The financial reaction function and the financial sustainability of South African households

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Abstract: Between 2002 and 2009 South African household debt as percentage of household disposable income increased from just over 50% to approximately 80%, while the household saving rate turned negative. These two developments raise the question: How sustainable is the financial position of South African households? To study the financial behaviour of households usually entails studying consumption or saving behaviour and establishing whether or not that behaviour fits with the postulates of the Permanent Income Hypothesis and the Life-cycle model. This paper is related to that approach but does so by drawing on fiscal reaction function literature initiated by Bohn (1995, 1998, 2007). Specifically, the paper specifies and estimates a saving reaction function for the South African household sector. The results point to households acting in a financially sustainable manner over the period 1969-2009. Nevertheless, the variance decomposition analysis as well as the Granger causality analysis does indicate that causality between the household debt/disposable income and house price/disposable income ratios might be bidirectional, so that changes in these two variables might be mutually re-enforcing, allowing for a debt-asset price spiral. However, given that households react sufficiently to changes in their debt and asset position implies that such spirals might themselves not be sustainable and fizzle out.

Key words: saving, household debt, house prices, reaction functions, Permanent Income Hypothesis and Life-cycle model

JEL code: E21,

1) Introduction

Between 2002 and 2009 household debt as percentage of household disposable income increased from just over 50% to approximately 80%. During the same period the household saving rate (i.e. the household saving/household disposable income ratio) turned negative. These two developments raise the question: How sustainable is the financial position of South African households? To study the financial behaviour of households usually entails studying consumption or saving behaviour and establishing whether or not that behaviour fits with the postulates of the Permanent Income Hypothesis and the Life-cycle model.[2]

Studies on South African household saving behaviour are few and far between. This is quite surprising for a country that experienced a significant drop in its household saving rate. Since 1999 research on saving in South Africa has been done mainly by Aron, Muellbauer and Prinsloo (2006a, 2006b, 2007), Aron and Muellbauer (1999, 2000, 2006), Muellbauer (2007), Prinsloo

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[2] For instance, see Muellbauer (1994) for a general discussion, and Aron and Muellbauer (2000) and Harjes and Ricci (2005) for South Africa, Aron, Muellbauer and Murphy (2006) for South Africa and the UK; also see Lettau and Ludvigson (2001) and Sousa (2010) for the UK and US.

(2000, 2002), Aron, Muellbauer and Murphy (2006), as well as Harjes and Ricci (2005).[3] Most of these studies focus on the balance sheet composition of South African households, the determinants of saving and the impact of financial liberalisation on saving in the South African context.

However, none of these approaches allow one to ascertain directly whether or not the financial behaviour of households is sustainable. Sustainable financial behaviour is a necessary, though not a sufficient condition for the empirical validity of the Permanent Income Hypothesis and the Life-cycle model.[4] To ascertain whether or not the financial position of households is sustainable the analysis in this paper estimates a *household saving reaction function*. In doing so the paper draws on the fiscal reaction function literature initiated by Bohn (1995, 1998, 2007). A fiscal reaction function is derived from the government budget constraint and measures the reaction of the government primary balance/GDP ratio to changes in the public debt/GDP ratio. A primary balance can also be calculated for the household sector. In addition, households own financial assets and have liabilities. Thus, in principle it is possible to estimate a household reaction function for the household sector akin to that of government that measures the reaction of the household primary balance/household disposable income ratio to changes in the net financial asset/household disposable income ratio. In South Africa, though, there is too little data to estimate this relationship.

However, as will be shown below, by subtracting net interest payments from both sides of the household reaction function, one obtains the household saving rate on the left-hand side and therefore an equation that measures the reaction of the household saving rate to changes in the household net financial asset/household disposable income ratio. This is the *household saving reaction function* and it will be used to ascertain whether or not the position of South African households is financially sustainable.

However, before estimating a household saving reaction function, one needs to establish the order of integration of the data. The Life-cycle model postulates that household consumption behaviour depends on life-time income as well as the assets with which a household is endowed – i.e. households act to maximise their life-time utility within their intertemporal budget constraint, which usually entails the smoothing of consumption over time. Therefore, in the Life-cycle model, household saving changes to facilitate the smoothing of consumption. With household consumption, and thus also saving, dependent on household disposable income in the long run (i.e. saving and disposable income and consumption and disposable income are expected to be cointegrated), it is fairly well accepted that the household consumption/disposable income ratio and the saving rate should be stationary variables.

However, testing consumption/disposable income ratios and saving rates to establish their orders of integration very often show that these variables are non-stationary. The same is true for South Africa, as the discussion below will show. When a non-stationary series is found, several authors explore whether such a series might not be stationary if one allows for one or

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[3] For studies before 2000, see the summary by Aron and Muellbauer (2000:512) who list the rather limited number of paper that appeared (four in total).

[4] Sustainability implies that households act within their intertemporal budget constraints (i.e. they are solvent). Households who act according to the Permanent Income Hypothesis and the Life-cycle model are financially sustainable and solvent, however, it is also possible for households to be financially sustainable and solvent while they are simultaneously not smoothing their consumption.

more structural breaks (*cf.* Sarantis and Steward 1999, Cook 2005, Romero-Ávila 2009). Therefore, if one removes the effect of the breaks, the series become stationary and one can perform normal regression analysis designed for stationary data. However, in the limit one could also argue that a random walk (i.e. non-stationary) process is 'a stationary series with a break in each and every period' (where the breaks behave stochastically with a normal distribution).[5] It therefore stands to be argued that most time series will be rendered stationary or at least trend stationary if one allows for enough breaks (the limiting case then being a break in every period, as represented by a stochastic trend). This paper, though, follows a different approach and shows in the next section that a non-stationary saving rate and net financial asset/income ratio can exist even if households act in accordance with the postulates of the Permanent Income Hypothesis and the Life-cycle model. With these ratios being non-stationary there is no need to detrend the data and one can apply cointegration analysis to the data, which allows for the long-run properties of the data not to get lost in the estimation process that follows.

2) Stationarity and household consumption, saving, wealth and net financial assets

One well-known prediction of the Permanent Income Hypothesis and the Life-cycle model is that households will attempt to smooth consumption over time (*cf.* Muellbauer 1994:8). Consumption is therefore not a function of just current income, but of lifetime wealth, defined as the present value of interest and non-interest income (i.e. income on human and non-human assets). Therefore (Muellbauer 1994):

$$C_t = W_t/\kappa_t \qquad (1)$$

where C: consumption, W: wealth (human and financial assets, so that $W = H + F$), $\kappa = 1+1/(1+i)$, where i is the nominal interest rate. In addition, if income is a random walk, then shocks to income are not transitory and changes in income, following the Permanent Income Hypothesis, will translate into changes in consumption. This yields the prediction by Hall (1978) that consumption is a random walk that shares the same shocks as income:

[5] Consider a stochastic trend in the form of a random-walk-with-drift process is: $U_t = a + U_{t-1} + \varepsilon_t$, which, if $U_0 = 0$, solves as $U_t = at + \Sigma\varepsilon_{t-i}$. Furthermore, $\Delta U_t = a + \varepsilon_t$. One can now specify Y as this stochastic trend process plus an additional stationary (white noise) process, so that $Y_t = U_t + \xi_t$. Therefore:

$$Y_t = at + \Sigma\varepsilon_{t-i} + \xi_t \qquad (1n).$$

A deterministic trend is: $Y_t = at + \xi_t$ (the first difference of which is $\Delta Y_t = a + \xi_t - \xi_{t-1}$). Should there be a shift in the deterministic trend, one gets: $Y_t = at + e_n d^P_{t-n} + \xi_t$, where e_n is the shift parameter and d^P_{t-n} is a permanent shift dummy taking the value 1 from t-n periods back. Should a break occur every period, then:

$$Y_t = at + e_0 d^P_t + e_1 d^P_{t-1} \ldots + e_{t-1} d^P_2 + \xi_t \qquad (2n)$$

If e_0 to e_{t-1} are drawn from a white noise process, then $e_0 d^P_t + e_1 d^P_{t-1} \ldots + e_{t-1} d^P_2$ is equivalent to $\Sigma\varepsilon_{t-l}$. Substituting $\Sigma\varepsilon_{t-l}$ into (2n) yields:

$$Y_t = at + \Sigma\varepsilon_{t-l} + \xi_t \qquad (3n)$$

Which is the same as (1n).

$$C_t = C_{t-1} + \varepsilon_t \quad (2)$$

where ε represents a white-noise shock. With both *C* and *Y* being random walks that share the same shocks, the ratio of *C* and *Y* will be stationary. This also represents a steady-state situation. Furthermore, in a steady state the W/Y ratio will also remain stationary. In addition, given the household budget constraint, saving can be related to consumption (Aron and Muellbauer 2000:516):

$$(S/Y)_t \approx -ln(1-(S/Y)_t) = -ln(C/Y)_t \quad (3)$$

Therefore, if (C/Y) is stationary, than (S/Y) will also be stationary. Another way of putting this: all these variables share the same underlying data generating process, hence their ratios are stationary.

However, stationarity tests performed on actual data indicate that these ratios very often are not stationary (household saving rates in South Africa and in countries such as the US declined sharply over the past three to four decades). Do these test results mean that the Permanent Income Hypothesis and the Life-cycle model do not hold? This section shows that a non-stationary saving rate does not necessarily imply the *in*validity of the Permanent Income Hypothesis and the Life-cycle model. Rather, the section shows that changes in preferences and other factors may cause non-stationarity due to frequent and persistent adjustments to the steady state, *even if the Permanent Income Hypothesis and the Life-cycle model hold*. If it can be shown that if the saving rate is non-stationary the Permanent Income Hypothesis and the Life-cycle model may still hold, it will also mean their policy implications continue to hold. The non-stationarity of the series then does not need to inspire attempts to save the Permanent Income Hypothesis and the Life-cycle model by rendering the data stationary through the removal of the effects of breaks in the data.

South Africa in the 1980s saw gradual financial market liberalisation, while the 1990s and 2000s saw further liberalisation as well as many new financial market entrants, who were previously denied access or earned too little income to enter the financial markets. Such events may affect the underlying data generating process of saving and net financial asset formation, but not that of income, thus calling for an augmentation of the underlying data generating process shared by wealth and saving, but not income.

To put this in more formal terms, consider the relationship between net financial assets and income. If F/Y is in a steady state, then, in the long run $E(\Delta(F/Y))=0$. A first note of interest is that theoretical discussions (as was done for Equations (1) to (3)) often abstract from the empirical finding that it is not *C*, *S*, *F* and *Y* that are random walks, but ln*C*, ln*S*, ln*F* and ln*Y*.

Suppose a steady state holds where households set their saving, S_t, as a fraction of the lag of their net financial assets, F_t, so that $S_t = \varphi F_{t-1}$. Hence, with household saving being the change in the net financial assets of households: $F_t = (\varphi)F_{t-1} + F_{t-1} = (1+\varphi)F_{t-1}$ so that: $F_{t-1} = (1/(1+\varphi))F_t$. Therefore:

$$S_t = (\varphi/(1+\varphi))F_t = \chi F_t \quad (4)$$

where $\chi = (\varphi/(1+\varphi))$

Now suppose in a steady state F_t is set as a ratio, ψ, of disposable income, Y_t (thus, saving will then indirectly also be a fraction of Y_t). Thus $F_t = \psi Y_t$, so that $S_t = \chi\psi Y_t$. If ln$Y$ is an I(1) process with a stochastic trend, then $lnY_t = \mu_{Yt} + \varepsilon_{Yt}$, where μ_{Yt} represents the stochastic trend, so that $\mu_{Yt} = \mu_{Yt-1} + a + \xi_{Yt}$. Thus, with $Y_t = e^{\mu_{Yt} + \varepsilon_{Yt}}$ and $F_t = \psi(e^{\mu_{Yt} + \varepsilon_{Yt}})$, $S_t = \chi\psi(e^{\mu_{Yt} + \varepsilon_{Yt}})$. One could then render the F_t/Y_t and S_t/Y_t ratios stochastic by including white noise terms, ζ_t and η_t:

$$F_t/Y_t = \psi + \zeta_t \qquad (5)$$

$$S_t/Y_t = \chi(\psi + \zeta_t) + \eta_t \qquad (6)$$

Household financial positions will be sustainable, while the saving/disposable income and net financial asset/disposable income ratios will be stationary. The stationarity of the two ratios is an indication of household financial sustainability. However, if households do not set F_t and S_t as ratios of disposable income, then:

$$F_t/Y_t = e^{\mu_{Ft} + \varepsilon_{Ft} - \mu_{Yt} - \varepsilon_{Yt}} \quad \text{and} \quad S_t/Y_t = e^{\mu_{St} + \varepsilon_{St} - \mu_{Yt} - \varepsilon_{Yt}} \qquad (7)$$

These ratio will be non-stationary and they will change exponentially so that differencing them (once or more) will not yield them stationary. (The log of the ratio will be an I(1) variable). Therefore, if the ratios cannot be rendered difference stationary, their numerator and denominator variables do not share a stochastic trend and are, therefore, not cointegrated.

Nevertheless, there is a third possibility, where the ratios are non-stationary, but difference stationary (i.e. after differencing it once or more). If the white noise term, ζ_t, is replaced by a stochastic trend in Equation (5). The stochastic trend represents changes in the preferences of households with regard to the level of net financial assets that they desire relative to their disposable income. Adding this stochastic trend turns the net financial asset/disposable income and saving/disposable income ratios into non-stationary variables:

$$F_t/Y_t = \psi + \mu_{Ft} \qquad (8)$$

$$S_t/Y_t = \chi(\psi + \mu_{Ft}) + \eta_t \qquad (9)$$

where:

$$\mu_{Ft} = \mu_{Ft-1} + \xi_{Ft} + \Sigma\delta\gamma_{Ft-i} \qquad (10)$$

where ξ and γ are white noise processes, $\delta=1/i$ so that $\delta i=1$, meaning $\Sigma\delta\gamma_{Ft-i}$ is a shock spread out equally over i number of years and then disappears. Then $F_t = (\psi + \mu_{Ft})Y_t$ and $S_t = \chi(\psi + \mu_{Ft})Y_t + \eta_t Y_t$.

Where would this stochastic trend, μ_{St}, come from? A number of possibilities exist. For instance, it was noted above that financial market liberalisation and the entrance of new households into the financial system has been an ongoing process since the 1980s in South Africa. Aron and Muellbauer (2000) and Prinsloo (2000) already noted these changes a decade ago. Furthermore, there might also be demographic factors that interact with differences in preferences and rates of access to financial markets between younger and older generations.

For instance, an older generation may have a lower discount rate than the younger generation when they decide between consumption now and consumption tomorrow, or because of lower income levels in the past they may have had less access to financial markets, which made them more dependent on own saving. However, these demographic changes occur very slowly and may be reflected in a drift of *C/Y*, *S/Y* and *F/Y* for a number of years as older households slowly fall away and younger households are added. In Equation (10) such a slow adjustment is represented by $\Sigma\delta\gamma_{St-i}$, a term that spreads out the effect of a shock over a number of years and, as such, adds a temporary drift term to equation (8). The effect of other shocks, that do not take as long to dissipate, is represented by ξ_{St} in Equation (10).

With μ_{Ft} Equations (8) and (9) become random walks, i.e. *F/Y* and *S/Y* are non-stationary processes even though households also set their saving and net financial assets as ratios of disposable income. What does the stochastic trend, μ_{Ft}, mean? It measures the extent to which households adjusts the level of their net financial assets relative to their disposable income. This is why μ_{Ft} in $F_t = (\psi + \mu_{Ft})Y_t$ acts as a stochastic (state-space) parameter. However, when expressing F_t as ratio of Y_t, as in Equation (8), it becomes a stochastic mean for the ratio. The same applies to the household saving/disposable income ratio. Because μ_{Ft} is difference stationary, *F/Y* and *S/Y* are also difference stationary. Furthermore, because S_t is set as a ratio of F_t, *F/Y* and *S/Y* will be cointegrated. Therefore, one can use the long-run properties of the data in a cointegrated framework and there is no need to detrend the data to clean it from possible structural breaks before one uses it in regression analysis.

Therefore, if *F/Y* and *S/Y* can be rendered difference stationary, it serves as an indicator that households set their net financial assets relative to their income, which implies that the Permanent Income Hypothesis and the Life-cycle model holds. However, because of the weak power of stationarity tests, *F/Y* and *S/Y* may turn up as I(1) in limited samples even if households do not act in accordance with the Permanent Income Hypothesis and the Life-cycle model. Thus, to establish whether or not *F/Y* and *S/Y* are cointegrated and what the nature of the cointegrated relationship is, will lend further support to a finding that households act in accordance with the Permanent Income Hypothesis and the Life-cycle model. Therefore, if *F/Y* and *S/Y* are I(1) variables and they are cointegrated, the financial position of households can be said to be sustainable and in accordance with the Permanent Income Hypothesis and the Life-cycle model.

3) Deriving the household reaction functions

The household reaction function is derived in analogy to the fiscal reaction function. A fiscal reaction function usually specifies the reaction of the primary balance (i.e. the budget balance excluding interest payments), expressed as a ratio of GDP, to changes in the one-period lagged public debt/GDP ratio (*cf.* Bohn 2007, 1998). The fiscal reaction function is a behavioural function where government's behaviour is informed by its budget constraint and the budget identity (*cf.* Bohn 1995, 1998, 2007, Gali and Perotti 2003, De Mello 2005). Similarly a household reaction function will be a behavioural function where household behaviour is informed by the household budget constraint and budget identity. The budget identity of government is specified as:

$$F_t = F_{t-1} + iF_{t-1} + B_t \qquad (11)$$

Where F: Net financial asset position of a household, i: Nominal interest rate (including the dividend rate and rent rate) and B: Primary balance of households (+ surplus; - deficit), defined as the difference between non-interest income and expenditure of households. Equation (11) can now be used to derive Equation (12):

$$\Delta(F/Y)_t = ((i_t - n_t)/(1 + n_t))(F/Y)_{t-1} + (B/Y)_t \qquad (12)$$

Where i: Nominal interest rate, n: Nominal growth rate of disposable income and Y: Nominal disposable income of households. Equation (12), in turn, can be used to derive the saving rate required to ensure that the net financial asset position of the household sector remains unchanged:

$$(S/Y)_t^{Required} = (B/Y)_t + (i_t/(1 + n_t))(F/Y)_{t-1} = (n_t/(1 + n_t))(F/Y)_{t-1} = \gamma^*(F/Y)_{t-1} \qquad (13)$$

Where S represents household saving and γ^* equals $n_t/(1 + n_t)$. This is a '*required* household sector saving reaction function', *required* in a steady state. Given that the 'required household sector reaction function' is derived from the budget constraint (Equation (11)), it is not unreasonable to assume that the household sector (and thus households in the aggregate and on average) use Equation (13) to inform their actual behaviour. To establish the actual behaviour of the household sector, and thus, what values γ took, one can estimate a reaction function of the form:

$$(S/Y)_t^{Actual} = \gamma(F/Y)_{t-1} + \varepsilon_{st} \qquad (14)$$

To allow for inertia in household sector behaviour and thus some smoothing over time, a lag of the primary balance/disposable income ratio or the saving rate is added to Equation (14). The output gap, $\hat{y}$, is also added to allow consumption smoothing over the business cycle, i.e. in boom times one would expect the saving rate to increase – hence its parameter is expected to be positive. If households are credit constrained, one would expect the parameter to be negative. A negative parameter within the long-run relationship indicates that the Life-cycle model holds (i.e. households smooth consumption over their lifetimes), though, since households may find it difficult to smooth their consumption in the shorter term (i.e. a business cycle), the Permanent Income Hypothesis might not hold in the shorter term. Furthermore, if households interpret all changes in income as changes in permanent income, one would expect the parameter to be zero. Lastly, a constant, γ_1 is added to Equation (14) to allow for an explicit or implicit net financial asset/disposable income or saving/disposable income target that may not equal zero. However, if the Life-cycle model holds, one would expect this parameter to equal zero. The basic *estimated* household sector saving reaction function is then specified as:

$$(S/Y)_t^{Actual} = \gamma_1 + \gamma_2(S/Y)_{t-1}^{Actual} + \gamma_3(F/Y)_{t-1} + \gamma_5(\hat{y})_{t-1} \qquad (15)$$

Net financial assets is the difference between financial assets, A, and debt, D, so that F in Equations (15) can be replaced by $(A - D)$:

$$(S/Y)_t^{Actual} = \gamma_1 + \gamma_2(S/Y)_{t-1}^{Actual} + \gamma_3(A/Y)_{t-1} - \gamma_4(D/Y)_{t-1} + \gamma_5(\hat{y})_{t-1} \qquad (16)$$

Equation (16) is the household sector saving reaction function estimated below.

4) Data used

The annual data used runs from 1969-2009. The paper uses the household saving/household disposable income ratio and the household debt/household disposable income ratio (Figures 1 and 2). For the output gap the paper estimates an output gap with a Kalman filter.[6]

Figure 1 - Household saving to household disposable income

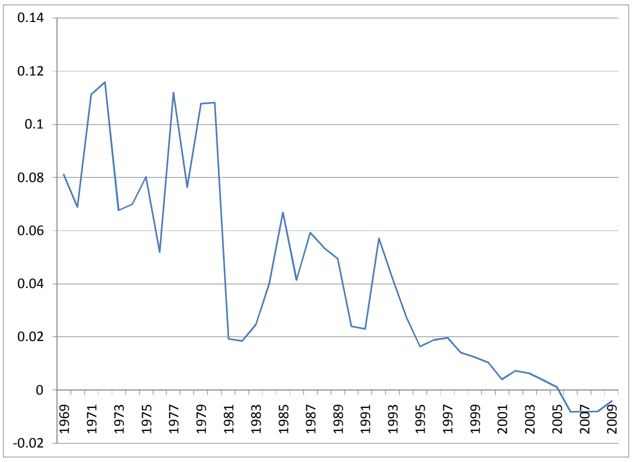
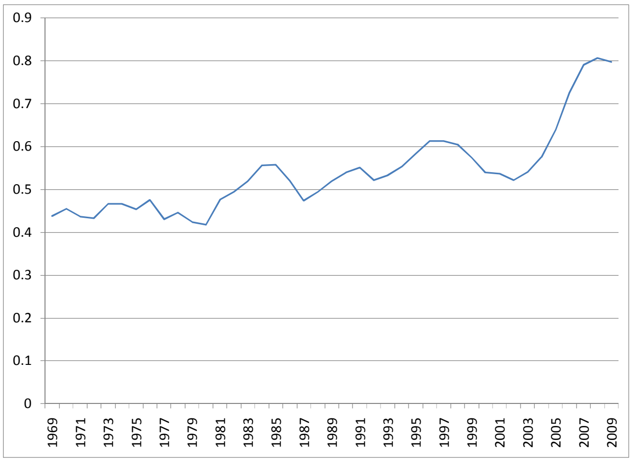


Figure 2 - Household debt to household disposable income



For household assets no aggregate data is available. However, given that for many households one of their most important assets is their house, the paper uses the ABSA house price index (Middle Group: Middle class houses - Total RSA: New & Old - All sizes - Purchase Price - Smoothed) divided by nominal per capita household disposable income. However, this house price series represents formal, middle class housing and one can therefore not argue that even though it is an average price, it is the average price of houses in the country.[7] Indeed, dividing the house price series by per capita household disposable income yields a ratio that varies between 18 and 40. However, assuming that the distribution (though not the level) of the house price/disposable income ratio is the same for everyone and that every household on average owns a house, one can use the distribution of the house price/disposable income ratio after

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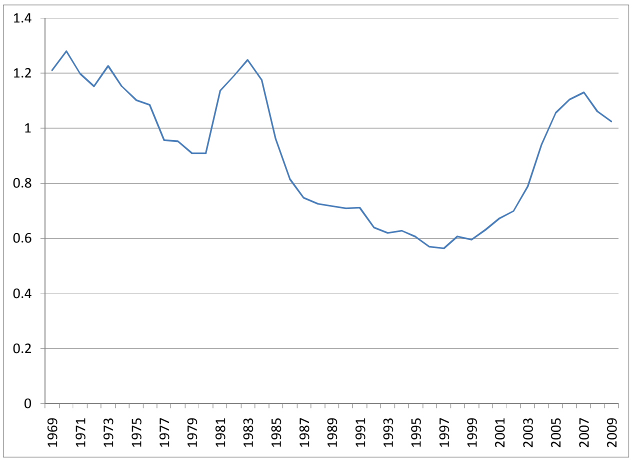
[6] Signal extraction using a Kalman filter allows for separating out an unobservable component from an observable component that contains noise. The model is $x_t = E_t x_t + \varepsilon$ and $E_t x_t = E_{t-1} x_{t-1} + v$ where x_t and $E_t x_t$ are respectively the observable variable and unobservable component (*cf.* Valente 2003:526).

[7] ABSA also has an 'affordable house' price series but it still represents middle class housing.

scaling the ratio to its proper value. To do this the analysis draws on Aron, Muellbauer and Prinsloo (2007).

Figure 3 - House prices to household disposable income



Using unpublished data from the South African Reserve Bank, ABSA, and Rode and associates that are not available in the public domain, as well as published data from Stats SA and the ABSA house price series (Aron *et al.* 2007:58-9), Aron *et al*. (2007:35) created a housing wealth/personal disposable income ratio series that peaks in the early 1980s at approximately 1.3. After that it fell, before increasing again since 2000.[8] The house price/disposable income ratio calculated above is scaled to that of Aron *et al*. (2007:61) using the 2005 value for their 'Residential buildings (including land)/personal disposable income' series. The distribution (and thus graph) of the Aron *et al*. (2007:35) series looks extremely similar to that of the house price/disposable income ratio calculated above. Table 1 reports the values of every fifth year reported by Aron *et al*. (2007:61), together with the scaled version of the house price/disposable income ratio, scaled to that of Aron *et al*. (2007:61) using their 2005 value.

Table 1 – House price/disposable income ratio

	Aron et al (2007)	Own calculations
1975	1.10	1.10
1980	0.97	0.91
1985	0.96	0.96
1990	0.75	0.71
1995	0.61	0.61
2000	0.62	0.63
2005	1.06	1.06

5) Estimation method

Because, as will be shown, all the time series except the output gap is I(1), Equation (16) cannot be simply estimated with, say, OLS or GMM. As a result, to allow for the variables to be non-stationary, the paper presents Vector Error-correction Models (VECM) estimated below using the Johansen procedure as well as an Autoregressive Distributed Lag (ARDL) model. However,

[8] Unfortunately Aron *et al*. (2007) only provide a graph of the full series and did not publish the data for the full series. They published only the data for every fifth year since 1975 (see Table 1).

when estimating a VECM one cannot estimate Equation (16) directly. Rather, the VECM takes the form of Equations (17):

$$\Delta X_t = \Pi X_{t-1} + \sum_{t=i}^{k} \Gamma_i \Delta X_{t-i} + \varepsilon_{kt} \qquad (17)$$

where $X_t = (S/Y, A/Y, D/Y, Ygap)$ is a 4x1 vector that includes the endogenous I(1) variables[9]), Γ_i is 4x4 short-run coefficient matrix and ε_{kt} are normally and independently distributed error terms.[10] No intercept is included as it was statistically insignificant in all models run with an intercept. However, as seen above, this is in line with *a priori* expectations. The trace and maximum likelihood tests are used to determine the number of cointegrated vectors. Π_i in Equation (17) can be decomposed as the following α and β' matrices:

$$\Pi X_{t-1} = \alpha\beta' X_{t-1} = \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \\ \alpha_{31} \\ \alpha_{41} \end{bmatrix} \begin{bmatrix} 1 & \beta_{21} & \beta_{31} & \beta_{41} \end{bmatrix} \begin{bmatrix} (S/Y)_{t-1} \\ (A/Y)_{t-1} \\ (D/Y)_{t-1} \\ (Ygap)_{t-1} \end{bmatrix} \qquad (18)$$

where α is a 4x1 matrix (four variables and one cointegrating relationship) that contains the error-correction (adjustment) parameters, and β' is a 1x4 matrix that contains the long-run parameters where:

$$(S/Y)_{t-1}^{Actual} = \beta_{21}(A/Y)_{t-1}^{Actual} + \beta_{31}(D/Y)_{t-1}^{Actual} + \beta_{41}(Ygap)_{t-1}^{Actual} \qquad (19)$$

is the long-run relationship. Equation (19) is the cointegrating relationship that can be interpreted as the long-run stance of the saving rate with respect to the asset/household disposable income and debt/household disposable income ratios, while the error correction term, α_{11}, in Equation (18) captures the reaction of *S/Y* to deviations from the long-run relationship as captured in Equation (19). Equation (17) (the VECM) can be used to retrieve Equation (16). This is done when the VECM is rewritten into its VAR format (this is done below).

6) Estimation results

The first step in the estimation process is to establish the order of integration of the data. Table 2 reports the results for the KPSS test. The KPSS test is a somewhat more robust test than the traditional ADF and PP tests. Note, however, that its null hypothesis is that a variable is stationary (unlike the null hypothesis of the ADF and PP tests, which is that a variable contains a unit root). The KPSS test shows that *S/Y*, *D/Y* and *A/Y* are I(1) variables at a 10% level, while the output gap is I(0). As the discussion above demonstrated, it is possible for *S/Y*, *D/Y* and *A/Y* to be I(1) while the Permanent Income Hypothesis and the Life-cycle model still hold true. The discussion also showed that it is possible for two variables to share a stochastic trend, though their ratio is still not stationary, as is the case with *S/Y*, *D/Y* and *A/Y*.

[9] The output gap, Y gap, is also included even though it is an I(0) variable. However, it is permissible to include I(0) variable in the long-term component of the model.

[10] In no model was the trend or intercept statistically significant. Thus, the paper presents the results for models without the trend and intercepts. In addition, the Johansen was also estimated without any exogenous variables.

Table 3 - KPSS test for stationarity

Variable	H_0 : D/Y is I(0) (Test statistic)	H_0 : D/Y is I(1) (Test statistic)
S/Y	0.750	0.147
D/Y	0.767	0.167
A/Y	0.360	0.166
Y GAP	0.225	

Critical values 1%, 5% and 10%: 0.739, 0.463 and 0.347.

Following the stationarity tests an unstructured VAR in levels is run and information criteria are used to establish the number of lags to include in the model. Table 4 shows that the optimal amount of lags is two.

Table 4 - VAR Lag Order Selection Criteria

Lag	LogL	LR	FPE	AIC	SC
0	87.346	NA	NA	1.46e-07	-4.387
1	214.269	220.447	220.447	4.29e-10	-10.225
2	249.874	54.344*	1.58e-10*	-11.257*	-9.705*
3	264.516	19.266	1.83e-10	-11.185	-8.944

* indicates lag order selected by the criterion
LogL: Log likelihood
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Next the Trace and the Maximum Eigenvalue tests are run to test for the presence of cointegration in a VAR (in levels) containing two lags, or, when specified in first differences, containing one lag. Both the Trace and Maximum-Eigenvalue tests indicate the presence of one cointegrating equation at the 1% level (see Table 5).

Table 5 - Cointegration tests (Trace and Maximum Eigenvalue tests)

Unrestricted Cointegration Rank Test (Trace)				
Hypothesized No. of CE(s)	**Eigenvalue**	**Trace Statistic**	**0.05 Critical Value**	**Prob.**
None *	0.543	51.419	40.175	0.003
At most 1	0.289	20.901	24.276	0.126
At most 2	0.142	7.619	12.321	0.268
At most 3	0.042	1.655	4.130	0.233
Unrestricted Cointegration Rank Test (Maximum Eigenvalue)				
Hypothesized No. of CE(s)	**Eigenvalue**	**Max-Eigen Statistic**	**0.05 Critical Value**	**Prob.**
None *	0.543	30.518	24.159	0.006
At most 1	0.289	13.282	17.797	0.210
At most 2	0.142	5.963	11.225	0.354
At most 3	0.042	1.655	4.130	0.233

* denotes rejection of the hypothesis at the 0.01 level

Table 6 subsequently reports the results for the VECM estimated with the Johansen method. Note that the cointegrating equation is stated in its vector format. Thus, the plus in front of *D/Y* and *Ygap* should be interpreted as a minus, while the minus in front of *A/Y* should be interpreted as a plus. The VECM is normalised on *S/Y*. What is notable from the VECM results are the very similar parameters for *D/Y* and *A/Y* in the long-run component of the model (while their parameters in the short-run dynamics are both statistically insignificant). This is an indication that households react to their *net* financial position ($A - D$) and not its individual components *D* and *A*.

The error correction term of *S/Y* is -0.111. It is negative and statistically significant, as is required for there to be a long-run relationship between the variables and for changes in *S/Y* to adjust to shocks in the long-run relationship. While the error-correction term for *D/Y* is statistically insignificant, those of *A/Y* and the output gap are statistically significant and have the right sign given their sign in the long-run relationship. Consistent with the statistical significance of the error correction terms of *S/Y*, *A/Y* and *Ygap*, and the statistical insignificant of the error correction term of *D/Y*, the weak exogeneity test indicates that the null hypothesis that *S/Y*, *A/Y* and *Ygap* are weakly exogenous, cannot be rejected, while the test fails to reject the null hypothesis that *D/Y* is weakly exogenous. With *S/Y* not being weakly exogenous indicates that one can normalise the long-run relationship on *S/Y*. The LM serial correlation test indicates no serial correlation at a 5% level.

Table 6 - VECM results – Johansen model

	Cointegrating Equation			
$(S/Y)_{t-1}$	1			
$(D/Y)_{t-1}$	0.255			
	[3.156]			
$(A/Y)_{t-1}$	-0.245			
	[-5.090]			
$(YGAP)_{t-1}$	4.751			
	[5.654]			
	Error Correction Equations			
	D(S/Y)	**D(D/Y)**	**D(A/Y)**	**D(YGAP)**
EC term	-0.111	0.023	0.355	-0.125
	[-2.647]	[0.491]	[3.316]	[-2.972]
$D(S/Y)_{t-1}$	-0.189	0.711	0.440	0.289
	[-0.979]	[3.350]	[0.891]	[1.485]
$D(D/Y)_{t-1}$	0.096	0.498	-1.074	-0.246
	[0.691]	[3.249]	[-3.011]	[-1.748]
$D(A/Y)_{t-1}$	-0.001	0.186	0.712	0.091
	[-0.019]	[2.823]	[4.654]	[1.517]
$D(YGAP)_{t-1}$	0.435	0.034	-1.197	0.183
	[2.388]	[0.167]	[-2.568]	[0.999]
Adj. R-squared	0.17	0.35	0.48	0.27
Weak exog test χ^2 (prob)	0.025	0.635	0.004	0.016
Instantaneous caus (prob)	0.000			
Serial corr LM (1)(prob)	0.08	**Serial corr LM (3)(prob)**		0.63
Serial corr LM (2)(prob)	0.36	**Serial corr LM (4)(prob)**		0.54
Variance Explained by:	**S/Y**	**D/Y**	**A/Y**	**Y GAP**
Variance Decomposition of:				
S/Y	31.38	16.03	0.87	51.72
D/Y	10.76	40.02	17.71	31.51
A/Y	0.60	16.84	37.67	44.89
Y GAP	9.37	8.26	7.38	74.99

Values in [ ] represent t values
H_0 for weak exogeneity test: Variable is not weakly exogenous
H_0 for instantaneous causality test: No instantaneous causality between "*D/Y*, *A/Y*, *Y GAP*" and "*S/Y*"
H_0 for serial correlation test: No serial correlation is present.
Cholesky ordering for the variance decomposition: *Y gap*, *A/Y*, *D/Y* and *S/Y*. Results pertain to variance decomposition after 10 periods.

The null hypothesis of no instantaneous causation running from *D/Y*, *A/Y* and *Ygap* combined, to *S/Y* can also be rejected. The variance decomposition shows that saving explains 31.38% of its own variance, while *D/Y* and *A/Y* together explain about 17% (with the contribution of *A/Y* being very small), with the output gap explaining 51.72%. What is also of interest is that the output gap explains a large proportion of the variation of the other three variables. Also note that *A/Y* explains 17.7% of the variation in *D/Y*, while *D/Y* explains 16.84% of the variation in

A/Y. These somewhat higher values that indicate that *D/Y* explains *A/Y*, and *vice versa*, can be expected, as a large part of credit extended to households is mortgage credit used to finance residential property. The parameters of the model are stable, as can be seen from the recursive estimation of the error correction terms and the eigenvalue as well as the result of the recursively estimated eigenvalue tau t-statistic where the estimated value lies well below the critical value (therefore one cannot reject the hypothesis that the eigenvalue is stable) (see appendix for Figures).

In addition to the above, the analysis also includes a VECM Granger causality test. Table 7 reports the results. It shows that in the short run and at a 5% level of significance, the output gap Granger causes *D/Y*. The relationship between *D/Y* and *A/Y* can also be observed as they both Granger cause each other. This bidirectional causation concurs with the results of the variance decomposition. Such bidirectional causation will also be a symptom of a system with a debt-asset-price spiral, where debt helps to augment demand for assets and thus cause the asset price to increase, while an increase in the asset price pushes up the collateral of a borrower, thereby allowing the borrower to borrow more. The significant increase in both household debt and house prices in the 2000s and the almost simultaneous halt in the increase of both would be in line with this type of bidirectional causation.

Table 7 - VECM Granger causality test (Probability)

Excluded	Dep variable: D(S/Y)	Excluded	Dep variable: D(A/Y)
D(D/Y)	0.490	**D(S/Y)**	0.373
D(A/Y)	0.985	**D(D/Y)**	0.003
D(YGAP)	0.017	**D(YGAP)**	0.010
All	0.113	**All**	0.000
	Dep variable: D(D/Y)		**Dep variable: D(YGAP)**
D(S/Y)	0.001	**D(S/Y)**	0.138
D(A/Y)	0.005	**D(D/Y)**	0.080
D(YGAP)	0.867	**D(A/Y)**	0.129
All	0.004	**All**	0.062

Following the above analysis, subset restrictions that eliminate statistically insignificant lags were also placed on the VECM to save degrees of freedom and to obtain estimates that possibly are more robust. As can be seen from Table 8 the results are substantially the same as those reported in Table 6. Both VECMs estimated with the Johansen technique will be used to calculate the VAR in levels that contains the saving reaction function of households.

Table 8 - VECM results – Johansen model – results with subset restrictions

	Cointegrating Equation			
$(S/Y)_{t-1}$	1			
$(D/Y)_{t-1}$	0.255 [3.381]			
$(A/Y)_{t-1}$	-0.245 [-5.452]			
$(YGAP)_{t-1}$	4.751 [6.056]			
	Error Correction Equations			
	D(S/Y)	**D(D/Y)**	**D(A/Y)**	**D(YGAP)**
EC term	-0.094 [-3.767]	0	0.326 [4.577]	-0.110 [-3.490]
$D(S/Y)_{t-1}$	-0.110 [-0.976]	0.591 [4.431]	0	0
$D(D/Y)_{t-1}$	0	0.570 [6.194]	-0.966 [-4.024]	-0.267 [-2.170]
$D(A/Y)_{t-1}$	0	0.179 [4.561]	0.670 [6.806]	0.056 [1.136]
$D(YGAP)_{t-1}$	0.471 [4.324]	0	-1.325 [-4.339]	0

Table 9 reports the VECM results estimated with the ARDL model for the relationship normalised on *d(S/Y)*. Its results are largely consistent with those of the VECM estimated with the Johansen technique. The estimation also does not suffer from serial correlation. Again the results for the long-run relationship should be read like a vector. Thus, the plus in front of *D/Y* and *Ygap* should be interpreted as a minus, while the minus in front of *A/Y* should be interpreted as a plus. The parameters of the long-run relationship (to be seen in the normalised coefficient column) are somewhat smaller than in the VECM estimated with Johansen, while the error correction term (read as the parameter on $(S/Y)_{t-1}$), is larger. However, the signs are all the same, while *D/Y* and *A/Y* are statistically significant in both estimations, with the output gap only being statistically significant in the VECM estimated with Johansen. Again, as with the VECM estimated with the Johansen technique, the parameters for *D/Y* and *A/Y* in the long-run relationship are virtually the same (while their parameters in the short-run dynamics are statistically insignificant). This is again an indication that households react to their *net* financial asset position (*A* – *D*) and not the separate components.

Table 8 - VECM results – ARDL model (Dependent variable d(S/Y))

Variable	Coefficient	Prob	Standardised coeff Coint Eq
$(S/Y)_{t-1}$	-0.505#	0.003 (t=-3.267)	1.000
$(D/Y)_{t-1}$	-0.059	0.018	0.116
$(A/Y)_{t-1}$	0.059	0.004	-0. 116
$(YGAP)_{t-1}$	-0.345	0.111	0.684
$D(S/Y)_{t-1}$	-0.210	0.254	
$D(D/Y)_{t-1}$	-0.066	0.701	
$D(A/Y)_{t-1}$	-0.084	0.195	
$D(YGAP)_{t-1}$	0.407	0.025	
Adjusted R-squared	0.27	**Akaike info criterion**	-4.699
Hannan-Quinn criter.	-4.576	**Schwarz criterion**	-4.357
Serial corr LM (1)(prob)	0.38	**Serial corr LM (2)(prob)**	0.92
JB test for normality (prob)	0.13	**Test (prob): $\beta_{11}=\beta_{21}=\beta_{31}=\beta_{41}=0$**	0.003

There are 39 observations, four regressors in the long-run relationship of which three are I(1), and there are eight deterministic regressors in the relationship. Thus $T^a = T - (2k - 1) - d = 39 - (2*3 - 1) - 8 = 26$. The t value of β_{11} (i.e. the parameter of $(S/Y)_{t-1}$, which also represent the error correction term, is -3.267, while the critical value using Ericsson and MacKinnon is -3.047 at a 5% level (Enders 2010:493). One can also use the bounds test of Pesaran *et al.* (2001) and test the null hypothesis that the parameters of the long-run component are together equals to zero (i.e. $\beta_1=\beta_2=\beta_3=\beta_4=0$). This null is rejected with the lower and upper bound values at 0.05 being 2.26 and 3.48, while the estimated F value is 3.99 (see Pesaran *et al.* 2001:300). In addition, the lower and upper bound critical t values for the error-correction term with four variables and no intercept at a 5% level of significance is -1.95 and -3.60, while at a 10% level it is -1.62 and -3.26. With a t value at -3.27 the error-correction term is significant at a 5% level for the lower bound and for both the lower and upper bound at 10%. Hence, the conclusion can be drawn that there is a long-run relationship, at least at a 10% level of significance.

The VECM results can now be used to derive the household financial reaction function, as expressed in Equation (32a). This is done, as mentioned above, by just rewriting the short-run dynamics of the VECM back into a VAR in levels. The same type of rewriting is done for the ARDL estimation. For instance an equation in the VECM written as:

$$\Delta Y_t = \alpha\beta_1 Y_{t-1} - \alpha\beta_2 X_{t-1} + \delta_1 \Delta Y_{t-1} + \delta_2 \Delta X_{t-1}$$

can be rewritten in levels as:

$$Y_t = (1+\alpha\beta_1+ \delta_1)Y_{t-1} - \delta_1 Y_{t-2} + (-\alpha\beta_2+ \delta_2)X_{t-1} - \delta_2 X_{t-2}$$

Table 9 presents the results for the Johansen VECM (columns two to five) and the ARDL VECM (column six). The top panel reports the Johansen VECM with no subset restrictions on the VAR to eliminate statistically insignificant lags, while the ARDL VECM shows all the parameters irrespective of their statistical significant. The bottom panel shows the results for the VECM with subset restrictions imposed to eliminate statistically insignificant lags, while the ARDL VECM shows the parameter values calculated with the statistically significant lags. Table 10 presents the sum of the two lags for each variable. It corresponds closest to Equation (16) and therefore reports the household saving reaction function. The signs of the parameters are in accordance with *a priori* expectations, with the size of the parameters being the same or very similar for *A/Y* and *D/Y*, again indicating that households react to their net financial asset position and not to debt and financial assets separately. There is also a high degree of inertia in behaviour, as can be seen with the parameter of the lagged *S/Y* term. The output gap has a negative parameter in the VECM estimation, which indicates the possibility that households face credit constraints, thus preventing them to smooth consumption over the business cycle.

Table 9 – The household financial reaction function

	Johansen – VAR representation – no subset restrictions				**ARDL – including stat insig lags**
	$(S/Y)_t$	**$(D/Y)_t$**	**$(A/Y)_t$**	**$(Y\ GAP)_t$**	**$(S/Y)_t$**
$(S/Y)_{t-1}$	0.700	0.733	0.795	0.164	0.285
$(D/Y)_{t-1}$	0.068	1.504	-0.984	-0.278	-0.125
$(A/Y)_{t-1}$	0.026	0.180	1.625	0.122	-0.026
$(Y\ GAP)_{t-1}$	-0.091	0.141	0.489	0.588	0.061
$(S/Y)_{t-2}$	0.189	-0.711	-0.440	-0.289	0.210
$(D/Y)_{t-2}$	-0.096	-0.498	1.074	0.246	0.066
$(A/Y)_{t-2}$	0.001	-0.186	-0.712	-0.091	0.084
$(Y\ GAP)_{t-2}$	-0.435	-0.034	1.197	-0.183	-0.407
	Johansen – VAR representation – with subset restrictions				**ARDL – excluding stat insig lags**
	$(S/Y)_t$	**$(D/Y)_t$**	**$(A/Y)_t$**	**$(Y\ GAP)_t$**	**$(S/Y)_t$**
$(S/Y)_{t-1}$	0.795	0.591	0.326	-0.110	0.495
$(D/Y)_{t-1}$	-0.024	1.570	-0.883	-0.295	-0.059
$(A/Y)_{t-1}$	0.023	0.179	1.590	0.083	0.059
$(Y\ GAP)_{t-1}$	0.022	0	0.222	0.479	0.407
$(S/Y)_{t-2}$	0.110	-0.591	0	0	0.000
$(D/Y)_{t-2}$	0	-0.570	0.966	0.267	0.000
$(A/Y)_{t-2}$	0	-0.179	-0.670	-0.056	0.000
$(Y\ GAP)_{t-2}$	-0.471	0	1.325	0	-0.407

Table 10 – The household financial reaction function – sum of parameters

	No subset restrictions		**With subset restrictions**	
	VAR	**ARDL**	**VAR**	**ARDL**
(S/Y)	0.889	0.495	0.905	0.495
(D/Y)	-0.028	-0.059	-0.024	-0.059
(A/Y)	0.027	0.059	0.023	0.059
(Y GAP)	-0.526	-0.345	-0.449	0.000

7) Conclusion

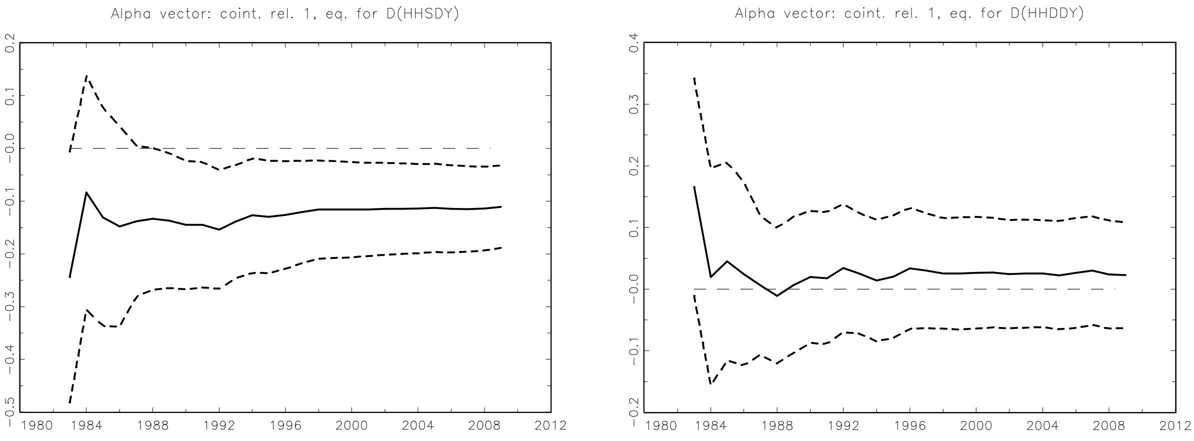
This paper shows that a non-stationary saving rate and net financial asset/income ratio can exist even if households act in accordance with the postulates of the Permanent Income Hypothesis and the Life-cycle model. With these ratios being non-stationary there is no need to detrend the data and one can apply cointegration analysis to the data, which allows for the long-run properties of the data not to get lost in the estimation process that follows.

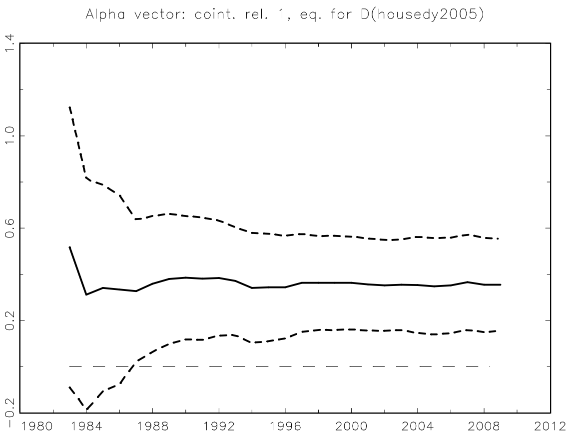
Drawing on fiscal reaction function literature initiated by Bohn (1995, 1998, 2007) the paper specifies and estimates a saving reaction function for the household sector. The estimated function measures the reaction of the household saving/disposable income ratio to changes in the household debt/disposable income ratio and house price/disposable income ratio. The results point to households acting in accordance with the Life-cycle model and thus act in a financially sustainable manner over the period 1969-2009. Therefore, households seem to act in a manner that prevents their debt and asset positions from landing on an explosive path.

However, in the Johansen estimation the parameter on the output gap shows that in the short run (i.e. a business cycle) the Permanent Income Hypothesis might not hold – this results though is not borne out by the ARDL model. In addition, the variance decomposition analysis as well as the Granger causality analysis, indicate that causality between *D/Y* and *A/Y* might be bidirectional, so that changes in these two variables might be mutually re-enforcing, allowing for a debt-asset price spiral. However, given that households react sufficiently to changes in their debt and asset position means that such spirals might themselves not be sustainable and therefore fizzle out. This is further borne out by *D/Y* being weakly exogenous in the VECM analysis.

Appendix 1

Figure A1 – recursively estimated values for error correction terms of *S/Y*, *D/Y*, *A/Y* and the output gap





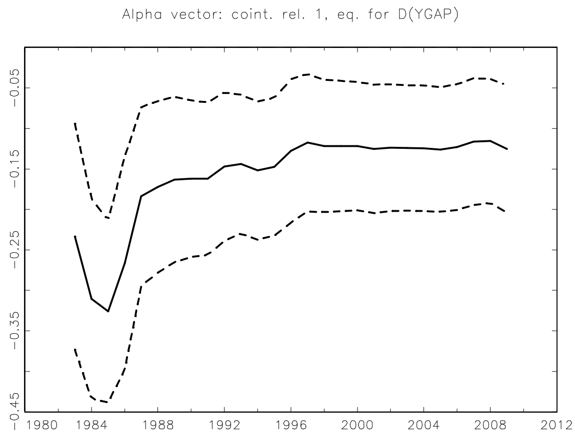


Figure A2 – Recursively estimated values for Eigenvalue

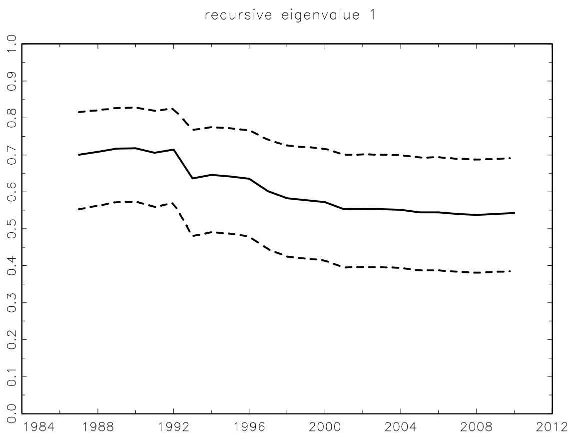
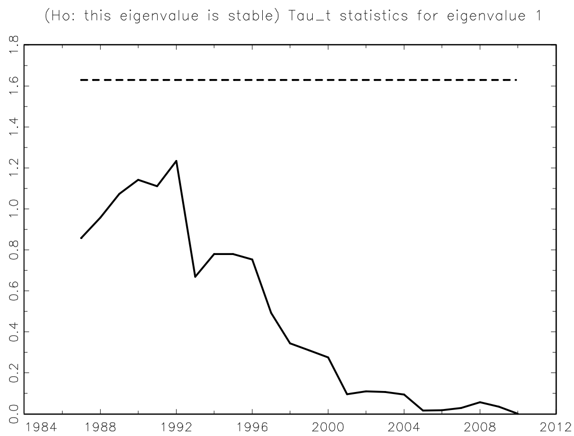


Figure A3 – Tau t statistic for recursively estimated values for eigenvalue



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