

---

# The Welfare Cost of Sovereign Default and Liquidity Injections

GUANGLING LIU

---

Stellenbosch Economic Working Papers: 12/14

KEYWORDS: SOVEREIGN DEFAULT, WELFARE COST, DEBT CRISIS, ROLLOVER RISK, LIQUIDITY  
JEL: E50, E58, E63, G18

GUANGLING LIU  
DEPARTMENT OF ECONOMICS  
UNIVERSITY OF STELLENBOSCH  
PRIVATE BAG X1, 7602  
MATIELAND, SOUTH AFRICA  
E-MAIL: GLIU@SUN.AC.ZA



UNIVERSITEIT  
STELLENBOSCH  
UNIVERSITY



A WORKING PAPER OF THE DEPARTMENT OF ECONOMICS AND THE  
BUREAU FOR ECONOMIC RESEARCH AT THE UNIVERSITY OF STELLENBOSCH

---

# The Welfare Cost of Sovereign Default and Liquidity Injections

Guangling Liu <sup>\*†</sup>

June 24, 2014

## Abstract

This paper develops a dynamic general equilibrium model with endogenous default on entrepreneur loans and funds borrowed from the central bank (liquidity injections) and investigates the welfare cost of sovereign default. The results show that sovereign default affects production through households' investment decisions and the bank's asset portfolio adjustment. The effect of sovereign default on entrepreneurs tends to be in favor of production. Sovereign default reduces the variability of the output gap and hence the welfare loss. Liquidity injections reduce the variability of the output gap and improve price stability during the period of sovereign debt crisis, resulting in an increase in households' welfare.

*JEL* codes: E50, E58, E63, G18

Keywords: sovereign default, welfare cost, debt crisis, rollover risk, liquidity

---

\*Department of Economics, University of Stellenbosch, Stellenbosch, 7602, South Africa. Tel: +27 21 808 2238 fax: +27 21 808 4637. E-mail address: davegliu@gmail.com

†This paper is based on research supported in part by the National Research Foundation (NRF) of South Africa (Grant number: 90577). The financial assistance of the NRF towards this research is hereby acknowledged. Financial support from Economic Research Southern Africa (ERSA) is also gratefully acknowledged. Opinions expressed and conclusions arrived at are those of the author and are not necessarily to be attributed to the NRF or ERSA.

# 1 INTRODUCTION

The European debt crisis, which arose as a consequence of the recent financial crisis, and the deterioration of fiscal imbalances in the U.S. have confronted policy makers with a major concern about government debt default. The general government debt to GDP ratio in the Eurozone has reached over 80% since 2010. Some European countries, such as Greece, Ireland, Portugal, Italy, and Spain, have already experienced sovereign debt crises. The U.S. fiscal fundamental has been even worse than that of some of the above mentioned European countries since the 2007/8 financial crisis. The U.S. government debt to GDP ratio reached 100% at the end of 2012.

In this paper I investigate the welfare cost of sovereign default using a dynamic general equilibrium model augmented with a banking sector, in which banks finance assets (entrepreneur loans and government debt) with bank capital and funds borrowed from the central bank (liquidity injections). The term 'sovereign default' is used to refer to default on domestic sovereign debt. I focus on domestic sovereign debt because such debt accounts for a large fraction of total public debt in most economies (Reinhart and Rogoff, 2008; D'Erasmus and Mendoza, 2012). Endogenous default on entrepreneur loans and funds borrowed from the central bank is also embedded in the model. To maximize their profits, in each period both entrepreneurs and banks choose their optimal default rates on their financial obligations. However, they have to pay a quadratic cost in terms of the amounts of the defaulted financial obligations.

There are at least two reasons for including liquidity injections in a commercial bank's balance sheet. Firstly, empirical evidence suggests that sovereign defaults and financial crises tend to occur simultaneously. This is because commercial banks are very often exposed not only to sovereign defaults but also to defaults on funds borrowed from the central bank. Sovereign default may therefore trigger banking crises. Secondly, the Federal Reserve Bank ('Fed' hereafter) has dramatically increased its holdings of mortgage-backed securities since 2009, as indicated by its balance sheet, and as Curdia and Woodford (2011) have pointed out. Moreover, since the failure of Lehman Brothers in 2008, the U.S. government has implemented various policies to prevent a recurrence of the financial crisis, including bailing out financial institutions and recapitalizing large commercial banks through the Trouble Asset Relief Program. In order to account for the vulnerability of the banking sector due to exposure to liquidity injections, I include liquidity injections in the bank's balance sheet in the model, which is in contrast to the Curdia and Woodford (2011) model, where the Fed became a direct lender to certain real sectors in the economy after the 2007/8 financial crisis.

In the literature on the dynamics of a sovereign debt crisis, Cole and Kehoe (2000) argue that sovereign debt crises may be self-fulfilling, and Arellano (2008) concludes that low income countries are more likely to default. In a study of sovereign default and business cycle dynamics, Mondeza and Yue (2012) develop a general equilibrium model with endogenous costs of default. They conclude that defaults on sovereign debt and entrepreneur loans tend to occur simultaneously, and they show that the model explains several features of cyclical dynamics involved in defaults, countercyclical spreads, high debt ratios, and key business cycle moments.

I develop a time consistent dynamic general equilibrium model based on Cole and Kehoe (2000), in which sovereign default may occur in each period. Five important modifications of the Cole and Kehoe

(2000) model are worth noting here. Firstly, I extend their real business cycle (RBC) model to a New Keynesian model. Secondly, I assume that households are not risk neutral but participate in the government debt market; that is, they accumulate assets in the form of physical capital and government debt in each period. Thirdly, I take into account the fact that banks finance assets with bank capital and funds borrowed from the central bank, and may default on the funds borrowed from the central bank. Fourthly, I recognize that entrepreneurs need to borrow bank loans to accumulate physical capital and finance their production costs, and may default on their bank loans. Finally, I do not assume that productivity drops permanently if the government defaults.

The study adopts the timing framework for sovereign default of Cole and Kehoe (2000), for the following reasons. I investigate the welfare cost of sovereign default using a utility-based welfare loss criterion based on a New Keynesian framework. Without exogenous costs of default, an outright default on the existing debt does not have any consequences for agents' decision making in the economy, nor for the welfare loss. In addition, this framework makes it possible to study the effect of altering the maturity structure (partial debt rollover).

Endogenous default on entrepreneurs' and banks' financial obligations in this study is related to De Walque et al. (2010), in which the authors develop an RBC model with endogenous default of Goodhart et al. (2006) and a heterogeneous banking sector to study financial (in)stability, bank supervision, liquidity injections and business cycles. In De Walque et al. (2010), entrepreneurs and banks may default on their financial obligations (entrepreneur loans and funds borrowed from the interbank market, respectively), subject to default costs. In the present study, I extend their model in three dimensions. Firstly, I extend their RBC model to a New Keynesian model. This is necessary for two reasons: an RBC model fails to account for price stability when evaluating the welfare cost of sovereign default, and, as De Walque et al. (2010) point out, a New Keynesian model is the right choice for investigating the implications of introducing liquidity injections in the bank sector. Secondly, since the focus of the present study is on the welfare cost of sovereign default, for simplicity I adopt a stylized banking sector without the interbank market. Thirdly, I use the term 'liquidity injections' to refer to funds borrowed from the central bank, and I assume that in equilibrium liquidity is determined by the bank's balance sheet, where De Walque et al. (2010) assume that liquidity is the difference between interbank borrowing and lending, and that in steady state liquidity is zero.

The contribution of this paper is twofold. Firstly, it develops a dynamic general equilibrium model and investigates the welfare cost of sovereign default using a utility-based welfare loss criterion based on a New Keynesian framework. The existing literature on sovereign default has focused mainly on the dynamics of sovereign default, very often using an ad hoc default cost in the form of a fractional loss of output (e.g. Cole and Kehoe, 2000; Arellano, 2008; Yue, 2010).<sup>1</sup> Moreover, most of the existing studies of sovereign default use either real business cycle models or overlapping generations models. To my knowledge, this paper is the first to study the welfare cost of sovereign default using a utility based welfare loss criterion based on a New Keynesian framework. Secondly, it investigates whether and to what extent liquidity injections may improve households' welfare, in particular during the period of sovereign debt crisis. This leads to a possible answer

---

<sup>1</sup>Mondeza and Yue (2012) are the first to introduce endogenous costs of sovereign default.

to an empirical question: why did governments continue to provide liquidity to financial sectors before and after the current financial crisis, knowing that governments themselves might default on their debts? To attempt to answer this question, I introduce liquidity injections in the model, and compare the welfare losses with and without these injections.

The main findings of the paper are as follows. The analysis for the model with liquidity injections suggests that sovereign default causes a significant decline in the variability of the output gap and hence the deadweight welfare loss. The sovereign default affects production through the household's investment decision and the bank's asset portfolio adjustment. The impact of sovereign default on entrepreneurs tends to be in favor of production, which results in a decline in the welfare loss. A comparison of the welfare cost of sovereign default with and without liquidity injections shows that these injections can reduce the variability of the output gap and improve price stability during the period of sovereign debt crisis and therefore improve households' welfare. The findings of the study are based solely on a utility based welfare loss criterion without an ad hoc exogenous default cost.

## 2 THE MODEL

The basic framework of the model is the standard medium-scale New Keynesian model (e.g. Smets and Wouters, 2007), in which I augment a stylized banking sector with endogenous default on entrepreneur loans and liquidity injections, and a consolidated government sector with the possibility of default. The model is a closed economy model, and there are no cross-border capital flows. It adopts Calvo-type sticky prices (Calvo, 1983). Wages are flexible. This section only presents new features and modifications of the model developed in the study - the rest is standard in the literature and not laid out here.

The timing of events within each period is as follows. Following Cole and Kehoe (2000), the model assumes that the government is able to issue new debt before retiring the old debt. That is, the recursive equilibrium is defined in an environment where there is no guarantee that the government will honor its debt and agents choose their actions sequentially in each period. If the government does not default, the government debt from the previous period is repaid, and new debt is issued. Households and commercial banks decide how much to invest in the government debt in the current period. Taxes, government expenditures, and liquidity injections are determined so that the government's budget constraint is maintained. If the government defaults, commercial banks are able to recover a certain fraction of the defaulted government debt from the insurance fund, upon paying an insurance premium, whereas households are able to recover the same without paying an insurance premium. Taxes, government expenditures, and liquidity injections are determined.

Cole and Kehoe (2000) assume that in each period an exogenous sunspot variable governs the default probability. The sunspot variable is independently and uniformly distributed on the interval  $[0, 1]$ . The probability of default depends on certain values of the government debt and the physical capital, and is arbitrary. In the present study the government's decision to default is exogenous too. I view government debt as a risk-free and liquid asset to both households and banks when times are normal. Sovereign default can never be optimal for the government, regardless of the level of government debt and the physical capital.

When sovereign default does occur, both households and banks suffer financial losses.

Sovereign default affects production only indirectly, and through the household's investment decision and the bank's balance sheet. The model assumes that entrepreneurs need loans from the bank to accumulate physical capital and finance their production costs. For simplicity, and explicitly motivating borrowing and lending, I assume that only entrepreneurs borrow from banks. If the government defaults, on the one hand this reduces the household's financial wealth and therefore the investment in the entrepreneur's physical capital; on the other, it shifts the household's investment from government debt towards the investment in physical capital. Sovereign default also affects production through the bank's balance sheet. Bank assets consist of entrepreneur loans and the government debt. If the government defaults, the bank will adjust its asset portfolio by increasing entrepreneur loans and decreasing government debt holdings. However, *ceteris paribus*, the bank will supply fewer entrepreneur loans because of its losses from investing in the government debt market. The supply of entrepreneur loans also depends on the repayment by the entrepreneur and the amount of funding that the bank can borrow from the central bank. It is therefore possible for production either to decrease or increase in response to sovereign default. This is, indeed, why I do not assume that productivity drops permanently if the government defaults, as Cole and Kehoe (2000) assume.

The borrowing and lending contract between entrepreneurs and banks plays an important role in bridging the two sectors. Without it, sovereign default would affect entrepreneurs and banks separately. Given that the government's objective is to maximize the household's utility - in other words, the welfare of the bank does not enter the welfare loss criterion - the banking sector would play no role in the welfare cost of sovereign default if this borrowing and lending contract did not exist.

## 2.1 Households

The economy consists of a continuum of households which have an infinite life span. The households consume consumption goods ( $C_t$ ), supply labor ( $N_t$ ) to entrepreneurs, and invest in the government debt ( $B_t$ ) market and physical capital. The representative household maximizes its expected discounted utility function (1), subject to the budget constraint (2) and capital accumulation (3),

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{\xi_{c,t} C_t^{1-\eta_c}}{1-\eta_c} - \frac{\xi_{n,t} N_t^{1+\eta_n}}{1+\eta_n} \right], \quad (1)$$

where  $\beta$  is the subjective discount factor, and  $\eta_c$  is the coefficient of the relative risk aversion for households.  $\eta_n$  is the inverse of the elasticity of work effort with respect to the real wage.  $\xi_{c,t} = \xi_{c,t-1}^{\rho_c} \exp(\epsilon_{c,t})$  and  $\xi_{n,t} = \xi_{n,t-1}^{\rho_n} \exp(\epsilon_{n,t})$  represent a preference shock and a shock to labor supply, respectively.

The representative household's budget constraint is as follows:

$$C_t + \omega X_t + \frac{\gamma_h B_t}{P_t} = \frac{W_t}{P_t} N_t + \frac{i_t^k K_{t-1}}{P_t} + T_t + \frac{I_{t-1}^b (1 - \mu^b) \gamma_h B_{t-1}}{P_t} + \frac{\zeta_b \mu^b \gamma_h B_{t-1}}{P_t} + \frac{\Pi_t^e}{P_t}, \quad (2)$$

where  $T_t$ ,  $W_t$ , and  $P_t$  represent the lump-sum tax, the nominal wage and the aggregate price, respectively.  $i_t^k$  is the nominal return on the physical capital, and  $K_t$  is the physical capital.  $\Pi_t^e$  is the profit or dividend payment received from entrepreneurs.  $I_t^b$  is the gross nominal return on government debt.  $\gamma_h \in [0, 1]$  is the

fraction of government debt that is held by households.  $\mu^b \in \{0, 1\}$  is the government's decision to default, and  $\zeta_b$  is the fraction of the defaulted government debt that is recovered. The model assumes that a fraction of investment  $\omega X_t$  is the direct investment by households, whereas entrepreneurs need to finance the rest with bank loans.

The capital accumulation equation is a standard one:

$$K_t = (1 - \delta_e)K_{t-1} + X_t. \quad (3)$$

The first-order conditions (FOCs hereafter) for  $N_t$ ,  $B_t$ , and  $K_t$  are:

$$\frac{W_t}{P_t} = \frac{\xi_{n,t} N_t^{\eta_n}}{\xi_{c,t} C_t^{-\eta_c}}, \quad (4)$$

$$1 = \beta E_0 \left[ \frac{\xi_{c,t+1}}{\xi_{c,t}} \left( \frac{C_{t+1}}{C_t} \right)^{-\eta_c} \frac{I_t^b (1 - \mu^b) + \zeta_b \mu^b}{\Pi_{t+1}} \right], \quad (5)$$

$$\omega = \beta E_0 \left[ \frac{\Pi_{t+1}}{I_t^b} \left( \omega (1 - \delta_e) + \frac{i_t^k}{P_{t+1}} \right) \right]. \quad (6)$$

In the case that the government does not default ( $\mu^b = 0$ ), the FOC for  $B_t$  (5) reduces to:

$$1 = \beta E_0 \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\eta_c} \frac{I_t^b}{\Pi_{t+1}} \right]. \quad (7)$$

Equation 7 is the Lucas asset pricing equation for the government debt, where inflation  $\Pi_{t+1} = P_{t+1}/P_t$ . However, if the government defaults ( $\mu^b = 1$ ), the asset pricing equation for the government debt becomes equation 5. If the government defaults, the gross nominal return on the government debt is equivalent to the recovery rate, resulting in a decline in the household's financial wealth.

## 2.2 Entrepreneurs

Entrepreneurs operate in a monopolistically competitive market. In each period, the representative entrepreneur  $j$  maximizes the periodic cashflow (profit), subject to the production technology (9) and the law of motion for physical capital (3),

$$\begin{aligned} \Pi_{j,t}^e &= E_0 \sum_{t=0}^{\infty} \beta_e^t \left\{ \frac{P_{j,t}}{P_t} Y_{j,t} - \frac{W_t}{P_t} N_{j,t} - \frac{i_t^k}{P_t} K_{j,t-1} \right. \\ &\quad \left. + \frac{L_{j,t}}{P_t} - \frac{I_{j,t-1}^e (1 - \mu_{j,t}^e) L_{j,t-1}}{P_t} - \frac{\Lambda_e (\mu_{j,t-1}^e L_{j,t-1})^2}{2 P_t} \right\}, \end{aligned} \quad (8)$$

where  $I_{j,t}^e$  is the gross nominal interest rate on bank loans  $L_{j,t}$ .  $Y_{j,t}$  is the output produced by the entrepreneur  $j$ , and  $P_{j,t}$  and  $P_t$  are the price for  $Y_{j,t}$  and the aggregate price level, respectively.

The model assumes that entrepreneur  $j$  can borrow from banks to accumulate physical capital and finance production costs, such as the wage bill and the rental cost of the physical capital. This implies that

$L_{j,t} \geq (1 - \omega)X_{j,t}$ , where  $\omega$  fraction of  $X_t$  is the direct investment from households.

Following Dubey et al. (2005), Elul (2008), and De Walque et al. (2010), I assume that in the case that entrepreneur  $j$  chooses to repay only  $1 - \mu_{j,t}^e$  fraction of previous loans, he bears pecuniary costs, due to, for instance, high search costs to obtain new loans in future because of his bad reputation. More specifically, he needs to pay a quadratic cost in terms of the amount of the defaulted loans, where  $\Lambda_e$  is the parameter governing the quadratic cost.

The production function is given as follows:

$$Y_{j,t} = \xi_{z,t} K_{j,t}^\alpha N_{j,t}^{1-\alpha}, \quad (9)$$

where the aggregate technology shock  $\xi_{z,t} = \xi_{z,t-1}^{\rho_z} \exp(\epsilon_{z,t})$  follows a stochastic AR(1) process.

The FOCs for  $N_{j,t}$ ,  $K_{j,t}$ ,  $L_{j,t}$ , and  $\mu_{j,t}^e$  are:

$$\frac{Y_{j,t}}{N_{j,t}} = \frac{1}{(1 - \alpha)MC_t} \frac{W_t}{P_t}, \quad (10)$$

$$Q_{j,t} = \beta_e E_0 [Q_{j,t+1}(1 - \delta_e) + \frac{i_t^k}{P_{t+1}}], \quad (11)$$

$$1 = \beta_e E_0 \left[ \frac{I_{j,t}^e (1 - \mu_{j,t+1}^e) + \Lambda_e (\mu_{j,t}^e)^2 L_{j,t}}{\Pi_{t+1}} \right], \quad (12)$$

$$I_{j,t-1}^e L_{j,t-1} = \beta_e E_0 \left[ \frac{\Lambda_e \mu_t^e}{\Pi_{t+1}} (L_{j,t})^2 \right]. \quad (13)$$

Equation 10 is the labor demand function derived from the entrepreneur's cost minimization problem, where  $MC_t$  is the real marginal cost. Equation 11 gives the asset pricing equation for the physical capital, where  $Q_{j,t}$  is the shadow price for the physical capital. The FOCs for  $L_{j,t}$  (12) and  $\mu_{j,t}^e$  (13) imply that the default rate of entrepreneur loans depends positively on the expected future default rate and inflation, and negatively on the amounts of loans borrowed and the bank loan rate.

### 2.3 Banks

The banking sector consists of identical, perfectly competitive commercial banks. Table 1 lays out the components of a commercial bank's balance sheet. In each period, the representative bank receives the repayment of previous loans to entrepreneurs and issues new loans. In the case that entrepreneurs only repay  $(1 - \mu_t^e)L_{t-1}$ , the bank is able to recover  $\zeta_e$  proportion of the amounts of loans defaulted from the insurance fund managed by a public authority, upon paying an insurance premium ( $\theta_e$  fraction of the expected defaulted entrepreneur loans). Without losing generality, the model assumes that the government debt is the only form of security held by commercial banks.<sup>2</sup> As happens with entrepreneur loans, in the case that the government defaults ( $\mu^b = 1$ ), the bank is able to recover  $\zeta_b$  proportion of the defaulted government debt, upon paying an insurance premium ( $\theta_b$  fraction of the expected defaulted government debt).

<sup>2</sup>I acknowledge that the interbank market plays an important role in a bank's operation, and that the amount of interbank borrowing and lending constitutes a considerable fraction of a bank's balance sheet. For simplicity, the model abstracts from interbank borrowing and lending and any other form of asset.



Table 1: A commercial bank's balance sheet

Assets	Liabilities
Loans $L$	Equity $\Psi^B$
Government debt $\gamma_b B$	Funds borrowed from central bank $F^{cb}$

Assets are funded by bank equity ( $\Psi^B$ ) and funds borrowed from the central bank ( $F^{cb}$ ). Analogically, commercial banks can choose to pay  $1 - \mu_t^f$  fraction of funds borrowed from the central bank in the previous period, knowing there would be a quadratic cost in terms of a fraction of the amounts of defaulted funds should they borrow from the central bank again in future, where  $\Lambda_g$  is the parameter governing the quadratic cost.

The representative bank therefore maximizes the expected payoff (profit):

$$\begin{aligned}
\Pi_t^b = & E_0 \sum_{t=0}^{\infty} \beta_b^t \left[ -\frac{L_t}{P_t} + \frac{I_{t-1}(1 - \mu_t^e)L_{t-1}}{P_t} + \frac{\zeta_e E_{t-1}(\mu_t^e L_{t-1})}{P_t} - \frac{\gamma_b B_t}{P_t} + \frac{I_{t-1}^b(1 - \mu^b)\gamma_b B_{t-1}}{P_t} \right. \\
& + \frac{\zeta_b E_{t-1}(\mu^b \gamma_b B_{t-1})}{P_t} + \frac{F_t^{cb}}{P_t} - \frac{I_{t-1}^f(1 - \mu_t^f)F_{t-1}^{cb}}{P_t} - \frac{\Lambda_f}{2} \frac{(\mu_{t-1}^f F_{t-1}^{cb})^2}{P_t} \\
& \left. + \frac{\Psi_t^B}{P_t} - \frac{I_{t-1}^\Psi \Psi_{t-1}^B}{P_t} - E_{t-1} \left( \frac{\theta_e \mu_t^e L_{t-1} + \theta_b \mu^b \gamma_b B_{t-1}}{P_t} \right) \right], \tag{14}
\end{aligned}$$

subject to the capital requirement constraint (15) and the balance sheet constraint (16):

$$\Psi_t^B \geq \kappa(\phi^e L_t + \phi^b \gamma_b B_t), \tag{15}$$

$$L_t + \gamma_b B_t = \Psi_t^B + F_t^{cb}, \tag{16}$$

where  $I_t^f$  and  $I_t^\Psi$  are gross nominal returns on funds borrowed from the central bank and bank equity, respectively.  $\gamma_b$  is the proportion of the total government debt held by commercial banks.  $\kappa$  is the minimum capital requirement imposed by the financial regulator, and  $\phi^e$  and  $\phi^b$  are risk coefficients imposed on entrepreneur loans and the government debt. I assume that the risk coefficient for entrepreneur loans is an increasing function of the steady state default rate,  $\phi^e = (\tilde{\phi}^e)^{\frac{1}{100\mu^e}}$ , where  $\tilde{\phi}^e$  is the risk coefficient imposed on entrepreneur loans according to Basel regulations.<sup>3</sup> This implies that the higher the default rate, the greater the risk coefficient for entrepreneur loans. The risk coefficient imposed on the government debt  $\phi^b$  equals either 0 according to Basel regulations if the government does not default, or 0.8, the same as  $\tilde{\phi}^e$ , if the government defaults.

Bank equity capital follows the law of motion:

$$\Psi_t^B = (1 - \delta_b)\Psi_{t-1}^B + \phi_\Pi \Pi_{t-1}^b. \tag{17}$$

<sup>3</sup>According to Basel,  $\tilde{\phi}^e = 0.8$ .  $\mu^e \in (0, 1)$  is the steady state default rate of entrepreneur loans in the model.

That is, in each period the bank decides to use  $\phi_\Pi$  fraction of previous profits to accumulate equity, and consumes  $\delta_b$  amount of equity (e.g. on labor costs and the physical capital depreciation).

The FOCs for  $B_t$ ,  $L_t$ ,  $F_t^{cb}$ , and  $\mu_t^f$  are:

$$0 = \beta_b E_0 \left[ \frac{I_t^b (1 - \mu^b) + (\zeta_b - \theta_b) \mu^b}{\Pi_{t+1}} + \frac{I_t^\Psi}{\Pi_{t+1}} + \lambda_{f,t} \kappa \phi_t^b \right], \quad (18)$$

$$0 = \beta_b E_0 \left[ \frac{I_t^e (1 - \mu_{t+1}^e) + (\zeta_e - \theta_e) \mu_{t+1}^e}{\Pi_{t+1}} + \frac{I_t^\Psi}{\Pi_{t+1}} + \lambda_{f,t} \kappa \phi_t^e \right], \quad (19)$$

$$1 = \beta_b E_0 \left[ \frac{I_t^f (1 - \mu_{t+1}^f) + \Lambda_f (\mu_t^f)^2 F_t^{cb}}{\Pi_{t+1}} \right], \quad (20)$$

$$I_{t-1}^f F_{t-1}^{cb} = \beta_b E_0 \left[ \frac{\Lambda_f \mu_t^f}{\Pi_{t+1}} (F_t^{cb})^2 \right], \quad (21)$$

where  $\lambda_{f,t}$  is the Lagrange multiplier of the capital requirement constraint. Equation 19 gives the optimal gross nominal return on entrepreneur loans. The FOCs for  $F_t^{cb}$  (20) and  $\mu_t^f$  (21) imply that default on funds borrowed from the central bank depends positively on the expected future default rate and inflation, and negatively on the amounts of funds borrowed and the interest rate charged.

## 2.4 Government

The government sector consists of a fiscal branch and a central bank branch, conducting fiscal and monetary policies respectively. The budget constraint for the fiscal branch is

$$G_t + \frac{I_{t-1}^b (1 - \mu_b) B_{t-1}}{P_t} = \frac{B_t}{P_t} + T_t, \quad (22)$$

and the budget constraint for the central bank is

$$\begin{aligned} \frac{(1 - \gamma_h - \gamma_b) B_t}{P_t} + \frac{F_t^{cb}}{P_t} &= \frac{I_{t-1}^b (1 - \gamma_h - \gamma_b) (1 - \mu_b) B_{t-1}}{P_t} \\ &+ \frac{I_{t-1}^f (1 - \mu_t^f) F_{t-1}^{cb}}{P_t}. \end{aligned} \quad (23)$$

Combing equations 22 and 23 gives the consolidated government-sector budget constraint<sup>4</sup>

$$\begin{aligned} G_t + \frac{I_{t-1}^b (\gamma_h + \gamma_b) (1 - \mu_b) B_{t-1}}{P_t} + \frac{F_t^{cb}}{P_t} &= T_t + \frac{(\gamma_h + \gamma_b) B_t}{P_t} \\ &+ \frac{I_{t-1}^f (1 - \mu_t^f) F_{t-1}^{cb}}{P_t}. \end{aligned} \quad (24)$$

Government expenditure  $G_t = G_{t-1}^{\rho_g} \exp(\epsilon_{g,t})$  is assumed to exogenously follow a stochastic AR(1) process.

<sup>4</sup>For simplicity, I assume that seigniorage revenues are turned over to the treasury, becoming part of the lump-sum transfer to/tax on households.

The government also manages the insurance fund, and acts as the regulator of the banking sector. To close the model, I assume that the insurance scheme is financed by the lump-sum tax:<sup>5</sup>

$$T_t = \frac{(\zeta_f - \theta_f)E_{t-1}(\mu_t^f F_{t-1}^{cb})}{P_t} + \frac{(\zeta_e - \theta_e)E_{t-1}(\mu_t^e L_{t-1})}{P_t} + \frac{\zeta_b E_{t-1}[\mu^b(\gamma_h + \gamma_b)B_{t-1}]}{P_t} - \frac{\theta_b E_{t-1}(\mu^b \gamma_b B_{t-1})}{P_t}. \quad (25)$$

## 2.5 Aggregate resource

The aggregate resource is described as follows:

$$Y_t = C_t + X_t + G_t + E_{t-1}\left(\frac{\theta_e \mu_t^e L_{t-1} + \theta_b \mu^b \gamma_b B_{t-1} + \theta_f \mu_t^f F_{t-1}^{cb}}{P_t}\right). \quad (26)$$

## 2.6 Equilibrium

At the beginning of each period, the government chooses  $\{B_t\}$  to maximize the welfare of households subject to equations (2), (3) and (24). The return on the government debt is determined. Later in the period, the government decides either to honor its debt ( $\mu_b = 0$ ) that matures in the current period, or to default ( $\mu_b = 1$ ). Then it determines its expenditure according to its budget constraint.

The representative household chooses  $\{K_t, \gamma_b B_t, N_t\}$  to maximize its utility (1), subject to (2) and (3). The bank chooses  $\{\gamma_b B_t, L_t, F_t^{cb}, \mu_t^f\}$  to maximize his profit (14), subject to (15) and (16). Once the household decides how much to invest in the physical capital and the bank decides the interest rate on loans, the representative entrepreneur chooses  $\{K_{j,t}, L_{j,t}, \mu_{j,t}^e\}$  to maximize his profit (8), subject to (9) and (3). Later, the household, the entrepreneur, and the bank re-optimize their individual problems in reaction to the government's default decision. The same applies to a case where the government partially rolls over its debt.

The market clearing condition implies that in equilibrium  $P_{j,t} = P_t$ ,  $K_{j,t} = K_t$ ,  $Q_{j,t} = Q_t$ ,  $Y_{j,t} = Y_t$ ,  $L_{j,t} = L_t$ ,  $I_{j,t}^e = I_t^e$ , and  $\mu_{j,t}^e = \mu_t^e$ .

## 3 QUADRATIC LOSS FUNCTION

The welfare cost of sovereign default is measured by a utility-based welfare loss criterion. The model assumes that the government's object is to maximize the household's utility function (1). Below is the quadratic loss function derived by conducting the second-order approximation to the utility function,

$$W_t = - \sum_{t=0}^{\infty} \beta_g^t \mathcal{L}_t + t.i.p. + O(\|\xi\|)^3, \quad (27)$$

---

<sup>5</sup>Note that only commercial banks pay an insurance premium on the amount of the government debt purchased ( $\gamma_b B_t$ ). However, if the government defaults, it needs to pay the coverage of its debt held by both households and commercial banks, that is,  $(\gamma_h + \gamma_b)B_t$ .

where *t.i.p.* are terms that are independent of the policy, and  $O(\|\xi\|)^3$  are terms of third or higher order. Taking the unconditional expectation of  $L_t$ , I obtain

$$\begin{aligned}\mathcal{L}_t = & \chi_y var(\hat{y}_t - \bar{y}) + \chi_x var(\hat{x}_t - \bar{x}) + \chi_k var(\hat{k}_t^e) \\ & + \chi_\pi var(\hat{\pi}_t) + \chi_{yx} cov(\hat{y}_t - \bar{y}, \hat{x}_t - \bar{x}),\end{aligned}\quad (28)$$

where a lower case letter with a hat represents the log-deviation of the variable, and a lower case letter with a bar represents the natural rate of the variable with the Woodford natural rate definition.  $\chi_y$ ,  $\chi_x$ ,  $\chi_k$ ,  $\chi_\pi$  and  $\chi_{yx}$  are the relative weights of the contributions from the variances of output, investment, physical capital, and inflation, and the covariance between output and investment.<sup>6</sup>

In Rotemberg and Woodford (1997) and Woodford (2003), the utility-based loss function depends only on the variance of the output gap and the variance of the inflation gap. In contrast, the deadweight loss (28) here is a weighted sum of the variabilities of aggregate output, investment, physical capital, and inflation around their natural rates, and the variability of the covariance between output and investment due to the inclusion of the endogenous physical capital accumulation in the model.<sup>7</sup>

The following explanation of the way that each welfare-related endogenous variable affects welfare will help to explain the results below. As indicated by equation 28, the welfare criterion is affected by four endogenous variables. *Ceteris paribus*, an increase in the output gap increases both the utility of consumption and the disutility of labor supply, where the net effect depends on some of the deep parameters from the model. Based on the model calibration, the relative weight of the output gap  $\chi_y = -3.637$ , indicating that an increase in the variability of the output gap decreases the overall utility for the household and hence increases the welfare loss. As in a standard New Keynesian model without endogenous physical capital accumulation, an increase in the inflation gap increases the welfare loss, *ceteris paribus*. The variabilities of the investment gap and the physical capital gap affect the welfare loss in the same manner as the output gap does. Intuitively, for instance, an increase in investment decreases consumption and hence decreases the household's utility. Moreover, an increase in the current investment results in an increase in the next period's physical capital. Therefore, an increase in both the variance of the investment gap and the variance of the physical capital gap increases the welfare loss. However, it is worth pointing out that, based on the calibration, the welfare implication of the variance of the investment gap ( $\chi_x = -0.00002$ ) is relatively insignificant compared with that of the physical capital gap ( $\chi_k = -0.156$ ). The deadweight loss is mainly due to the variance of the output gap and the variance of the inflation gap ( $\chi_\pi = -1.61$ ). Finally, the co-movement of output and investment gaps has a minor negative effect ( $\chi_{yx} = 0.01$ ) on the welfare loss.

<sup>6</sup>All variables are in gap terms, e.g. output:  $\hat{y}_t - \bar{y}$ . An uppercase letter without a time subscript stands for the steady state of the variable. Where,  $\chi_y = -\frac{1}{2}[\frac{\eta_m + \alpha}{1-\alpha} + \frac{\eta_c}{C/Y}]$ ,  $\chi_x = -\frac{1}{2}[\frac{\eta_c}{C/Y}(\frac{\alpha\beta\omega\delta_e}{1-\beta\omega(1-\delta_e)})^2 + \alpha\beta\omega^2\delta_e^2]$ ,  $\chi_k = -\frac{1}{2}[\frac{\eta_c}{C/Y}(\frac{\alpha\beta\omega(1-\delta_e)}{1-\beta\omega(1-\delta_e)})^2 + \frac{\alpha^2(1+\eta_m)}{1-\alpha}]$ ,  $\chi_\pi = -\frac{1}{2}[\frac{\epsilon_p}{1-\alpha}(1 + \frac{\epsilon_p\alpha}{1-\alpha})\frac{\theta_p}{(1-\theta_p)(1-\beta\theta_p)}]$  and  $\chi_{yx} = \frac{\eta_c}{C/Y}\frac{\alpha\beta\omega\delta_e}{1-\beta\omega(1-\delta_e)}$ . An uppercase letter without a time subscript stands for the steady state of the variable.

<sup>7</sup>The derivation of the quadratic loss function is closely related to the derivation by Edge (2003) of a utility-based welfare criterion in a model with endogenous capital accumulation, as in the present study.

## 4 CALIBRATION

The model is calibrated using quarterly U.S. data over the period 1992Q1 to 2012Q3. Calibration of the real sector is based on the national account data and calibration of the financial sector on the commercial bank balance sheet and the micro-finance data.<sup>8</sup> The calibration results are reported in Tables 2 and 3.

### 4.1 Financial sector

In the balance sheets of all U.S. commercial banks, the steady-state ratio of the bank profit to capital equals 0.107, and the bank capital depreciation rate equals 0.10. In each period, half of the bank's profits are used to accumulate bank capital. Based on the data, the average minimum capital requirement  $\kappa$  for all U.S. commercial banks is 0.11. According to Basel regulations, the risk coefficient imposed on entrepreneur loans  $\tilde{\phi}^e$  is set to 0.8, whereas the risk coefficient for the government debt  $\tilde{\phi}^b$  is set to 0 for the case of no sovereign default.

The average quarterly nominal return on equity for all U.S. banks equals 3.1%. The average quarterly market prime loan rate over the sample period is 6.3%. The gross nominal return on the government debt is calibrated to 1.03 based on the three-month treasury bill rate, implying a discount factor of 0.975 for commercial banks. The model implies that the discount factor for households is also 0.975. The average quarterly effective Fed funds rate over the sample period is 3.3%. The return on funds borrowed from the central bank is assumed to be the effective Fed funds rate less 1%.

The public holds approximately half of the U.S. government debt, i.e.  $\gamma_h = \frac{1}{2}$ , and the Fed and local governments hold about one-third of this debt. The model assumes that the rest of the total government debt is held by commercial banks, i.e.  $\gamma_b = \frac{1}{6}$ . As in the literature (e.g. De Walque et al., 2010), the recovery rate for sovereign debt in default  $\zeta_b$  is set to 0.<sup>9</sup> That is, if the government defaults, households and commercial banks are unable to recover any loss. However, 30% of defaulted entrepreneur loans and 80% of defaulted funds borrowed from the central bank can be reimbursed by the insurance fund. Insurance premiums for entrepreneur loans and the government debt that commercial banks need to pay are set to 0.5% and 0.1%, respectively. The insurance premium for funds borrowed from the central bank is set to 0.2%.

### 4.2 Real sector

Calibration of the real sector (households, entrepreneurs, and government) is mainly based on macroeconomic aggregates: the capital share  $\alpha$  in the production function is calibrated to 0.3; the physical capital depreciation rate  $\delta_e$  is 0.025; the steady-state entrepreneur loan to physical capital ratio is 0.011. The steady-state default rate of entrepreneur loans is set to 5%, as in De Walque et al. (2010). The fraction parameter of the direct investment from households is assumed to be 0.33, implying that entrepreneurs need to borrow bank loans equivalent to at least two-thirds of the total investment. Steady-state ratios of consumption to output, investment to output, total government expenditures to output, total entrepreneur loans to output, the total

---

<sup>8</sup>See Appendix A for the data source and variables used for calibration.

<sup>9</sup>For modeling purposes I set  $\zeta_b = 0.0001$ .

Table 2: Calibration: financial sector

Parameter	Description	Value
$\delta_b$	Bank capital depreciation rate	0.10
$\phi_\Pi$	Fraction of profits devoted to bank capital accumulation	0.5
$\frac{\Pi^b}{\Psi B}$	Steady state bank profit to capital ratio	0.107
$\kappa$	Capital requirement ratio	0.11
$\tilde{\phi}^e$	Risk coefficient on entrepreneur loans according to Basel	0.8
$\tilde{\phi}^b$	Risk coefficient on the government debt according to Basel	0
$\Lambda_f$	Parameter governing the quadratic search cost for banks	2
$I^b$	Steady state gross nominal return on the government debt	1.03
$I^e$	Steady state gross nominal entrepreneur bank loan rate	1.063
$I^f$	Steady state gross nominal return on funds borrowed from the central bank	1.023
$I^\Psi$	Steady state gross nominal return on bank equity	1.031
$\gamma_h$	Fraction of the government debt held by public	$\frac{1}{2}$
$\gamma_b$	Fraction of the government debt held by commercial banks	$\frac{1}{6}$
$\zeta_b$	Proportion of the defaulted government debt recovered	0
$\zeta_e$	Proportion of defaulted entrepreneur loans recovered	0.3
$\zeta_f$	Proportion of defaulted funds borrowed from the central bank recovered	0.8
$\theta_b$	Insurance premium on the government debt	0.001
$\theta_e$	Insurance premium on entrepreneur loans	0.005
$\theta_f$	Insurance premium on funds borrowed from the central bank	0.002
$\frac{L}{\Pi^b}$	Steady state loan-to-profit ratio for banks	13.53
$\frac{B}{\Pi^b}$	Steady state government debt to profit ratio for banks	100.877
$\frac{F^{cb}}{\Pi^b}$	Steady state funds borrowed from the central bank to profit ratio for banks	11.357

Table 3: Calibration: real sector

Parameter	Description	Value
$\eta_n$	Inverse of the elasticity of work effort with respect to real wage	1
$\eta_c$	Coefficient of the relative risk aversion	2
$\alpha$	Capital share in production function	0.3
$\delta_e$	Physical capital depreciation rate	0.025
$\omega$	Fraction of investment from households	0.33
$\Lambda_e$	Parameter governing the quadratic search cost for entrepreneurs	5
$\mu^e$	Steady state fraction of entrepreneur loans defaulted	0.05
$\Pi$	Steady state gross inflation rate	1.005
$\theta_p$	Calvo sticky price coefficient	0.75
$\epsilon_p$	Price elasticity of demand for intermediate goods	0.5
$\frac{C}{Y}$	Steady state consumption to output ratio	0.68
$\frac{X}{Y}$	Steady state investment to output ratio	0.2
$\frac{G}{Y}$	Steady state total government expenditures to output ratio	0.1
$\frac{L}{Y}$	Steady state total entrepreneur loans to output ratio	0.088
$\frac{B}{Y}$	Steady state total government debt to output ratio	0.673
$\frac{F^{cb}}{Y}$	Steady state total liquidity injections to output ratio	0.072
$\frac{F^{cb}}{T}$	Steady state total liquidity injections to total taxes ratio	1.482
$\frac{L}{T}$	Steady state total entrepreneur loans to total taxes ratio	1.776
$\frac{B}{T}$	Steady state total government debt to total taxes ratio	13.17

government debt to output, and total liquidity injections<sup>10</sup> to output are 0.68, 0.2, 0.1, 0.088, 0.673, and 0.072, respectively. Steady-state ratios relative to total taxes are 1.482 for liquidity injections, 1.776 for total entrepreneur loans, and 13.17 for the total government debt. Some standard parameter values are calibrated within the range of the New Keynesian literature. For instance, the coefficient of the relative aversion  $\eta_c$  is set to 2, and the inverse of the elasticity of work effort with respect to the real wage is set to 1.

## 5 RESULTS

This section reports the welfare cost of sovereign default. I compare the welfare losses in the model with and without liquidity injections. I also look at the welfare losses if the government partially rolls over its debt.

### 5.1 Welfare cost of sovereign default

Table 4 reports the welfare losses conditional on a positive productivity shock (one percent standard deviation). The standard deviation of output ( $y$ ), investment ( $x$ ), physical capital ( $k$ ) and inflation ( $\pi$ ), and the covariance of  $y$  and  $x$  are reported in columns 3 to 7, and the deadweight welfare losses in terms of percentage units of steady state consumption are reported in the last column.

If the government defaults, the deadweight welfare loss decreases 43% (from 2.867 to 1.644). This decrease is mainly due to the 30% decrease in the variability of the output gap, which, together with the highest weight among other arguments in the welfare criterion, results in a significant decline in the deadweight welfare loss. Even though there is a huge decrease in the variability of the investment gap, its contribution towards the deadweight welfare loss can be ignored because of its minimal weight in the welfare criterion. However, the variability of the physical capital gap increases 33%, which only marginally offsets the decrease in the deadweight welfare loss because of its minimal weight in the welfare loss criterion. The variability of the inflation gap is surprisingly small, making no significant contribution to the changes of the deadweight welfare loss. Finally, the variability of the covariance of output and investment can be ignored since both their magnitudes and their weights in the welfare loss criterion are very small.

Using an alternative version of the model, I analyze the welfare loss caused by sovereign default when there are no liquidity injections in the model. The results are reported in table 5. Sovereign default causes large fluctuations in the physical capital (140%) and the inflation gap (30%). However, the significant increase in the variabilities of the physical capital and inflation gaps results in only an 8% increase in the deadweight welfare loss. This is because there are no significant changes in the variability of the output gap, which carries a high weight in the welfare loss criterion. It is worth noting that investment remains the most volatile variable in both cases (with and without liquidity injections).

---

<sup>10</sup>I use the data of mortgage-backed securities held by the Fed as the proxy for liquidity injections to commercial banks. The Fed only started purchasing mortgage-backed securities in 2009. I nonetheless use the available data to calculate the ratios related to liquidity injections.

Table 4: The welfare cost of sovereign default

Government	y	x	$k^e$	$\pi$	yx	Welfare loss
No default	0.830	3.620	0.360	0.480	0.030	2.867
Default	0.580	0.520	0.490	0.490	0.003	1.644
50% Rollover	0.880	4.670	0.390	0.560	0.041	3.295

Note: numbers are in percentage.

Table 5: The welfare cost of sovereign default (without liquidity injections)

Government	y	x	$k^e$	$\pi$	yx	Welfare loss
No default	1.010	3.090	0.300	0.360	0.031	3.902
Default	1.030	5.980	0.720	0.470	0.062	4.234
50% Rollover	0.900	2.640	0.230	0.280	0.024	3.057

Note: numbers are in percentage.

## 5.2 Debt rollover

In this section I extend my analysis one step further by investigating the welfare cost in a case where the government partially rolls over its debt. I do this by repeating the same exercise as in the previous section, but setting  $\mu_b = 0.5$ . That is, the model assumes that the government rolls over half of its existing debt. The results are reported in the last row of Tables 4 and 5. For the model with liquidity injections, the deadweight welfare loss increases 15%. In contrast to the case of default, if the government rolls over half of its existing debt, the variabilities of both the output and investment gaps are negligible. However, for the model without liquidity injections, the deadweight welfare loss falls by 23% because of the decrease in the variabilities of both the output and inflation gaps.

## 5.3 Remarks

The analysis for the model with liquidity injections suggests that sovereign default results in a significant decline in the variability of the output gap and hence a significant decrease in the deadweight welfare loss. However, if the government rolls over half of its existing debt, the deadweight welfare loss increases slightly. Intuitively, as a consequence of sovereign default the government might reduce taxes, which would to some extent compensate households for their financial losses. Both sovereign default and tax deduction have an immediate effect on the household's investment decision as regards the government debt and the physical capital. This is why investment is the most volatile variable of the endogenous arguments in the welfare loss criterion. After default, to keep the budget constraint binding, the government might also increase the supply of liquidity injections if necessary. This in turn would provide more funds for banks to supply entrepreneur loans, *ceteris paribus*.

The impact of sovereign default on entrepreneurs tends to be in favor of production. Should the government default, there will be neither a permanent decline in production, as Cole and Kehoe (2000) argue, nor a large decline in output. If the government defaults, the deadweight welfare loss decreases, because of the



decrease in the variability of the output gap. This is consistent with the finding by Cole and Kehoe (2000) that it is optimal for the government to default to exit the crisis zone, which leads to investment, consumption, and output rising. However, if the government partially rolls over its existing debt, the variabilities of the output, physical capital, and inflation gaps remain stable. There is, however, a slight increase in the deadweight loss (15%) because of the marginal increases in the variabilities of the output and inflation gaps. The results with debt rollover tend to support the argument that sovereign default affects production through the household's investment decision and the bank's asset portfolio adjustment and, more importantly, results in a decrease in households' welfare.

The analysis for the model without liquidity injections suggests exactly the opposite. The deadweight welfare loss increases slightly (mainly because of the increase in the variabilities of the physical capital and inflation gaps) if the government defaults, and moderately decreases (mainly because of the decrease in the variabilities of the output and inflation gaps) if the government rolls over half of its existing debt. Intuitively, without the access to liquidity injections from the central bank, if the government defaults, banks will not have any funding sources other than equity capital. At the same time, banks will suffer losses from investing in the government debt. The supply of entrepreneur loans is therefore lower in this case than in the case where banks can access liquidity injections. One interesting finding here is that the variability of the inflation gap increases 30%, suggesting that sovereign default may result in price instability. In other words, liquidity injections improve price stability during the period of sovereign debt crisis and hence households' welfare. The results further show that liquidity injections reduce the variability of the output gap during the period of sovereign debt crisis. If the government defaults, the variability of the output gap decreases about 30% in the model with liquidity injections, whereas it remains extremely stable in the model without these injections.

## 5.4 Business cycle properties

To complement the quantitative analysis, this section reports the business cycle properties of several key variables from the models with and without liquidity injections, Model 1 and Model 2 in Table 6, respectively. I compare each variable's second moment to that of output ( $\frac{\sigma(X)}{\sigma(Y)}$ ) and its correlation between output from the data with those from the model. The model results are obtained from simulations driven by a positive technology shock (one percent standard deviation) over the sample period of 1992Q1 to 2012Q3.

The model reproduces the second moments for consumption, investment, inflation, and entrepreneur loans<sup>11</sup> remarkably well, except for a high correlation between entrepreneur loans and output. The correlation of inflation with output is rather too high as well. Over the sample period, returns on government debt and bank equity, and the Fed fund rate, are more volatile than those observed in the literature with a much longer sample period. This is because rates became very volatile during the Great Recession, resulting in a relatively large standard deviation on average, given such a short sample period. For the same reason, a negative correlation between government debt and output is observed. During the Great Recession, output declined sharply, accompanied by skyrocketing total government debt. The model simulation reproduces the second moment for the government debt, but fails to account for the negative correlation between the

---

<sup>11</sup>The measurement of entrepreneur loans is a stock in data and a flow in the model.

Table 6: Cyclical properties

Variable	Data		Model1		Model2	
	$\frac{\sigma(X)}{\sigma(Y)}$	Corr(X,Y)	$\frac{\sigma(X)}{\sigma(Y)}$	Corr(X,Y)	$\frac{\sigma(X)}{\sigma(Y)}$	Corr(X,Y)
Output (Y)	1	1	1	1	1	1
Consumption	0.82	0.69	0.54	0.52	1.03	0.71
Inflation	0.33	0.04	0.58	0.65	0.22	0.98
Investment	5.14	0.80	4.36	0.93	3.55	0.72
Entr. loans	2.15	0.15	2.93	0.93	2.38	0.72
Entr. default rate	1.68	-0.06	5.64	-0.59	4.59	-0.43
Bank capital	2.93	-0.04	8.69	0.00	1.11	0.08
Govt. debt	2.15	-0.42	2.34	0.40	1.44	0.52
Govt. debt rate	2.26	0.35	1.08	0.32	0.69	0.65
Return to equity	6.79	0.48	1.10	0.32	0.61	0.88
Fed fund rate	3.33	0.27	1.11	0.32	0.62	0.88

Note: All aggregate variables are in real per capita term, and detrended by log-differencing.

government debt and output. Finally, the model yields a relatively strong negative correlation between the entrepreneur default rate and output.<sup>12</sup>

## 6 CONCLUSION

This paper investigates the welfare cost of sovereign default using a dynamic general equilibrium model with endogenous default on entrepreneur loans and liquidity injections. Using a utility-based welfare criterion, without an ad hoc exogenous default cost, I show that sovereign default affects production through the household's investment decision and the bank's asset portfolio adjustment. The impact of sovereign default on entrepreneurs tends to be in favor of production and hence leads to a decline in the deadweight welfare loss. Liquidity injections can reduce the variability of the output gap and improve price stability during the period of sovereign debt crisis and hence increase households' welfare.

<sup>12</sup>I use the delinquency rate on business loans (DRBLACBS) as the proxy for entrepreneur default rate.

## References

- Arellano, C., 2008. Default risk and income fluctuations in emerging economies. *American Economic Review* 98, 690–712.
- Calvo, G., 1983. Staggered prices in a utility-maximizing framework. *Journal of Monetary Economy* 12 (3), 383–398.
- Cole, H. L., Kehoe, T. J., 2000. Self-fulfilling debt crises. *Review of Economic Studies* 67, 91–116.
- Curdia, V., Woodford, M., 2011. The central-bank balance sheet as an instrument of monetary policy. *Journal of Monetary Economics* 58, 54–79.
- De Walque, G., Pierrard, O., Rouabah, A., 2010. Financial (in)stability, supervision and liquidity injections: a dynamic general equilibrium approach. *The Economic Journal* 120, 1234–1261.
- D’Erasmus, P., Mendoza, E. G., 2012. Domestic sovereign default as optimal redistributive policy. Memo.
- Dubey, P., Geanakoplos, J., Shubik, M., 2005. Default and punishment in general equilibrium. *Econometrica* 73 (1), 1–37.
- Edge, R., 2003. A utility-based welfare criterion in a model with endogenous capital accumulation. Federal Reserve Board, Working Paper Series no. 68.
- Elul, R., 2008. Collateral, credit history, and the financial decelerator. *Journal of Financial Intermediation* 17, 63–88.
- Goodhart, C., Sunirand, P., Tsomocos, D., 2006. A model to analyse financial fragility. *Economic Theory* 27, 107–142.
- Mondeza, E., Yue, V., 2012. A general equilibrium model of sovereign default and business cycles. *Quarterly Journal of Economics* 127, 889–946.
- Reinhart, C. M., Rogoff, S., 2008. The forgotten history of domestic debt. NBER Working Paper Series no. 13946.
- Rotemberg, J., Woodford, M., 1997. An optimization-based econometric framework for the evaluation of monetary policy. In: Bernanke, B., Rotemberg, J. (Eds.), *NBER Macroeconomics Annual*. The MIT Press, Cambridge.
- Smets, F., Wouters, R., 2007. Shocks and frictions in us business cycles: a bayesian approach. *American Economic Review* 97 (3), 586–606.
- Woodford, M., 2003. *Interest and prices*. Princeton University Press. Princeton.
- Yue, V., 2010. Sovereign default and debt renegotiation. *Journal of International Economics* 80, 176–187.

## Appendix: Data and sources

Data source: from the St. Louis Federal Reserve Economic Data (FRED).

1. RGDP: Real Gross Domestic Product, 1 Decimal (GDPC1), Billions of Chained 2005 Dollars, Quarterly, Seasonally Adjusted Annual Rate.
2. Consumption: Real Personal Consumption Expenditures (PCECC96), Billions of Chained 2005 Dollars, Quarterly, Seasonally Adjusted Annual Rate.
3. US population: Civilian Noninstitutional Population (CNP16OV), Thousands of Persons, Quarterly, Not Seasonally Adjusted.
4. Wages: Non-farm Business Sector: Real Compensation Per Hour (COMPRNFB), Index 2005 = 100, Quarterly, Seasonally Adjusted (equivalent to AHETPI).
5. Inflation: Gross Domestic Product: Implicit Price Deflator (GDPDEF), Index 2005 = 100, Quarterly, Seasonally Adjusted.
6. Producer Price Index: Finished Goods: Capital Equipment (PPICPE), Index 2005 = 100, Quarterly, Seasonally Adjusted.
7. Nominal interest rate: Effective Federal Funds Rate (FEDFUNDS), Percent, Quarterly, Not Seasonally Adjusted.
8. Loan rate to entrepreneurs: Moody's seasoned Baa corporate bond yield (BAA), Percent, Quarterly, Not seasonally Adjusted.
9. Loan rate to households: 30-Year Conventional Mortgage Rate (MORTG), Percent, Quarterly, Not Seasonally Adjusted.
10. Loans to households: Total Liabilities - Balance Sheet of Households and Nonprofit Organizations (TLBSHNO), Billions of Dollars, Quarterly, Not Seasonally Adjusted - includes mortgage sector and consumer credit sector (equivalent to CMDEBT).
11. Loans to entrepreneurs: Total Liabilities - Balance Sheet of Non-farm Nonfinancial Corporate Business (TLBSNNCB), Billions of Dollars, Quarterly, Not Seasonally Adjusted.
12. Entrepreneur default rate: Delinquency Rate On Business Loans, All Commercial Banks (DRBLACBS), Percent, Quarterly, Seasonally Adjusted
13. Equity price: Standard and Poor 500 Index (SP500), Index, Quarterly, Not Seasonally Adjusted.