there were 15 common items in both the student and teacher test (Ross, et al., 2005, p. 126), teacher test scores could be rescaled so as to be comparable with student test scores.

The main benefit of using Rasch analysis is that items are weighted differently based on their difficulty and a student’s performance on those items. This is in contrast to simpler measures such as “percentage correct” which weights all items equally. For the purposes of the present analysis we do not use Rasch scores, partially because these scores have already been reported for South Africa (Moloi & Chetty, 2011, p. 43), but more importantly, because they are difficult to interpret intuitively and are thus less helpful for policymaking purposes. For example, knowing that the South African average grade 6 mathematics teacher test score is 764 is not particularly illuminating since these scores have little conceptual purchase outside of those familiar with SACMEQ. While this may be useful for comparing the *relative* performance of provinces (for example the average score in the Western Cape was 852 while in Mpumalanga it was 700), it tells us little in relation to mathematical topics, curriculum levels, or traditional mathematical benchmarks. We therefore use the ‘percentage correct’ approach in this paper.

Moving to the more comprehensive analyses, Shepherd (2013) employs a within-pupil across-subject analysis and finds that: ‘Teacher knowledge is only estimated to have a significant positive impact on performance when considering the wealthiest quintile of schools’ in South Africa’ (Shepherd, 2013, p. 1). She also cautions that this effect is removed after controlling for teacher unobservables and that these are likely to be correlated with teacher content knowledge. Consequently, and due to the difficulty of ascribing causality with cross-sectional data, the results from this analysis remain tentative. Furthermore, Shepherd uses the overall mathematics teacher test score and does not disaggregate the items by content-strand or grade-level as we do here.

In their analysis of the SACMEQ 2007 data, Taylor & Taylor (2013) also use the ‘percentage correct’ approach. Their primary interest in analyzing the data is to compare learner and teacher performance on common items. Thus, their methodology identified three broad cases of items: (1) ‘transmission’ - items on which teacher and learner scores were relatively high, (2) ‘knowledge impedance’ - items on which teacher and learner scores were relatively low, and (3) ‘complex impedance' - items on which teacher scores were relatively high but learner scores were relatively low. These three categories follow from their model of teacher knowledge as comprised of three aspects: disciplinary knowledge, subject knowledge for teaching, and classroom competence (2013, p.206). Of the 15 items that were common across the teacher and learner tests, eight items fell into the 'knowledge impedance' category and a further five items fell into the 'complex impedance' category.

Taylor & Taylor (2013) also included a breakdown of teacher test items by mathematical strands (arithmetic operations; fractions, ratio and proportion; algebraic logic; rate of change; and shape and space), and included some commentary on item-specific language demands. A key finding from their analysis was that many teachers exhibited particular weaknesses related to fractions, ratio and proportion and shape and space. Comparing student and teacher performance on common items, they also argued that content knowledge gaps and inadequate subject knowledge for teaching seemed to contribute to poor learner level results. In the closing section of their chapter they conclude:

> "The subject knowledge base of the majority of South African grade 6 mathematics teachers is simply inadequate to provide learners with a principled understanding of the discipline…providing teachers with a deep conceptual understanding of their subject should be the main focus for both pre- and in-service teacher training" (Taylor & Taylor, 2013, p. 230).

Taylor & Taylor's analysis (2013) highlights that content knowledge, while necessary, is not sufficient for coherent teaching. While this understanding can lead to questions about why the emphasis on content knowledge continues, the converse statement relating to a lack of content knowledge has much less contention associated with it. Given that 8 of the 15 common items indicated 'low' teacher performance, Taylor & Taylor's findings stress that emphasis on fundamental content knowledge remains very important in the South African terrain.

4. Frameworks for considering South African teachers' mathematical content knowledge

The preceding overview points to different ways in which teachers' disciplinary knowledge of mathematics is viewed and/or measured across the studies that have been overviewed here. Moloi & Chetty (2011) use a particularly low benchmark for what constitutes an adequate level of content knowledge by using a framework that was developed for assessing early numeracy learning. The NSES study adopts a 'minimal' approach to assessing teachers' mathematical knowledge with their five-item (grade 6 curriculum-related) test. The limited nature of this test is acknowledged (Taylor, 2011), but remains useful for drawing attention to content knowledge gaps at, or close to, the levels at which the teachers' are actually teaching.

Taylor & Taylor (2013) provide a framework linked to mathematical topic strands rather than hierarchical levels of mathematics, aligning with their key interest in comparing teachers' performance on items with learners' performance. Carnoy et al. (2012) include both content knowledge and pedagogical content knowledge in their teacher testing, and as such, present a broader view of the knowledge base required for teaching primary mathematics.

The key point to note about these different configurations of teachers' mathematical content knowledge is the extent to which (even in the Carnoy et al study) they differ from the international literature base on 'mathematical knowledge for teaching'. Even if we restrict our attention to content knowledge per se, rather than including pedagogic content knowledge, frameworks in the international literature base take a broader and deeper perspective on the knowledge base required for teaching. We illustrate this with two important examples from the international literature on mathematical knowledge for teaching, both drawn from research groups focused on primary level mathematics teaching. Ball, Thames & Phelps' (2008, p.389) interest is in characterizing 'professionally oriented subject matter knowledge in mathematics'. Within their subject matter knowledge category (i.e. leaving out their additional focus on pedagogic content knowledge), these authors distinguish between 'common content knowledge' (CCK), 'specialized content knowledge' (SCK) and 'horizon content knowledge' (HCK) – each described in the following terms:

> CCK: 'the mathematical knowledge known in common with others [i.e. not teachers] who know and use mathematics.' (p.403, our bracketed addition for clarity)
>
> SCK: 'the mathematical knowledge and skill unique to teaching. […] In looking for patterns in student errors or in sizing up whether a nonstandard approach would work in general, … teachers have to do a kind of mathematical work that others do not.' (p400)
>
> HCK: 'an awareness of how mathematical topics are related over the span of mathematics included in the curriculum. (p.403)

Focusing on the notion of 'knowledge packages', Ma (1999) describes the need for primary mathematics teacher knowledge to exhibit what she terms as 'profound understanding of fundamental mathematics'. In contrasting the knowledge base of the Chinese and American teachers in her study, Ma noted the greater incidence of a knowledge of fundamental mathematics that was 'broad, deep and thorough' amongst the Chinese teachers, seen in awareness of connections between concepts and of progressions of key ideas and allowing for flexible movements between representations and related ideas. This contrasts with the disconnections evident in teaching at all levels of the South African schooling system (Venkat & Adler, 2012; Adler & Venkat, 2014). Ma (1999) further notes that while there was a higher prevalence of advanced mathematics course taking among the American teachers, this did not appear to translate into better 'profound understanding of fundamental mathematics'.

Across both of these frameworks, we see indicators that extend more broadly and more deeply in mathematical terms than the frameworks underlying the construction and analysis of the items/examples in the South African terrain. The overview of South African teacher performance indicates gaps at the level of what Ball et al (2008) describe as common content knowledge. Gaps at this level raise questions about whether attention to more advanced mathematics is an *immediate* priority in the South African teacher education landscape, in spite of advocacy in the international literature.

We note these differences in order to point out the extent to which, even at the broader levels considered by Carnoy *et al.* (2011), the South African evidence on primary teachers' mathematical knowledge sits at a different base to the international literature on what might constitute strong mathematical knowledge for teaching. The situation indicated by these datasets is a long way from the 'mastery' of content being taught, and indeed 'mastery' at some grade levels beyond the level of teaching that is suggested as required for good mathematics teaching in many international policy reports - e.g. the Conference Board of Mathematics Sciences (2001, p. 7) – and in the mathematics education research literature.

These contrasts help us to understand, in pragmatic terms, the emphasis on common content knowledge within South African primary mathematics teacher knowledge research. In this paper, we retain this focus using a breakdown into mathematical topic strands (similar, but not identical to the topic strands used by Taylor & Taylor, 2013). Our interest in policy levers and teacher development activity, however, presents a further need. Much of the South African teacher knowledge research base presents evidence of what teachers cannot do, but this does not provide us with information on the level at which interventions to develop primary mathematics knowledge for teaching should begin. Thus our analysis attaches a topic strand *and* grade band allocation to each item. While mathematical topics have been considered in prior analyses, thinking about these in relation to curricular grades has not been a central concern. We deal in some depth with all 42 items in the SACMEQ 2007 teacher test, rather than just with the 15 items overlapping between the teacher and learner tests. From the perspective of policy making in relation to teacher development, both mathematical topic strands and grade-related curriculum specifications are particularly salient, and thus we use a content domain/curriculum grade-level based mapping to understand teacher performance in relation to this map.

5. Data: SACMEQ 2007

The Southern and Eastern African Consortium for Monitoring Educational Quality (SACMEQ) is a consortium of education ministries, policy-makers and researchers who, in conjunction with UNESCO's International Institute for Educational Planning (IIEP), aims to improve the research capacity and technical skills of educational planners (Moloi & Strauss, 2005, p. 12; Murimba, 2005) and to provide policy-relevant information on the quality of education in 14 participating countries. To date, it has conducted three nationally representative school surveys in participating countries, specifically SACMEQ I (1996), SACMEQ II (2000), and SACMEQ III (2007). These surveys collect extensive background information on the schooling and home environments of students, and in addition, test students and teachers in both numeracy and literacy (Ross *et al.,* 2005). Although there were teacher tests administered in the two most recent waves of SACMEQ (2000 and 2007), South

African teachers were only tested in SACMEQ 2007, not in SACMEQ 2000, due to teacher union objections to the tests in 2000.

The SACMEQ 2007 survey used complex two-stage cluster sampling including weighting adjustments to compensate for variations in the probability of selection (Hungi, et al., 2010). In calculating the standard errors we take this into account by using STATA's SVY command with stratification by province and clustering by school in accordance with the sampling structure used in the SACMEQ survey.

Table 1 below provides the overview background and performance for the SACMEQ 2007 dataset. It shows that there were 498 grade 6 mathematics teachers included in the SACMEQ 2007 South Africa survey and that of these, 401 (81%) wrote the mathematics teacher test. The table also shows the breakdown by three important subgroups: (1) province, (2) school location, and (3) quintile of school socioeconomic status, with additional information on average teacher age, the percentage of the sample that were female and the mean percentage correct on the 42 items included in the SACMEQ 2007 mathematics teacher test (corrected for guessing).

Table 1 shows large discrepancies between teacher response rates by province, with 93% of grade 6 mathematics teachers in the Free State and Limpopo provinces writing the test, but only 64% of teachers in the Western Cape. It is unclear why some teachers did not write the test. Given that the Western Cape and Gauteng provinces are frequently shown to be the best performing provinces in the country (Reddy *et al.,* 2012), it is possible that the lowest response rates in Gauteng (70%) and the Western Cape (64%) could relate to competent teachers refusing to write the test. Given that this is unverifiable, we do not stress the differences in province-level differences in teacher content knowledge in this paper.

Table 1: Background information on SACMEQ 2007 South Africa Mathematics teacher sample

		Number of Gr-6 Maths teachers	Proportion who wrote the test	Average age (years)	Percentage female	Percentage correct on 42 item test (corrected for guessing)	
						Mean	*Std. Err.*
Province	ECA	51	82%	41	78%	39%	*2.7%*
	FST	45	93%	41	47%	50%	*3.5%*
	GTN	57	70%	42	73%	52%	*4.4%*
	KZN	91	85%	39	64%	46%	*3.2%*
	LMP	40	93%	44	37%	44%	*3.2%*
	MPU	41	83%	41	40%	32%	*2.9%*
	NCA	51	84%	41	38%	53%	*3.7%*
	NWP	46	80%	42	51%	47%	*4.1%*
	WCA	76	64%	41	50%	63%	*3.4%*

School location	Rural	188	89%	41	53%	39%	*1.7%*
	Urban	310	75%	41	64%	54%	*2.0%*
Quintile of school socioeconomic status	Q1 (Poorest 20%)	83	90%	40	60%	38%	*2.3%*
	Q2	89	88%	41	58%	40%	*2.9%*
	Q3	101	80%	42	51%	40%	*2.4%*
	Q4	98	80%	43	51%	47%	*3.6%*
	Q5 (Richest 20%)	127	70%	40	71%	67%	*2.5%*
National		**498**	**81%**	**41**	**58%**	**46%**	***1%***

6. Analytical framework: SACMEQ III Teacher mathematics test

In order to provide an analytical framework to assess the levels and distribution of mathematics teacher content knowledge in South Africa, it was first necessary to classify items into broad content domains. Rather than using Rasch analysis to classify items we used the South African Curriculum Assessment Policy Statements (DBE, 2011a, 2011b, 2011c) and classified each item in the SACMEQ teacher test by grade level and broad content-related strands. Given that the SACMEQ items were not designed with the South African curriculum as guide, we based our content-related strands on the following four broad areas of mathematics: 1) Number and operations, 2) Fractions, decimals, and proportional reasoning, 3) Patterns, graphical reasoning and algebra, and 4) Shape and space. Our classification thus differs somewhat from Taylor & Taylor's (2013) classification, while taking cognizance of their comment about the overlaps between algebraic thinking about change and proportional reasoning in particular. Given the multiple choice format of the test with items specified in a four-choice format, we have also corrected for random guessing using Frary's (1988) formula[2]. All results reported in this paper have been corrected for guessing using this formula.

A consequence of the SACMEQ test not being written with the South African curriculum in mind was that some items integrated content from more than one content area or grade, or incorporated content 'explored' in an earlier grade, but explicitly specified in a later grade. Where this occurred, we allocated the item to the higher or highest explicit grade specification and content area. A point to note within a curriculum grade level categorization is that while we view mathematical learning as broadly cumulative, it is entirely possible to construct more straightforward procedural items related to higher grades' content, and more complex analytical items drawing from content in lower grades (e.g. see De Lange, 1999) – which makes anomalies in relation to grade specification relatively likely. A further consequence of working with a test that was not designed within the South African

[2] Frary's formula is FS = R - W/(C – 1) where "FS" is the corrected formula score, "R" is the number of items answered correctly, "W" is the number of items answered incorrectly, and "C" is the number of choices per item (Frary, 1988: 33).

curriculum frame is an imbalanced distribution of items across our content strands and grade levels. In particular, the ‘number and operations’ items are predominantly clustered at grade 4/5 level (6 items) with only one item at each of grade 6/7 and grade 8/9 levels. In contrast, the ‘proportional reasoning’, ‘pattern, graphical interpretation and algebra’ and ‘shape and space’ items were predominantly spread between grades 6/7 and 7/8. In Appendix A, we include a summary table of items matched to mathematical strands and curriculum grades.

Of the 42 questions in the teacher test, nine items could be matched to the grade 4 or 5 curriculum statements, 19 items could be matched to the grade 6 or 7 curriculum statements, and 14 items could be matched to the grade 8 or 9 curriculum statements[3]. The aim of grouping two grades together was to ensure that there were enough items per grouping and to identify the extent to which teacher performance indicated knowledge of grade-related mathematical content.

Preliminary analyses revealed generally strong performance on the grade 4/5 related items, with overall mean scores standing at 69% (SE 1.8%). We interpret teacher performance in cumulative terms (unless data indicated contradictory evidence – explained below). Our key grade level distinctions yielded the following mutually exclusive categories:

1. less than grade 4 and 5 content knowledge
2. grades 4 and 5 content knowledge,
3. grades 4, 5, 6 and 7 content knowledge,
4. grades 4, 5, 6, 7, 8 and 9 content knowledge

We classified teachers as attaining the content knowledge for a particular grade if they scored an average of 60% or higher on the items at that level after correcting for random guessing. As noted earlier, this cut off stands some way off the ‘mastery’ recommended in the international literature. For example, the American Conference Board of Mathematical Sciences (2001, p. 7) explains that mathematics teachers should have, “a thorough mastery of the mathematics in several grades beyond that which they expect to teach, as well as of the mathematics in earlier grades.” Rather, our classification is a pragmatic way of looking at a content strand in relation to grade levels prior to exploring teacher knowledge through the associated sample of items. A small proportion of teachers’ performance (39 out of the 401 teachers in the sample) confounded the cumulative view taken in our categorization, with attainment at above 60% in grade 8/9 level content domains, while below this in the related content domain at grades 6/7. Across all these cases, these teachers had attained well over 60% on grades 4/5 related items. We chose to add these instances into the grade 4/5 categorization to fit with our literature-based view that mathematical knowledge for teaching rests on deep and connected knowledge of mathematics. As Silverman & Thompson (2008, p. 501) have noted:

[3] The exact classification is as follows grade 4 or 5 items (Q1, Q2, Q3, Q4, Q5, Q7, Q11, Q12, Q14), grade 6 or 7 items (Q6, Q8, Q9, Q10, Q13, Q15, Q17, Q20, Q21, Q22, Q24, Q26,Q27, Q28, Q29, Q32, Q33, Q34, Q35), grade 8 or 9 items (Q16, Q18, Q19, Q23, Q25, Q30, Q31, Q36, Q37, Q38, Q39, Q40, Q41, Q42)

> ‘the work of teaching for understanding is predicated on coherent and generative understandings of the big mathematical ideas that make up the curriculum’

A further level of analysis that we incorporate within our findings is attention to disaggregating findings on grade band levels of teacher knowledge by school socioeconomic quintile. We do this given the evidence, emphasized in a range of South African studies (Fleisch, 2008; Taylor, 2011; Spaull, 2013a), of the highly misleading nature of national averages in a context that is riven with high levels of inequality.

7. Findings

In this section, we use the analytical categories of grade-band level, mathematical topic and socioeconomic quintile outlined above to present our findings. These findings are discussed in the commentary section that follows.

We classified teachers into one of the four mutually exclusive categories. Figure 1 below reports the proportions of South African grade 6 mathematics teachers in each category. Results show that the vast majority (79%) of South African grade 6 mathematics teachers were classified as having content knowledge levels below grade 6. That is to say that they could not achieve 60% correct or higher on the grade six/seven items in the test. Given that the SACMEQ survey was sampled so as to be nationally representative, this suggests that one can interpret the proportions in Figure 1 as follows:

- 17% of grade 6 students in South Africa were taught by maths teachers who had content knowledge below a grade 4 or 5 level,
- 62% of grade 6 students were taught by maths teachers who had a grade 4 or 5 level of content knowledge,
- 5% of grade 6 students were taught by maths teachers who had a grade 6 or 7 level of content knowledge, and
- 16% of grade 6 students were taught by maths teachers who had at least a grade 8 or 9 level of content knowledge.

Figure 1: Proportion of South African grade 6 mathematics teachers by content knowledge (CK) group - SACMEQ 2007 (with 95% confidence interval)

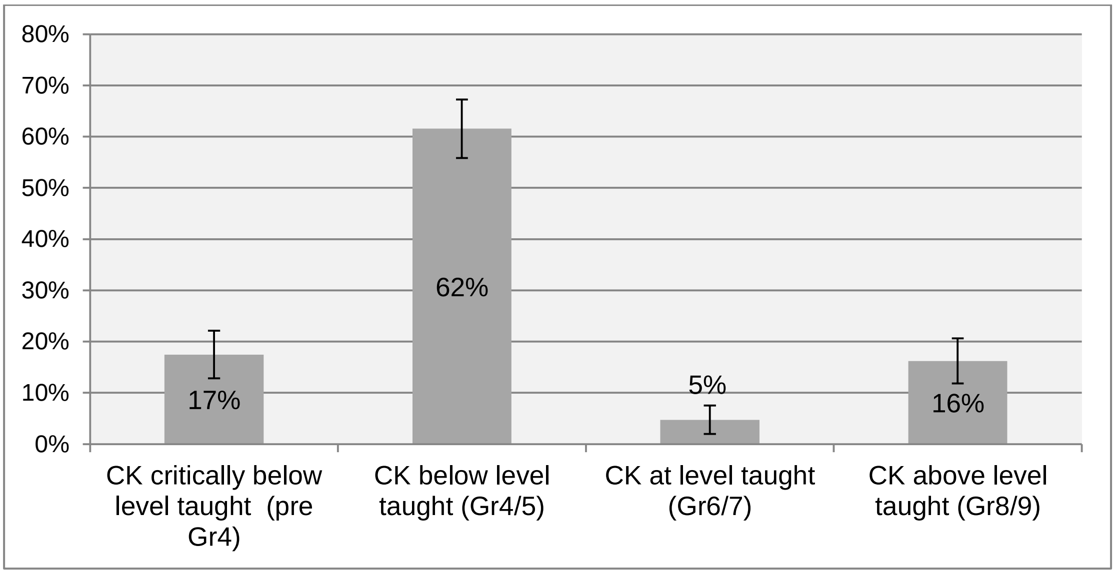


Table 2 below reports the average score for each group on each of the three sets of items. The average scores found in the table show that teachers with the highest levels of content knowledge (grade 8 or 9 and above) scored highest on all three groups of items averaging 95% on grade 4/5 items, 78% on grade 6/7 level items and 77% on grade 8/9 items. Equally predictably, teachers with the lowest content knowledge (pre grade 4/5) had the lowest average score on each of the three groups averaging 45% on grade 4/5 items, 16% on grade 6/7 level items and 19% on grade 8/9 level items.

Table 2: Average score on grade-level item groupings by teacher category (corrected for guessing)

	Grade 4/5 level questions (9 items)		**Grade 6/7 level questions (19 items)**		**Grade 8/9 level questions (14 items)**	
	Mean	*Std. Err.*	Mean	*Std. Err.*	Mean	*Std. Err.*
Teachers with pre-Gr4/5 content knowledge	45%	*2.0%*	16%	*2.4%*	19%	*2.7%*
Teachers with Gr4/5 content knowledge	86%	*0.9%*	30%	*1.2%*	34%	*1.6%*
Teachers with Gr6/7 content knowledge	88%	*3.9%*	70%	*1.7%*	43%	*2.8%*
Teachers with Gr 8/9+ content knowledge	95%	*1.1%*	78%	*1.3%*	77%	*1.8%*

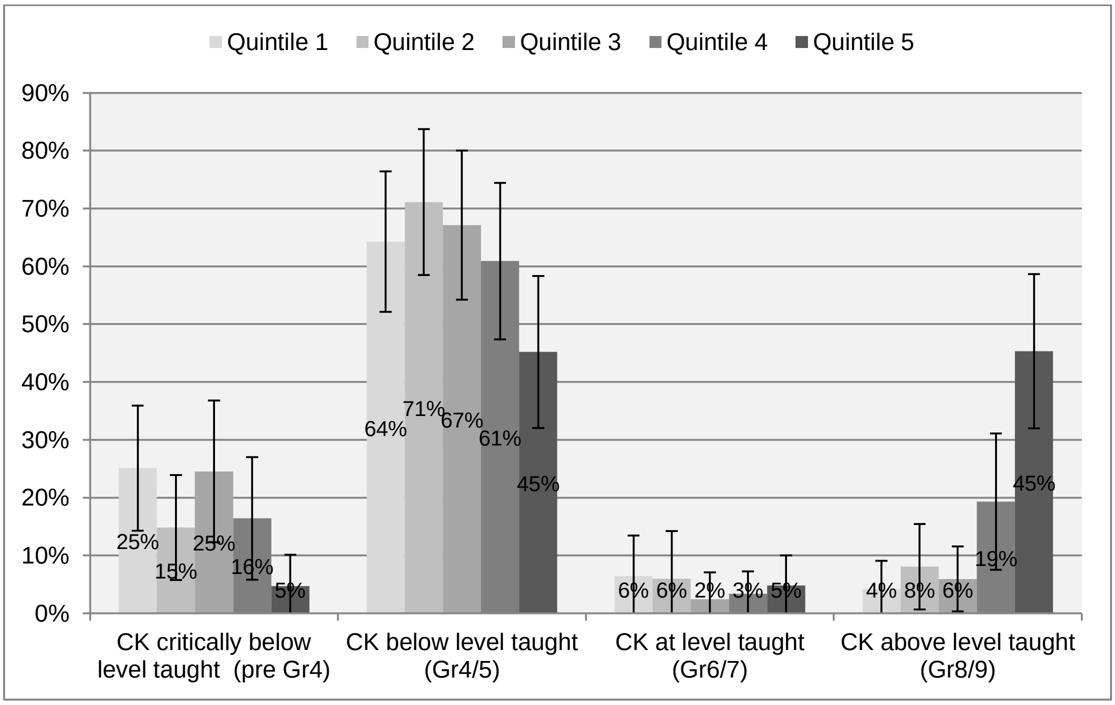
In order to ensure robustness we report on categories only where there were at least three items in a particular curriculum grade-level band (See Appendix A). Table 3 shows the average facility across each of the content-strand and grade-level bands. The results, while reflecting the broad decline in average scores across grade level bands seen in Table 2, point to some anomalies as well. For example, the average scores on the proportional reasoning items at the grade 6/7 level are substantially lower than those at the grade 8/9 level. We discuss this anomaly further in the commentary section that follows.

Table 3: Average score correct on selected item groups by grade-level and content-strand (SACMEQ 2007) (corrected for guessing)

Grade and content strand (number of items)	Mean	*Std. Err.*
Gr 4/5 Number & operations (6 items)	71%	*1.9%*
Gr 6/7 Proportional reasoning (8 items)	28%	*1.5%*
Gr 6/7 Pattern, graphical interpretation & algebra (5 items)	31%	*1.8%*
Gr 6/7 Shape and space (5 items)	44%	*1.9%*
Gr 8/9 Proportional reasoning (3 items)	47%	*2.1%*
Gr 8/9 Pattern, graphical interpretation & algebra (7 items)	25%	*1.5%*
Gr 8/9 Shape and space (3 items)	46%	*2.2%*

In disaggregating the grade band level analysis by school socioeconomic quintile, it is clearly apparent that teachers with relatively high levels of mathematical content knowledge are highly inequitably distributed and highly concentrated in the wealthiest 20% of schools (i.e. Quintile 5) – see Figure 2.

Figure 2: Proportion of Grade 6 mathematics teachers by CK grouping and quintile of school socioeconomic status (SACMEQ 2007) - with 95% confidence intervals



By further disaggregating the overall mathematics teacher score and classifying teachers based on the four-category curriculum mastery scale outlined above, it is possible to show the content-knowledge grouping of teachers by school socioeconomic quintile. After calculating a measure of wealth based on the average household asset wealth of students in a school, we classified schools into one of five quintiles. Quintile 1 represents the poorest 20% of schools in our sample, quintile 2 the second-poorest 20% of schools etc., all the way to quintile 5 which represents the wealthiest 20% of schools in our sample (Figure 2). When calculating the proportion of teachers in each grade-level of content knowledge by school socioeconomic quintile, there is a large increase in the standard errors, as one might expect with shrinking sample size. Notably the teachers' scores for quintiles 1, 2, 3 and 4 are not statistically significantly different from each other. The most striking feature of the socioeconomic distribution of knowledgeable teachers is the large spike in the proportion of quintile 5 teachers with content knowledge above the level taught. Once again, while the broad patterns of content knowledge broken down by grade band level seen in Figure 1 are reflected here, the quintile categorization highlights that almost half (45%) of the quintile 5 teachers were (non-anomalously) able to handle grade 8/9 items compared to less than 10% of teachers in Quintile 1, 2 and 3 schools.

8. Commentary on policy suggestions

Several features of interest arise in relation to this re-analysis of the SACMEQ 2007 dataset. As outlined in our opening sections, our analysis is driven by interests in developing specific policy and primary mathematics teacher development mechanisms for addressing concerns.

Firstly, there is confirmation of the earlier finding of significant gaps for middle-years teachers of content related to topics they are teaching. While the NSES study sample suggested gaps in teacher content knowledge at the Grade 6 level, i.e. 'close' to the level of teaching for grade 4 and 5 teachers, our analysis points to some gaps **below** the grade 6/7 level for significant proportions of grade 6 teachers. In policy terms, this suggests that development activity would need to begin with a focus on consolidating and extending towards 'profound understandings' of content in the early Intermediate years (grades 4 and 5 in South Africa) as a way of building a solid foundation for extensive attention to grades 6/7 level content. This implication follows from the fact that even in the wealthiest quintile of schools (Quintile 5), 50% of the teachers' performance indicated content knowledge below, or critically below, the grade 6/7 level. In the poorer parts of the schooling system, this proportion extends to the majority of teachers. The percentages of teachers showing content knowledge below Grade 6 level are: quintile 1 (89%), quintile 2 (86%), quintile 3 (92%) and quintile 4 (77%). The low average scores at grade 6/7 levels across content strands also suggest the need for consolidating and extending the base of grade 4/5 content and representations in order to provide the breadth needed to function effectively with grade 6/7 level content.

The broader South African classroom observation evidence, backed by the international literature, would suggest that this content needs to be presented in ways that are well attuned to the nature of mathematical working. This entails attention within teacher development activity to a problem-solving orientation in which given information is used to find unknown information, and with a focus on developing teachers' capacity for mathematical explanations. In this way, common content knowledge can be built through an orientation that is focused on specialized content knowledge – i.e. knowing and doing mathematics in ways that are helpful for teaching mathematics.

While the first point above deals with the level at which interventions should be pitched, our content strand analysis also indicates priority areas. Here, our analysis concurs with the findings of Taylor & Taylor (2013) who argue for an emphasis on proportional reasoning. An extensive body of literature in mathematics education suggests that moving from an 'additive' relational sense of numbers to a multiplicative relational sense is difficult for learners, and thus requires careful selections and sequencing within curriculum and pedagogy (Lamon, 2005). Lamon describes proportional reasoning as central to multiple mathematical topic areas: fractions, percentages, ratio, and many covariation situations. Thus, problems with proportional reasoning in the middle years frequently lead to problems with later, more complex percentages and ratios, as well as creating difficulties for later algebraic and function ideas of covariation – a pattern seen across our analysis. While the grade 8/9 proportional reasoning items showed higher performance, these three grade 8/9 items could all be characterized in relatively procedural terms, in contrast to some of the grade 6/7 items in this strand which were non-routine and less amenable to solution by commonly taught procedures.

Our final point relates to the finding, seen in earlier research (Fleisch, 2008; Spaull, 2013a), that teacher performance, across mathematical strands and grade band levels, is hard to distinguish between Quintiles 1-4, i.e. across the poorest 80% of schools. A consequence of this broadly poor performance, particularly when allied to our earlier noting of gaps among the Quintile 5 teacher population, is that interventions at the level of mathematical content knowledge are likely to have to be broad based, rather than targeted at narrower socioeconomic fractions of the primary teacher population.

The general motif that runs through much of the South African mathematics literature is that a lack of teacher content knowledge is a major impediment to learning. Despite this consistent finding, there is currently a paucity of rigorously evaluated in-service teacher training programs in South Africa showing growth in mathematics teacher content knowledge at any significant scale. While there are localized initiatives that have been successful at improving teachers' content knowledge (e.g. see Mogamberry, 2011) it is unclear whether these programs are scalable from capacity, cost and/or program-design perspectives, and unclear too whether they make a difference to classroom teaching. In the United States context, Borko (2004) has argued for a three phase process in which successful professional development projects are researched at scaled-up levels. In scaling up studies a central goal is to 'determine whether a professional development program can be enacted with integrity in different settings and by different professional development providers' (Borko 2004, p.9). In phase 3, multiple scaled up interventions are compared on impact and cost variables, leading to robust professional development policy.

The double predicament that faces South African policy-makers is the presence of a well-established problem (the overwhelming majority of South African primary school mathematics teachers lack the content knowledge needed to provide their students with the knowledge and skills necessary to succeed in mathematics), coupled with limited current evidence even at Borko's phase 1 level where 'well specified professional development programs' are required that have been shown to be effective at raising teachers' mathematical content knowledge.

9. Conclusion

Given the findings presented in this paper, it is not an overstatement to say that there is a crisis relating to primary school mathematics teacher content knowledge in South Africa. The local evidence base supporting this conclusion is large, consistent and unambiguous. Our paper has contributed to this literature by re-analyzing the SACMEQ 2007 teacher test data and classifying items (and then teachers) into broad grade-level categories with some additional content-strand analysis. The results showed that the vast majority (79%) of South African grade 6 mathematics teachers could not score an average mark of 60% correct on a range of grade 6/7 level items. Furthermore, given the distribution of performance across the various content-strands, we argue that a focus on ideas related to proportional reasoning is critical within middle years' in-service mathematics teacher education.

We concur with the international conclusion that improving teachers' (common) content knowledge is unlikely, on its own, to raise the quality of mathematics instruction. Our analysis, linked with the broader literature on mathematics teacher knowledge, suggests the need for urgent focus on 'mathematical knowledge for teaching' in the context of primary mathematics. The current absence of any national plan that brings the key stakeholders (provinces, university teacher educators, teacher unions and professional organizations) together to establish a linked 'Phase 1' (Borko, 2004) research and development agenda is particularly problematic in this regard. We know that a lot of teacher development work is happening in South Africa – through provincial interventions, the university and the NGO sectors, but there appears to be no rigorous guiding agenda for this work that would allow for reporting on their impact at content knowledge, pedagogic content knowledge, or classroom practice levels. There is very little evidence in the South African terrain of impact at these levels on the basis of 'well specified professional development programs'. Without this evidence base, we would argue that raising student outcomes in mathematics remains a distant pipe dream in South Africa.

10. References

Adler, J., & Venkat, H. (2014). Teachers' mathematical discourse in instruction: Focus on examples and explanations. In H. Venkat, M. Rollnick, J. Loughran & M. Askew (Eds.), Exploring mathematics and science teachers' knowledge: Windows into teacher thinking (pp. 132-146). Abingdon, Oxon: Routledge.

Ally, N. & Christiansen, I. (2013). Opportunities to develop mathematical proficiency in Grade 6 mathematics classrooms in KwaZulu-Natal. Perspectives in Education, 31(3), 106-121.

Ball, D., Thames, M.H., & Phelpps, G. (2008). Content knowledge for teaching: What makes it special? Journal for Teacher Education, 59(5), 389-407.

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. Educational Researcher, 33(8), 3-15.

Carnoy, M., & Chisholm, L. (2008). *Towards Understanding Student Academic Performance in South Africa: A Pilot Study of Grade 6 Mathematics Lessons in South Africa.* Pretoria: HSRC.

Carnoy, M., Chisholm, L., & Chilisa, B. (2012). *The Low Achievement Trap: Comparing Schooling in Botswana and South Africa.* Cape Town: HSRC Press.

Carnoy, M., Chisholm, L., Addy, N., Arends, F., Baloyi, H., Irving, M., et al. (2011). *The Process of Learning in South Africa: The Quality of Mathematics Teaching in North West Province.* HSRC Press.

Conference Board of the Mathematical Sciences. (2001). The Mathematical Education of Teachers. Washington D.C.: American Mathematics Society and Mathematical Association of America.

Council on Higher Education. (2010). Report on the National Review of Academic and Professional Programmes in Education Higher Education Monitor (Vol. 11). Pretoria: CHE.

DBE. (2011a). *Curriculum and Assessment Policy Statement (CAPS): Foundation Phase Mathematics, Grades R-3.* Pretoria: Department for Basic Education.

DBE. (2011b). *Curriculum and Assessment Policy Statement (CAPS): Intermediate Phase Mathematics, Grades 4-6.* Pretoria: Department for Basic Education.

DBE. (2011c). *Curriculum and Assessment Policy Statement (CAPS): Senior Phase Mathematics, Grades 7-9.* Pretoria: Department for Basic Education.

Ensor, P., Hoadley, U., Jacklin, H., Kuhne, C., Schmitt, E., Lombard, A., & Van den Heuvel-Panhuizen, M. (2009). Specialising pedagogic text and time in Foundation Phase numeracy classrooms. Journal of Education, 47, 5-30.

De Lange, J. (1999). Framework for classroom assessment in mathematics. Utrecht: Freudenthal Institute & the National Center for Improving Student Learning and Achievement in Mathematics and Science.

Fleisch, B. (2008). *Primary Education in Crisis: Why South African schoolchildren underachieve in reading and mathematics.* Cape Town. : Juta & Co.

Frary, R. (1988). *Formula Scoring of Multiple Choice tests (Correction for Guessing).* Instructional Topics in Educational Measurement (ITEMS).

Hoadley, U. K. (2006) Analysing pedagogy: the problem of framing. Journal of Education, 40, 15–34.

Hungi, N., Makuwa, D., Ross, K., Saito, M., Dolata, S., van Capelle, F., et al. (2011). *SACMEQ III Project Results: Levels and Trends in School Resources among SACMEQ School Systems.* Paris: Southern and Eastern Africa Consortium for Monitoring Educational Quality.

Lamon, S. (2005). Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers (2nd ed.). Mahwah, NJ: Erlbaum.

Ma, L. (1999). Knowing and teaching elementary mathematics: Teachers' understandings of fundamental mathematics in China and the United States. Mahwah, NJ: Lawrence Erlbaum Associates.

Mogamberry, M. (2011) AMESA's role in relation to research and development in mathematics education – conversations with past presidents. Proceedings of the 17th National Congress of the Association for Mathematics Education of South Africa. University of the Witwatersrand, 11-15 July, 62-67.

Moloi, M., & Chetty, M. (2011). *The SACMEQ III Project in South Africa: A Study of the Conditions of Schooling and the Quality of Education.* Pretoria: Department of Basic Education.

Moloi, M., & Strauss, J. (2005). *The SACMEQ II Project in South Africa: A study into the conditions of schooling and the quality of education.* Harare: SACMEQ.

Murimba. (2005). The Southern and Eastern African Consortium for Monitoring Educational Quality (SACMEQ): Mission, approach and projects. *Prospects, XXXV*(1).

Prestage, S., & Perks, P. (2001). Models and Super-Models: ways of thinking about professional knowledge in mathematics teaching. *Research in Mathematics Education*, 3, 101-114.

OECD. (2005). *Teachers Matter: Attracting, Developing and Retaining Effective teachers.* Paris: OECD Publications.

Reddy, V., Prinsloo, C., Visser, M., Arends, F., Winnaar, L., Rogers, S., et al. (2012). *Highlights from TIMSS 2011: The South African perspective.* Pretoria: HSRC.

Reeves, C., & Muller, J. (2005). Picking up the pace: variation in the structure and organization of learning school mathematics. Journal of Education, 37, 103-130.

Ross, K., Saito, M., Dolata, S., Ikeda, M., Zuze, L., Murimba, S., et al. (2005). The Conduct of the SACMEQ III Project. In E. Onsomu, J. Nzomo, & C. Obiero, *The SACMEQ II Project in Kenya: A Study of the Conditions of Schooling and the Quality of Education.* Harare: SACMEQ.

Sayed, Y. (2004). The case of teacher education in post-apartheid South Africa: politics and priorities . In L. Chisholm, *Changing Class: education and social change in post-apartheid South Africa.* Pretoria: HSRC Press.

Schollar, E. (2008). Final Report (Short Version) The Primary Mathematics Research Project 2004-2007: Towards evidence-based educational development in South Africa. Johannesburg: Eric Schollar and Associates.

Shepherd, D. (2013). The impact of teacher subject knowledge on learner performance in South Africa: A within-pupil across-subject approach. *International Workshop on Applied Economics of Education*, (pp. 1-32). Cantanzaro.

Silverman, J., & Thompson, P.W. (2008). Toward a framework for the development of mathematical knowledge for teaching. Journal of Mathematics Teacher Education, 11, 499-511.

Sorto, A. & Sapire, I. (2011). The teaching quality of mathematics lessons in South African schools. *Journal of Education*, 51, 1-22.

Spaull, N. (2013a). Poverty & priviledge: Primary school inequality in South Africa. *International Journal of Educational Development, 33*, 436-447.

Spaull, N. (2013b). South Africa's Education Crisis. Centre for Development and Enterprise. Johannesburg.

Taylor, N. (2011). The National School Effectiveness Study (NSES): Summary for the synthesis report. Johannesburg: JET Education Services.

Taylor, N., & Taylor, S. (2013). Teacher knowledge and professional habitus. In N. Taylor, S. Van der Berg, & T. Mabogoane, *What makes schools effective? Report of the National Schools Effectiveness Study* (pp. 202-232). Cape Town: Pearson Education South Africa.

Taylor, N., & Vinjevold, P. (1999). Getting Learning Right: Report of the President's Education Initiative Research Project. Joint Education Trust.

Taylor, N., & Moyana, J. (2005). *Khanyisa Education Support Programme: Baseline study part 1: communities, schools and classrooms.* Johannesburg: JET Education Services.

Taylor, N., Van der Berg, S., & Mabogoane, T. (2013). *What makes schools effective? Report of the National Schools Effectiveness Study.* Cape Town: Pearson.

Taylor, S. (2011). Uncovering indicators of effective school management in South Africa using the National School Effectiveness Study. Stellenbosch Economic Working Papers 10/11, 1-51.

Van der Berg, S. et al (2011). Low quality education as a poverty trap. Stellenbosch: Stellenbosch University.

Van der Sandt, S., & Nieuwoudt, H. (2003). Grade 7 Teachers' and Prospective Teachers' Content Knowledge of Geometry. *South African Journal of Education, 23(3)*, 199-205.

Venkat, H. (2013). Curriculum development minus teacher development ≠ mathematics education. Paper presented at the Proceedings of the 19th Annual National Congress of the Association for Mathematics Education of South Africa, 24-28th June, University of the Western Cape.

Venkat, H., & Adler, J. (2012). Coherence and connections in teachers' mathematical discourses in instruction. Pythagoras, 33(3), 25-32.

Appendix A

Questions mapped by grade-level and content-strand

	Number & operations	**Proportional reasoning**	**Pattern, graphical interpretation & algebra**	**Shape & space**
Grade 4/5 items	Q1 - Gr4 Q2 - Gr4 Q4 - Gr4 Q7 - Gr4 Q14 - Gr4 Q11 - Gr5	Q3 - Gr5	Q12 - Gr4 Q5 - Gr5	
Grade 6/7 items	Q6 - Gr6	Q29 - Gr6* Q9 - Gr7 Q15 - Gr7 Q22 - Gr7 Q26 - Gr7 Q27 - Gr7 Q28 - Gr7 Q35 - Gr7	Q17 - Gr7 Q20 - Gr7 Q21 - Gr7 Q24 - Gr7 Q32 - Gr7	Q8 - Gr6 Q10 - Gr7 Q13 - Gr7 Q33 - Gr7 Q34 - Gr7
Grade 8/9 items	Q31 - Gr8	Q23 - Gr8 Q19 - Gr8 Q36 - Gr9	Q30 - Gr8 Q37 - Gr8 Q38 - Gr8 Q39 - Gr8 Q18 - Gr9 Q41 - Gr10 Q42 - Gr8	Q40 - Gr8 Q16 - Gr9 Q25 - Gr9

*For example "Q29 – Gr6" shows that Question 29 was allocated a grade 6 level and falls under the "Proportional reasoning" content strand.