Energy market dependence - Vine Copula application

Stellenbosch University/ Eighty20

How I learned to stop worrying about linear dependence and love the Copula

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Introduction

Why energy markets?

- Homogeneity among market movements post-GFC have become a growing topic in literature
- Large sell-off of commodities after GFC due to uncertainty over global economic growth
- Conventional markets dynamics does not have such a strong realisation within the real economy
- As industrial sector strives towards renewable energy, traditional energy firms change stucturally and require correct co-dependence analysis

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Revisiting dependence

- In financial econometrics, we utilize a vast array of financial models to measure dependence
  - Basic ARIMA models for the mean equation
  - GARCH extensions to deal with heteroscedasticity
  - Multivariate GARCH models that deal with dependence modeling
- Theoretical problem arises when we talk about **dependence**
  - Capturing co-movement between financial asset returns with linear correlation has been the staple approach in modern finance since the birth of Harry Markowitz's portfolio theory
  - But linear correlation is only appropriate when the dependence structure (or joint distribution) follow a normal distribution

Goal

- To introduce to you an extension in the field of dependence measurement (contagion)
- Grasp basic concepts and generators within the field of copulas
  - Learn to walk, before we can run
  - Revisit your statistics
- Understand the field of copulas to such an extent that you might go on to do a PhD in this field ;-)

Fields where copulas are applied

- Quantitative finance
  - Dependence modeling
  - "Downside/crisis/panic regimes" where extreme downside events are important
  - Pool of asset evaluation
  - Latest development: Vine Copulas
  - Hot research page here
- Civil engineering
- Warranty data analysis
- Medicine

Introduction to copulas

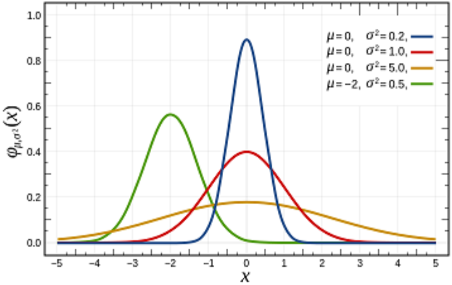
- Copula stems from the latin verb copulare; bond or tie.
  - Regulated financial institutions are under pressure to build robust internal models to account for risk exposure
  - Fundamental ideology of these internal models rely on joint dependency among whole basket of mixed instruments
  - This issue can be addresed through the copula instrument
  - It functions as a linking mechanism between uniform marginals through the inverse cdf
- Copula theory was first developed by Sklar in 1959 Nelsen (2007).

Introduction to copulas (Sklar)

- Sklar's theorem forms the basis for copula models as:
  - It does not require identical marginal distributions and allows for n-dimensional expansion, allowing for the estimation of the marginals and joint distribution to happen seperately
- Let X be a random variable with marginal cumulative distribution function:
  - $F_X(x) = \mathcal{P}(X \leq x)$
  - If we now denote the inverse CDF (Quantile funtion) as F_x^{-1}
  - For a given probability in the probability distribution of a random variable, the value at which the probability of the random variable is less than or equal to the given probability.

Transformations

- PDF
- CDF
- CDF^{-1}



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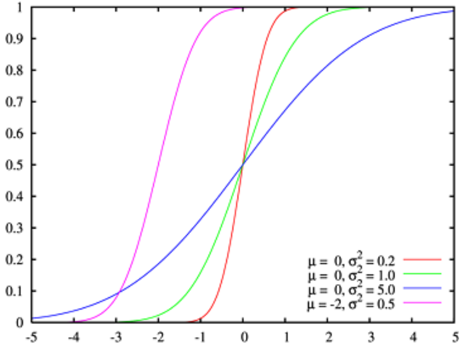

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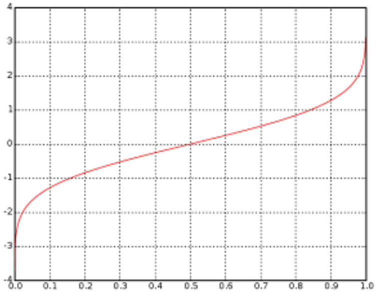


Definitions and basic properties (cont)

Definition (Copula): A d-dimensional copula is the distrubutiom function $\mathcal{C}$ of a random vector U whose components U_k are uniformly distributed

$$C(u_1, \dots, u_d) = P(U_1 \leq u_1, \dots, U_d \leq u_d), (u_1, \dots, u_d) \in (0,1)^d \tag{1}$$

Thus Sklar's theorem states:

$$\begin{aligned} C(F_1(x_1), \dots, F_d(x_d)) &= P(U_1 \leq F_1(x_1), \dots, U_d \leq F_d(x_d)) \\ &= P(F_1^{-1}(U_1) \leq x_1, \dots, F_d^{-1}(U_d) \leq x_d) \\ &= F(x_1, \dots, x_d) \end{aligned} \tag{2}$$

Joint distribution function:

- This represents the joint distribution function F can be expressed in terms of a copula C and the marginal distribution $(F_1, \ldots, F_d)$. Modeling them seperately
- **Easy Def:** A Copula is a function that couples the joint distribution function to its univariate marginal distribution
- For copulas, we use Kendall's Tau - non-linear concordance measure

Coming back to the energy markets

	Universe	Unique Stocks
1	AUS	45
2	CAN	114
3	JALSHAII	8
4	KOSPI	7
5	SP500	64
6	SPEURO	10
7	TAIEX	13
8	UK	12

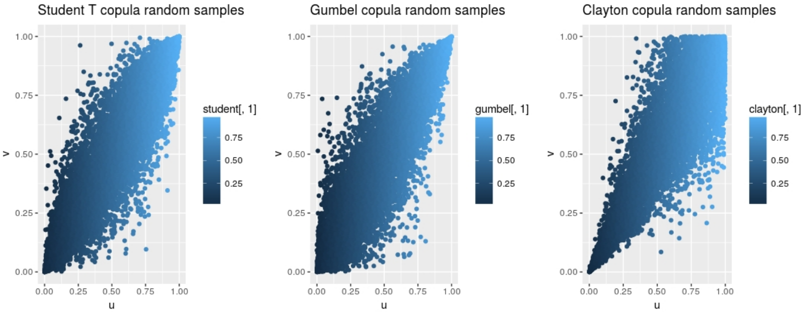
Estimation of the marginals

It was Bollerslev (1986) who proposed a generalization of Engle (1982)'s ARCH(q) model, which resulted in a more flexible lag structure that controls for autocorrelation persistence:

$$h_t = \omega_0 + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} + \epsilon_t \tag{3}$$

with h_t denoting the conditional variance, ω the intercept, α_j the news effect, β_j the momentum carried forward from past variance and ε_t the residuals. Using the standardised residuals we can now fit a copula between a bivariate pair

What does this visually look like?



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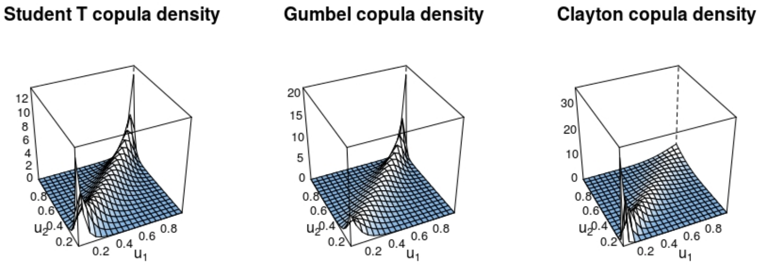


Vine-Copulas

- A vine is a graphical tool for labeling constraints in high-dimensional probability distributions
- Regular Vines from part of what is know as pair copula construction
- Trees are constructed between copulas based on what is know as maximum spanning degree (Or concordance measure)
- Under suitable differentiability conditions, any multivariate density $F_{1 \ldots n}$ on n variables may be represented in closed form as a product of univariate densities and (conditional) copula densities on any R-vine V

Vine copulas kurowicka2006

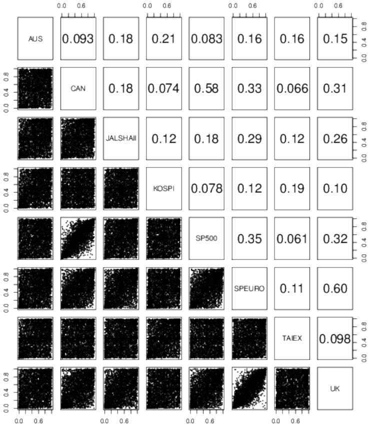
The R-vine copula density is uniquely identified according to

Theorom 4.2 of @kurowicka2006:

$$c(F_1(x_1), \cdots, F_d(x_d)) = \prod_{i=1}^{d-1} \prod_{e \in E_i} c_{j(e),k(e)|D(e)} \left(F(x_{j(e)}|\mathbf{x}_{D(e)}) \right) \tag{4}$$

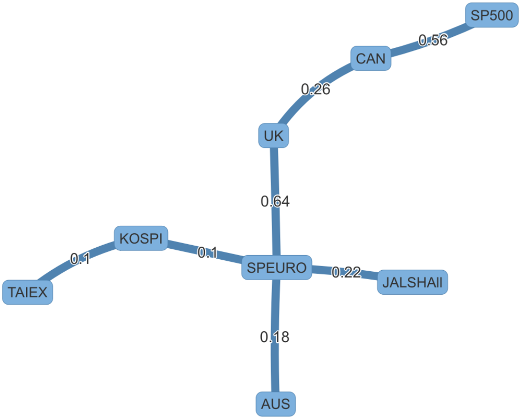
- Introduction to VineCopula
- Website for the research here

A look into energy market depenendence using Vine Copula



Energy market through Vine Visualization

- Pre - GFC
- GFC
- Post - GFC



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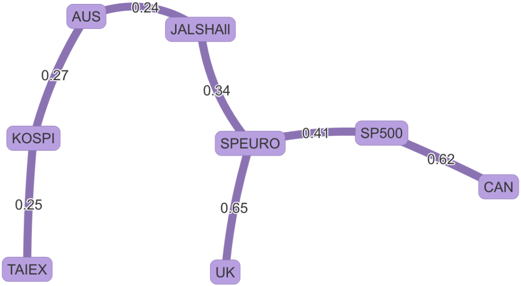

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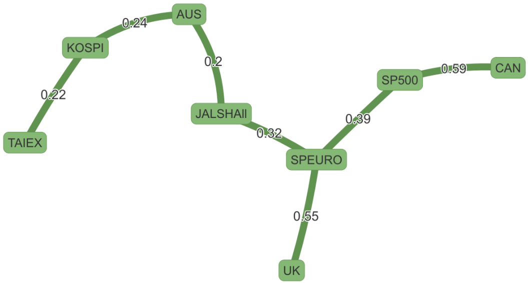
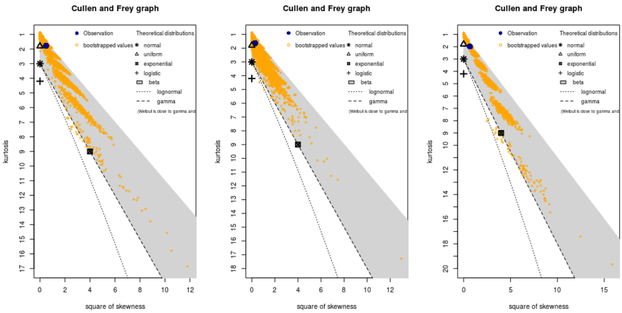


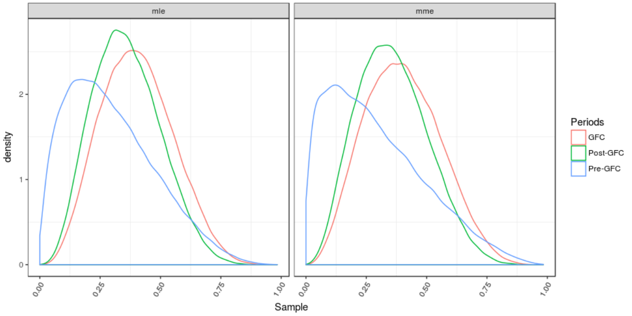
In depth look at the copula estimation

	Edges	Family	Par	Par2	τ	λ_U	λ_L
Pre-GFC	CAN - SP500	t	0.77	6.43	0.56	0.35	0.35
	SPEURO - JALSHAII	t	0.34	6.69	0.22	0.09	0.09
	KOSPI - TAIEX	SG	1.12	0.00	0.10	-	0.14
	SPEURO - AUS	N	0.28	0.00	0.18	-	-
	SPEURO - KOSPI	C	0.23	0.00	0.10	-	0.05
	UK - CAN	t	0.39	22.22	0.26	-	-
	UK - SPEURO	t	0.84	17.16	0.64	0.23	0.23
GFC	KOSPI - TAIEX	SG	1.34	0.00	0.25	-	0.32
	AUS - KOSPI	SG	1.38	0.00	0.27	-	0.34
	SP500 - CAN	t	0.83	4.79	0.62	0.49	0.49
	JALSHAII - AUS	N	0.36	0.00	0.24	-	-
	SPEURO - JALSHAII	N	0.51	0.00	0.34	-	-
	SPEURO - SP500	t	0.60	6.30	0.41	0.21	0.21
	UK - SPEURO	t	0.86	4.84	0.65	0.53	0.53
Post-GFC	KOSPI - TAIEX	t	0.34	14.62	0.22	0.01	0.01
	AUS - KOSPI	t	0.38	17.47	0.24	0.01	0.01
	JALSHAII - AUS	t	0.31	18.91	0.20	-	-
	SP500 - CAN	t	0.80	5.67	0.59	0.42	0.42
	SPEURO - JALSHAII	t	0.48	9.52	0.32	0.08	0.08
	SPEURO - SP500	t	0.58	9.88	0.39	0.12	0.12
	UK - SPEURO	t	0.76	5.64	0.55	0.38	0.38

Quantifying dynamic dependence



Quantifying dynamic dependence



Final results

	hypothesis	fit Type	estimate	statistic	p.value	conf.low	conf.high	alternative
1	Pre-GFC/GFC	MLE	-0.11	1296773158.00	1.00	-0.11	Inf	greater
2	Pre-GFC/GFC	MME	-0.11	1360073661.00	1.00	-0.11	Inf	greater
3	Pre-GFC/Post-GFC	MLE	-0.07	1718227541.00	1.00	-0.07	Inf	greater
4	Pre-GFC/Post-GFC	MME	-0.07	1727208393.00	1.00	-0.07	Inf	greater

Table : Mann-Whitey location test results

Conclusion

- Increase in dependence among the energy markets post-GFC
  - Rvine - estimation helps to capture the complex dynamics through a felxible estimation procedure
  - Tail dependence as an indication of contagion is very prevelent
  - Structurely student T copulas are preferred (symmetric tail)
- Opens the doors to practitioners (such as risk managers), to be better equiped in dealing with modern day finance
- Exchanges with a large energy sector constituency are highly exposed to fluctuations - especially as commodity prices look to remain volatile in the immediate future

Interesting afterthoughts

- How would a 8-dimensional Student T copula fit to the data?
- What would happen if the vine had free reign to choose from all 40+ copulas?
- Is there scope for option pricing in the one tailed distribution?

Nelsen, Roger B. 2007. *An Introduction to Copulas*. Springer Science & Business Media.