Systemic Interaction Risk*

Toni Ahnert[†], Co-Pierre Georg[‡]

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DRAFT - COMMENTS WELCOME

Abstract

The financial crisis emphasized the role of systemic risks for financial (in-)stability. While different forms of systemic risk have been studied in isolation, we develop a unified model of interbank contagion, a common shock, and informational spillovers. We document a substantial, non-trivial interaction effect between the different forms of systemic risk. We also demonstrate that the interation effect increases with increasing measures of financial distress. Thus, our findings not only highlight the importance of a joint analysis of different forms of systemic risks, but also have strong implications for macro-prudential regulation and capital adequacy requirements.

Keywords: contagion, common shocks, systemic risk

JEL Classification: D53, G01, G21, G33

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†London School of Economics and Political Science, Department of Economics, Houghton Street, London WC2A 2AE and Centre for Economic Performance, Houghton Street, London WC2A 2AE. The author acknowledges financial support by the ESRC and Studienstiftung des deutschen Volkes. E-Mail: `t.ahnert@lse.ac.uk`

‡Graduate School "Foundations of Global Financial Markets - Stability and Change", Friedrich-Schiller-Universität Jena, Bachstraße 18, D-07743 Jena. The author acknowledges financial support by the Graduate School "Global Financial Markets – Stability and Change", which is funded by the Stiftung "Geld und Währung". Part of this research was conducted when the author was a visiting scholar at the New York University Stern School of Business. E-Mail: `co.georg@uni-jena.de`

1 Introduction

Systemic risk and systemic crises, in which a large part of the financial system fails, have gained much attention in recent years. The literature typically considers three different forms of systemic risk that contribute to the probability of a systemic crisis. First, interbank contagion refers to a situation in which banks lend funds among themselves to insure against liquidity shocks. This makes the banking system susceptible to the default of one bank, triggering default of other banks due to existing interbank loan default. Second, correlated or common shocks materializes when the banks' asset returns are positively correlated and a negative realization of one particular asset or asset class hits several banks at once. Third, informational spillovers take place when news about one bank are useful for the prediction of another banks health. Then, bad news about one bank affects all related banks.

While there exists a large body of theoretical work on the individual forms of systemic risk, a unified framework is still missing. The present paper closes this gap by developing a banking model that features interbank contagion, a common shock, and informational spillovers. We analyse each form of systemic risk's individual contribution to financial (in-) stability. Furthermore, we study the interaction between individual forms of systemic risks, termed as *systemic interaction risk*.

While establishing an independent role for each form of systemic risk, we demonstrate the relevance of the interaction effect. In the baseline calibration, for example, systemic interaction risk constitutes about ten percent of the total probability of systemic crisis. We also examine how the size of systemic interaction risk depends on bank interconnectedness and asset return volatility, a measure of financial crisis. Notably, the systemic interaction risk increases with the volatility of asset returns. Our results therefore document a substantial interaction effect between individual forms of systemic risk and suggest that systemic risks ought to be analysed in a unified model.

Several policy conclusions arise from our analysis. First, effective prudential regulation must consider all forms of systemic risk jointly, thus taking systemic interaction risk into account. This is underlined by the procyclicality of systemic interaction risk. Whereas the systemic interaction risk is small – or even negative – in tranquil times, emphasizing the insurance character of interbank networks, it is large in times of financial distress. Second, overall capital adequacy requirements may need to be adjusted substantially. The new Basel III capital requirements strengthen the quality and quantity of regulatory capital, while the risk weights used to calculate the amount of required capital are almost unchanged. This incentivizes banks to hold financial assets and effectively increases the interconnectedness in the financial system. Our results show that in this case the systemic interaction risk is most severe.

In sum, we demonstrate the importance of the unified model of systemic risk in the analysis of financial stability. The size of the interaction effect suggests strong implications for macro-prudential regulation of systemic risk, particularly capital adequacy requirements.

1.1 Related Literature

Our paper is related to the literatures on individual forms of systemic risk. First, we build on the interbank contagion literature pioneered by Allen and Gale (2000) who extended the seminal contribution of Diamond and Dybvig (1983) to study the role of interbank connections for financial fragility. Banks insure themselves against liquidity shocks by interbank lending. While achieving efficiency, this arrangement is financially fragile as a positive liquidity demand shock may travel through the entire financial system. The authors demonstrate that the size and the interconnectedness of the banking system matter, with complete loan structures being more robust than incomplete ones. A similar approach is taken by Freixas et al. (2000) who consider spatial instead of intertemporal uncertainty about liquidity needs. Dasgupta (2004) uses global games methods to obtain contagion in a rational expectations equilibrium. Considering different levels of bank interconnectedness, he finds that stable banking systems with rare defaults display the most severe impact of an individual bank's default. This result is confirmed by Gai and Kapadia (2009) who use a network model of a banking system and show that with increasing connectivity the risk of systemic crisis is reduced, while the crisis' impact increases. We extend that literature by considering several forms of systemic risk including interbank lending, which allows us to examine the interaction of systemic risks.

Second, common shocks are another form of systemic risk recently studied. In Acharya (2009), systemic risk is modelled as the endogenously chosen correlation of returns on assets held by banks. Because of limited liability and the negative externality of one bank's failure on the health of other banks, all banks undertake correlated investments, thereby increasing economy-wide aggregate risk, a systemic risk-shifting incentive.See also Georg and Poschmann (2010) who compare interbank contagion and common shocks in a dynamic multi-agent simulation, Cifuentes et al. (2005) who considers systemic risk arising from portfolio holdings of financial institutions and Adrian and Shin (2010). Following Acharya, we incorporate common shocks in our paper but take them as exogenous for tractability. As before, considering several forms of systemic risk goes beyond the existing literature as it allows us to analyse the interaction between these forms. Finally, we also incorporate informational spillovers. They arise when the insolvency of one bank carries a signal about the health of similar institutions. See Acharya and Yorulmazer (2008), Nier et al. (2007), and Cipriani and Guarino (2008) for examples.

2 The Model

2.1 Households

Households are modelled as in Diamond and Dybvig (1983). The economy extends over three periods $t = 0, 1, 2$ and there exists an all-purpose consumption and investment good. There is one bank and a large number of ex-ante identical households, $i \in [0, 1]$. Households are endowed with one unit in $t = 0$ only. There are two types of households: early consumers value consumption in $t = 1$ only, while late consumers value consumption in either period. There are two regions $k \in \{A, B\}$, see also Allen and Gale (2000) and Dasgupta (2004). The ex-ante probability of being an early consumer is identical across consumers and given by λ_k (which is also the share of early consumers in that region).

2.2 Regional liquidity shocks

There are negatively correlated regional liquidity shocks that motivate interbank insurance [1] Excess liquidity in one region is associated with liquidity shortage in the other region,

[1] Freixas et al. (2000) motivate this assumption by allowing for interregional travel of depositors who learn the location of their liquidity demand at the beginning of the first period one. See also

with an equal probability of being the high liquidity demand region. We study negatively correlated liquidity shocks of equal size to exclude bank runs that are merely driven by aggregate liquidity surplus or shortage.

probability	region A	region B
$\frac{1}{2}$	$\lambda_A = \lambda + \eta$	$\lambda_B = \lambda - \eta$
$\frac{1}{2}$	$\lambda_A = \lambda - \eta$	$\lambda_B = \lambda + \eta$

Note that $\eta > 0$ is the size of the regional liquidity shock and $\lambda_H \equiv \lambda + \eta$ and $\lambda_L \equiv \lambda - \eta$ denote high and low liquidity demand, respectively.

2.3 Assets

There are two assets. First, a risk-free short asset matures after one period and yields a return of one, interpreted as a storage technology. Second, a risky long asset matures after two periods. It yields a uniformly distributed gross return $R \sim U\left[\mu - \sigma, \mu + \sigma\right]$ if held to maturity, where $\mu > 1$ is the expected return and $\sigma > 0$ a volatility measure. Premature liquidation after one period yields an inferior return $\beta \in [0, 1)$ only. The payoffs are summarized as follows:

Asset	$t = 0$	$t = 1$	$t = 2$
Short $(0 \to 1)$	-1	1	0
Long $(0 \to 2)$	-1	β	R

2.4 Signals

Before making their withdrawal decision in $t = 1$ households receive a private signal about the long asset return in both regions. Signals take the following form:

$$S_{ik} = R_k + E_{ik} \ , \tag{1}$$

where $E_{ik} \sim U[-\chi, \chi]$ is an identically and independently distributed noise that is independent from any region's fundamental, R_k.

Allen and Gale (2000) and Dasgupta (2004).

2.5 Banks and Deposit Contracts

Banks have a comparative advantage in managing funds because of greater access to assets. That is, households have only access to the short asset, whereas banks may invest in either the short or the long asset. A second role for banks comes from the provision of liquidity. Banks offer deposit contract to the households in their region. A contract specifies withdrawals c_1, c_2 in $t = 1, 2$. The non-observability of the idiosyncratic liquidity shock prevents the deposit contract between the bank and the household to be contingent on the household's liquidity shock. Banks pay out deposits c_1 in $t = 1$ if there is sufficient liquidity available. If not, it declares insolvency, liquidates all assets and pays an equal amount to all depositors.

If the bank survives, it settles its liabilities in the interbank market and pays out its remaining resources to depositors in $t = 2$. Note that c_2 is contingent on the realization of the investment return R_k, the regional liquidity shock λ_k, and both bank's default probabilities. We denote the binary default decision as $d_k \in \{0, 1\}$ and the probability of default as as $a_k \equiv \Pr\{d_k = 1\}$.

Banks make zero profit motivated by free-entry. Therefore, they choose the period-one withdrawal level c_1, investment into liquidity, y, and interbank insurance b to maximize the household's ex-ante expected utility. The literature commonly considers the optimal contract in some baseline scenario and studies its implications in a perturbed setting. Examples for baseline scenarios are the absence of investment risk (Morris and Shin (2000)) or a no-default environment (Allen and Gale (2000)). Likewise, period-one withdrawals are often set to the first-best consumption level. We follow that approach and consider the optimal contract with no investment risk ($\sigma = 0$) as our benchmark.[2]

2.6 Timeline

Period Zero: First, households in both regions receive their endowment. Next, banks offer a deposit contract to the households in their region. Third, the households decide whether or not to deposit their endowment.[3] Finally, each bank chooses the level of in-

[2]This implies $y = \lambda$, $b = \eta$, and $c_1 = 1$ for log utility. See also Dasgupta (2004).

[3]This gives rise to a participation constraint: $\mathrm{E}[U] \geq u(1)$, where E is the expectation operator that

vestment in the short and long asset $(y, 1-y)$ and agree upon the amount of interbank insurance b at price $\phi > 1$.[4] The interbank loan b will be transferred from the liquidity surplus region to the liquidity shortage region at the beginning of $t = 1$ after the realization of the liquidity shock, to be repaid at the beginning of $t = 2$.

Period One: First, the short asset matures. Next, the regional liquidity shocks λ_k are drawn and publicly observed, initiating the interbank transfer.Households in both regions draw their individual liquidity demands and observe them privately.[5] Third, households receive a signal of the the long asset returns from both regions. They update their expectations about the long asset return and optimally decide whether or not to withdraw.

Finally, banks pay out withdrawing depositors using a *strict pro-rata mechanism*. They collect the liquidity demands and compare it to their liquidity. If desired withdrawals exceed the bank's liquidity, the bank declares bankruptcy, liquidates its long assets and interbank claims. The bank then not only pays an equal share to households that wish to withdraw (weak pro-rata) but to all depositors (strict pro-rata). The strict pro-rata assumption buys the uniquess of equilibria without resorting to global games techniques.

Period Two: First, the long asset matures and pays the return as realized in $t = 1$. Then, banks settle their interbank positions. This includes the repayment to an insolvent counterpart but no repayment from an insolvent bank takes place (interbank contagion), reflecting the seniority of deposits over interbank claims.[6] Finally, all surviving banks pay out deposits.

The complete timeline of the model is depicted in Figure (9).

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takes into account regional liquidity shocks, investment risk, and the joint probability distribution of banks' default. The participation constraint specifies a joint constraint on the model parameters that can be interpreted as a constraint on the lower bound of the average return μ. The maintained assumption is that the households' participation constraints are satisfied. Each household will therefore deposit in full.

[4]There are two forms of interbank insurance. First, the interbank gross interest rate ϕ is set by a central bank and banks take it as given. Alternatively, the interbank insurance can take the form of depositexchange, which implies $\phi = R_k$.

[5]An example of such a timing is the UK spending cuts. It is already clear that jobs will be cut (aggregate result), while it has not been decided, which jobs will vanish (individual result).

[6]See article 11(d) FDICA.

3 Equilibrium

3.1 The Baseline Case

The baseline case abstracts from interbank linkages. Liquidity demand shocks are absent, $\eta = 0$, and long asset returns are uncorrelated across regions, $\rho \equiv corr(R_A, R_B) = 0$. We use this case to illustrate some of the model's mechanisms and to demonstrate the equilibrium solution technique.

3.1.1 Strict pro-rata and uniqueness of equilibria

The seminal contribution of Diamond and Dybvig (1983)considers a demand deposits contract in a world of non-verifiable idiosyncratic liquidity demand. This setup gives rise to strategic complementarity between depositors implying multiplicity of equilibria. One equilibrium improves upon the competitive equilibrium allocation because of improved risk-sharing, whereas the second equilibrium features an inefficient bank-run. Multiplicity arises from *weak pro-rata*, that is the resources of a bank run are distributed among those depositors who run only. Hence, any depositor's incentive to run increases with the number of depositors running. Maintaining weak pro-rata, Morris and Shin (2000) demonstrate the uniqueness of equilibrium can be restored when some sufficiently precise idiosyncratic information about the economy's fundamentals, such as the second period's asset return, is introduced (global games). See also Angeletos and Werning (2006) for a recent critique.

By contrast we consider a *strict pro-rata* allotment. That is, all depositors, not only those who ran the bank, receive an equal share upon default of a bank. This appears to be an appropriate description of the legal arrangement in many countries, including the US[7] and Germany.[8]

Strict pro-rata excludes strategic complementarities between late consumers. Inspecting the payoff structure in figure (10), we observe that late consumers now have a weakly dominant strategy in the Bayesian Nash threshold equilibrium - irrespective of the other

[7]See US Federal Deposit Insurance Corporation Act and article 11(d)(11) in particular. Also: `http://www.law.cornell.edu/uscode/html/uscode11/usc_sup_01_11.html`.

[8]See for the legal arrangement: `http://www.gesetze-im-internet.de/inso/`.

late consumers. Then, the uniquesness of equilibria is guaranteed even with region-specific non-idiosyncratic signals. Hence, we do not need to resort to global games techniques. Finally, the indifference between withdrawing and waiting yields the threshold of the long asset's conditional expectation. For the deposit contract considered it is given by:

$$\hat{R} = \lambda + \beta(1 - \lambda) \tag{2}$$

3.1.2 Updating and signal thresholds

We start by deriving the signal's distribution. Let $E_{\min} = -\chi$, $E_{\max} = +\chi$, $R_{\min} = \mu - \sigma$, and $R_{\max} = \mu + \sigma$ be the lower and upper bounds of the noise and the asset return, respectively. Assuming that a signal-to-noise ratio of above one, $\sigma > \chi$, we partition the support of S:

$$\begin{aligned} S_{\min} &\equiv R_{\min} + E_{\min} = \mu - \sigma - \chi && (3) \\ \underline{S} &\equiv R_{\min} + E_{\max} = \mu - \sigma + \chi && (4) \\ \overline{S} &\equiv R_{\max} + E_{\min} = \mu + \sigma - \chi && (5) \\ S_{\max} &\equiv R_{\max} + E_{\max} = \mu + \sigma + \chi && (6) \end{aligned}$$

By convolution, the signal's probability density $f(S)$ is (see appendix A.2 for a proof):

$$f(S) = \begin{cases} \frac{S - S_{\min}}{4\chi\sigma} & \text{for} \quad S \in [S_{\min}, \underline{S}] \\ \frac{1}{2\sigma} & \text{for} \quad S \in [\underline{S}, \overline{S}] \\ \frac{S_{\max} - S}{4\chi\sigma} & \text{for} \quad S \in [\overline{S}, S_{\max}] \end{cases} \tag{7}$$

Second, we determine the expected asset return conditional on the signal. In the baseline case households use their region's signal only since the signal about the other region's asset return carries no information. The conditional expectation $E[R|S]$ is (see appendix

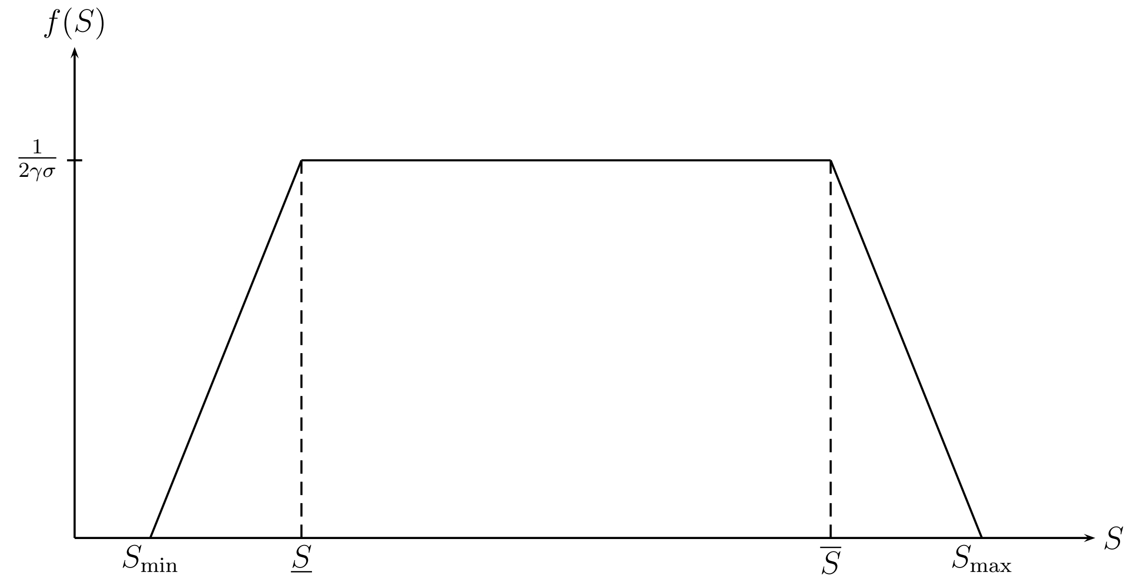

Figure 1: The signal's probability density ($\sigma > \chi$).

A.2 for a proof and Figure 11 for a depiction):

$$E[R|S] = \begin{cases} \frac{1}{2}(S + \underline{S}) & \text{for} \quad S \in [S_{\min}, \underline{S}] \\ S & \text{for} \quad S \in [\underline{S}, \overline{S}] \\ \frac{1}{2}(S + \overline{S}) & \text{for} \quad S \in [\overline{S}, S_{\max}] \end{cases} \tag{8}$$

Third, we determine the signal's threshold $\hat{S}$ up to which households will optimally withdraw. We assume a non-trivial withdrawal decision, that is a late household will always withdraw upon receipt of the lowest possible signal $S_{\min}$ and will never withdraw upon receipt of the highest possible signal $S_{\max}$. This implies a parameter constraint $R_{\min} \leq \hat{R} \leq R_{\max}$. To capture the low empirical bank default frequency, we focus on withdrawals for low signal levels: $S \in [S_{\min}, \underline{S}]$.

Then, the signal's threshold is:

$$\hat{S} = 2\hat{R} - \underline{S} \tag{9}$$

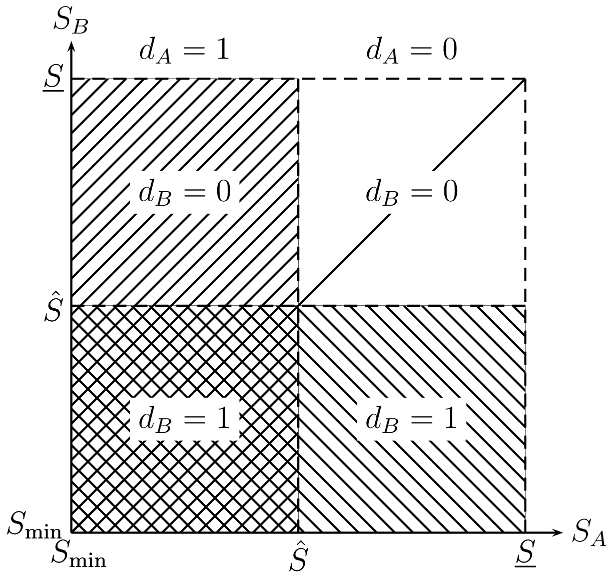

Figure 2: The bank's default areas (baseline case)

3.1.3 Bank default and systemic crisis

Given the symmetry of equilibrium, default of a bank takes place if and only if $S \leq \hat{S}$. Thus, a bank's default probability is:

$$a^{(1)} \equiv \Pr\{S \leq \hat{S}\} = \frac{\left(\hat{R} - R_{\min}\right)^2}{2\chi\sigma} , \tag{10}$$

where the superscript (1) refers to scenario (1), the baseline case.

We define a *systemic crisis* as the default of both banks. The probability of a systemic crisis is then the ex-ante probability of joint default, denoted as a_D, and given by:

$$a_D^{(1)} \equiv \Pr\{d_A = 1, d_B = 1\} = a^{(1)}\, a^{(1)} , \tag{11}$$

where the second equality arises from independence of the two default events. Let a_N denote the probability of default in region A and survival in region B, a measure useful for an evaluation of positive or stabilizing contagion effects. It is:

$$a_N^{(1)} \equiv \Pr\{d_A = 1, d_B = 0\} = (1 - a^{(1)})\, a^{(1)} . \tag{12}$$

Figure (2) illustrates the default situation for different signals.

3.2 Interbank Contagion

We now consider interbank linkages in the form of interbank loans, caused by liquidity fluctuations $\eta > 0$. We proceed with backward induction as before. First, households in the high liquidity demand region H receive their signal S_H, update their expectations $E[R_H|S_H]$ about the long asset's return, and decide strategically whether or not to withdraw at the end of $t = 1$. All early consumers will withdraw and consume, whereas late consumers will withdraw and store for $t = 2$ if and only if the expected asset return conditional on the signal falls short of a threshold level $\hat{R}_H$, determined by the indifference between withdrawing in $t = 1$ and waiting for repayment in $t = 2$. Note that the threshold differs from the baseline case because of the presence of interbank loans. Withdrawal of households and bank default are synonymous and will take place if and only if $S_H < \hat{S}_H$.

Second, households in region L observe whether or not the bank in H has defaulted and receive their signal S_L. Signal thresholds are determined as before, but now depend on whether default in region H occurred (state N for no default and D for default). Hence, thresholds are denoted as $\hat{S}_{L,N}$, $\hat{S}_{L,D}$ for signals and $\hat{R}_{L,N}$, $\hat{R}_{L,D}$ for the conditional expectation about the long asset's return. As before, households in region L then strategically decide whether or not to withdraw.

Consider households in region H who move first. Note that their payoffs are independent of the withdrawal decision of households in region L. They receive $y + \beta(1 - y) + b$ if they withdraw and $\frac{1}{1-\lambda_H}\left[E[R_H|S_H](1 - y) + (y - \lambda_H + b) - \phi b\right]$ otherwise. The cutoff values of the expected asset return is:

$$\hat{R}_H = \lambda_H + \beta(1 - \lambda_H) + \frac{\phi - \lambda_H}{1 - y} b \tag{13}$$

Now consider a household in region L who has observed a default of region H's bank. Then, not withdrawing yields $\frac{1}{1-\lambda_L}\left[E[R_L|S_L](1 - y) + (y - b - \lambda_L)\right]$, while withdrawing yields $y + \beta(1 - y) - b$. This leads to a threshold:

$$\hat{R}_{L,D} = \lambda_L + \beta(1 - \lambda_L) + \frac{\lambda_L}{1 - y} b \tag{14}$$

Likewise, in the case of no default in H, withdrawing yields $y + \beta(1 - y) - b + \beta\phi b$, whereas

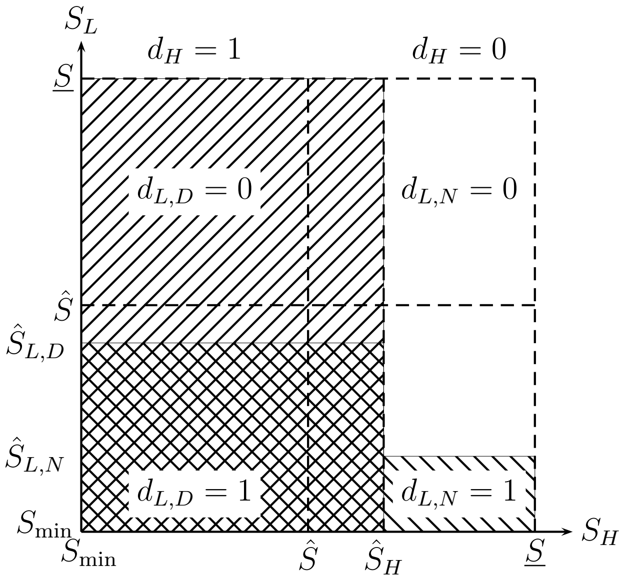

Figure 3: The support of the joint density $g(S_H, S_L)$ if there is interbank contagion, but no informational contagion.

not withdrawing yields $\frac{1}{1-\lambda_L}\left[E[R_L|S_L](1-y)+(y-\lambda_L-b)+\phi b\right]$. Hence, the threshold of the long asset's conditional expectation is given by:

$$\hat{R}_{L,N} = \lambda_L + \beta(1-\lambda_L) + \frac{\lambda_L - \phi\left[1-\beta(1-\lambda_L)\right]}{1-y}b \quad < \quad \hat{R}_{L,D} \tag{15}$$

The larger cutoff value in region L after a default in region H is intuitive: the default reduces the available assets in region L in period $t = 2$ and induces late consumers to withdraw prematurely for a larger range of expected asset returns. This is the classical contagion case.

Any threshold of the conditional expectation translates uniquely into a threshold of the signal S:

$$\hat{S}_j = 2\hat{R}_j - \underline{S} \quad \text{for } j \in \{H, LD, LN\} \tag{16}$$

There are parameter constraints as in the baseline case: $\hat{R}_{L,N} \geq R_{\min}$ and $\hat{R}_H \leq R_{\max}$. The default decisions are depicted in Figure (3).

To compare the default probabilities ot the baseline case, we first define the individual default probabilities:

$$a_j^{(2)} \equiv \Pr\{S \leq \hat{S}_j\} = \frac{1}{2\chi\sigma}\left[\hat{R}_j - R_{\min}\right]^2 \quad , \quad j \in \{H, (L,D), (L,N)\} \tag{17}$$

Comparing this to the baseline case without interbank loans, it can be seen that the area where a joint default occurs increases, since $\hat{S}_H > \hat{S}$ while $\hat{S}_{L,D}$ can be larger or smaller than $\hat{S}$ and $\hat{S}_H > \hat{S}_{L,D}$). At the same time, the probability of a default of the bank in L decreases if there is no default in H.

The probability of a systemic crisis reads as:

$$\overline{a}_D^{(2)} \equiv \Pr\{d_H = 1, d_{L,D} = 1\} = a_H^{(2)} a_{L,D}^{(2)} \tag{18}$$

While the impact on the probability of a systemic crisis is in general ambiguous, imposing equilibrium conditions, a sufficient condition for the increase in systemic risk is:

$$\frac{\lambda_L b}{1-y} \geq (1-\beta)\eta \tag{19}$$

which is satisfied, unless liquidation values are tiny and liquidity shocks are huge.

Likewise, the probability of default in region L after no default in region H is given as:

$$\overline{a}_N^{(2)} \equiv \Pr\{d_H = 0, d_{L,N} = 1\} = (1 - a_H^{(2)}) a_{L,N}^{(2)} \ll a^{(1)}\,(1 - a^{(1)}) = \overline{a}_N^{(1)} \tag{20}$$

In the case of interbank loans the probability of default upon survival in region H falls due to lower survival probability in region H $(1 - a_H^{(2)} < 1 - a_H^{(1)})$ and lower default probability when region H's bank did not default $(a_{L,N}^{(2)} < a_L^{(1)})$. This is the mutual insurance character of interbank loans, a positive effect of interbank linkages.

3.3 Informational spillovers and common shocks

Informational contagion poses another form of systemic risk. If asset fundamentals are correlated, the observation of region H's signal helps households in region L to infer their own asset fundamentals. In particular, region H's signal may not only trigger a bank run

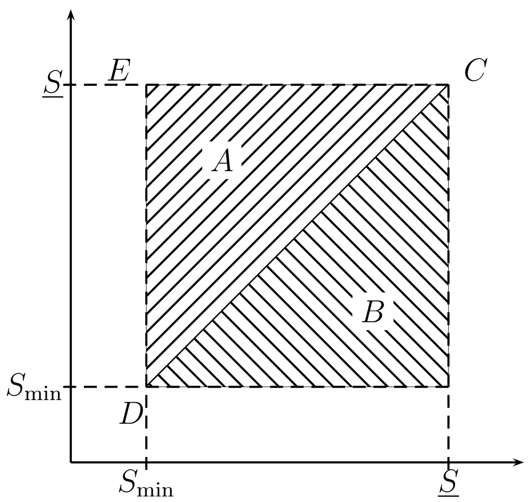

Figure 4: Support of $g(S_H, S_L)$ with the partition into two regions A, B.

in this region but may suffice to induce a run in region L as well, even in the absence of interbank markets. Let $\rho \equiv corr(R_H, R_L) \in [0, 1]$ denote the correlation of long assets, where $\rho = 0$ excludes informational contagion altogether. For tractability we focus on the case $\rho = 1$, a perfect common shock, that implies $R_L = R_H$. [9]

We start by calculating the two-dimensional density $g(S_H, S_L)$. The pdf, depicted in Appendices (12), is fully symmetric but differs from the baseline and interbank contagion case's pdf. We maintain our focus on low signal levels, that is $S_k \in [S_{\min}, \underline{S}]$ to match the emprirical relative frequency of bank defaults. The support in this area is depicted in figure (4). The joint density in this area is given as (see Appendix (A.2) for a proof):

$$g(S_H, S_L) = \begin{cases} \frac{1}{8\chi^2\sigma}(S_H - S_{\min}) & \text{in Region A} \\ \\ \frac{1}{8\chi^2\sigma}(S_L - S_{\min}) & \text{in Region B} \end{cases} \tag{21}$$

Households in region L use both signals to update their expectation about the long asset's return. The conditional expectation $E[R|S_H, S_L]$ is then given as (see Appendix (A.2.4)

[9] Up to a scaling factor that is set to unity for simplicity.

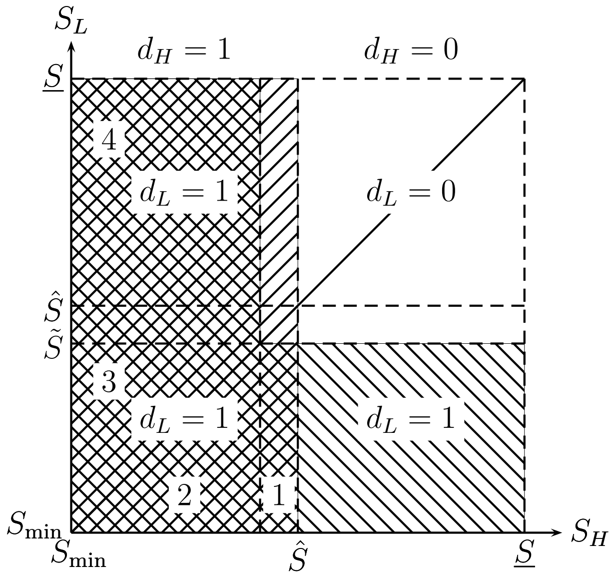

Figure 5: Support of the joint density $g(S_H, S_L)$ in the case of pure informational contagion.

for a proof):

$$E[R|S_H, S_L] = \begin{cases} S_H + \chi & \text{in Region A} \\ \\ S_L + \chi & \text{in Region B} \end{cases} \tag{22}$$

As the payoffs for early households are unchanged from the baseline case, the thresholds for the conditional expectations are unchanged, $\hat{R}_H = \hat{R}$. The signal's threshold is $\hat{S}_H = \hat{S}$. Hence, $d_H = 1$ if and only if $S_H \leq \hat{S}$. Households in region L take both signals into account and thus use the conditional expectation $E[R|S_H, S_L]$, which leads to a signal threshold $\tilde{S} = \hat{R} - \chi$. Therefore, households in region L *always* withdraw given that the signal in region H is smaller than $\tilde{S}$, illustrating the power of informational contagion. Figure (5) displays the default pattern in the presence of informational contagion.

To determine default probabilities, it is useful to find the individual probabilities of the areas 1,2,3, and 4 as marked in Figure (5):

$$a_2^{(3)} = a^{(1)} \frac{1}{24\chi}(\hat{R} - R_{\min}) = a_3^{(3)} \tag{23}$$

$$a_1^{(3)} = a^{(1)} \frac{1}{8\chi}(\hat{R} - R_{\min}) \tag{24}$$

$$a_4^{(3)} = a^{(1)} \left[\frac{1}{4} - \frac{1}{8\chi}(\hat{R} - R_{\min})\right] \tag{25}$$

where the first equality holds because of symmetry. The relevant default probabilities are then:

$$\bar{a}_N^{(3)} \equiv \Pr\{d_L = 1, d_H = 0\} = a_1^{(1)} \left[\frac{1}{4} - \frac{1}{8\chi}(\hat{R} - R_{\min})\right] \tag{26}$$

$$\bar{a}_D^{(3)} \equiv \Pr\{d_L = 1, d_H = 1\} = a^{(1)} \left[\frac{1}{4} + \frac{1}{12\chi}(\hat{R} - R_{\min})\right] \tag{27}$$

Comparing these default probabilities to the ones of the baseline case, we find for the joint probability of no default in region H and default in region L:

$$\bar{a}_N^{(3)} \leq \bar{a}_N^{(1)} \Leftarrow \mu > \lambda + \beta(1 - \lambda) \tag{28}$$

which is always satisfied. Finally, we compare the setup of information contagion with the baseline case regarding the probability of systemic crisis:

$$\bar{a}_D^{(3)} \geq \bar{a}_D^{(1)} \Leftarrow \mu - \sigma > 1 - \sqrt{\frac{\chi\sigma}{2}} \tag{29}$$

which is a mild condition and likely to be satisfied. This shows that informational contagion acts stabilizing if no default occurs, destabilizing otherwise, and is thus pro-cyclical.

3.4 Unified model of systemic risk

We now consider the unified model of systemic risk. It features interbank insurance ($\eta > 0$), common shocks ($\rho = 1$), and informational spillovers. The signals' density $g(S_H, S_L)$ and the conditional expectation $E[R|S_H, S_L]$ are unchanged from the information spillover case, whereas the asset return thresholds are as in the interbank contagion case.

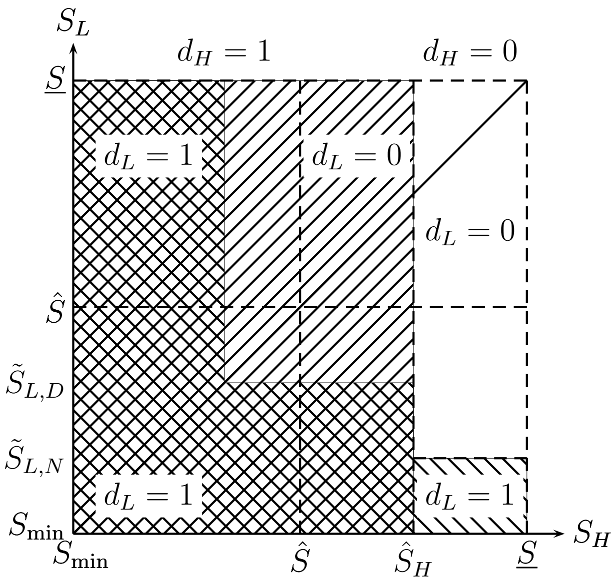

Figure 6: Partitioning of the joint density $g(S_H, S_L)$ in the presence of both forms of contagion. Gains and losses are relative to the case of pure informational contagion.

The households' withdrawal decisions are depicted in (6). Households in region H default if and only if $S_H \leq \hat{S}_H$ as before. Given no default in region H, households in region L default if and only if $S_L \leq \tilde{S}_{L,N}$. Given default in region H, households in region L default if and only if $S_L \leq \tilde{S}_{L,D}$ in area B and if and only if $S_H \leq \tilde{S}_{L,D}$ in area A.

The relevant default probabilities are again found by appropriately dividing the support and are given as:

$$\bar{a}_N^{(4)} = a_{L,N}^{(2)} \left[\frac{1}{4} - \frac{1}{4\chi} (\hat{R}_H - R_{\min}) \right] \leq \bar{a}_N^{(2)} \tag{30}$$

$$\bar{a}_D^{(4)} = a_{L,D}^{(2)} \left[\frac{1}{4} + \frac{1}{4\chi} \hat{R}_H + \frac{1}{12\chi} (\hat{R}_{L,D} - R_{\min}) \right] \tag{31}$$

The probability of default in region L and survival in region H is smaller than in the case of pure interbank contagion. This illustrates the stabilizing or positive effect of informational contagion, as $\tilde{S} \leq \hat{S}$. Survival in region H is good news for households in region L as their expected asset return is higher than without the news, making default in region L less likely.

4 Results

4.1 Systemic Interaction Risk

Having studied the models with a single form of systemic risk as well as the unified model of systemic risk, we are now ready to examine the interaction effect between the individual forms of systemic risk. We term it *systemic interaction risk*, denoted as Δ. It measures the contribution to the probability of a systemic crisis in excess of the sum of the individual contributions from information contagion and interbank contagion:

$$\Delta \equiv \underbrace{(a_D^{(4)} - a_D^{(1)})}_{\text{total effect}} - \underbrace{(a_D^{(3)} - a_D^{(1)})}_{\text{inform. contagion}} - \underbrace{(a_D^{(2)} - a_D^{(1)})}_{\text{interb. contagion}} \tag{32}$$

$$\Delta = (a_D^{(4)} - a_D^{(3)}) - (a_D^{(2)} - a_D^{(1)}) \ , \tag{33}$$

where the second line decomposes the systemic interaction into the difference of two components. The first one is the increase in the probability of a systemic crisis when interbank lending is added to a model of informational contagion. The second component refers to the increase in the probability of a systemic crisis arising from the pure interbanking contagion case. As each component used the same density, it makes the change comparable.

4.2 Numerical Results

Figure (7) shows the probability of a systemic crisis in each of the four cases. As a baseline calibration we chose the following set of parameters: $\mu = 1.1$, $\chi = 1/3$, $\gamma = 0.5$, $p_H = 0.5$, $\beta = 0.15$, and $\phi = 1.1$. Systemic risk is plotted against the volatility of the risky asset, $\sigma \in [0.5, 0.9]$, and a measure of the volatility of liquidity demand, $\eta \in [0.0, 0.08]$. Larger volatility of the long asset return is interpreted as measure of crisis. Under sufficient conditions, larger regional liquidity shocks map into larger interconnectedness on the interbank market.

As there is no interbank lending in the baseline case and in the case of informational spillover, the probability of a systemic crisis is independent of η but rises with our measure of the financial crisis. While the probability of systemic crisis rises after the introduction of each form of systemic risk, the effect is larger for the case of informational contagion

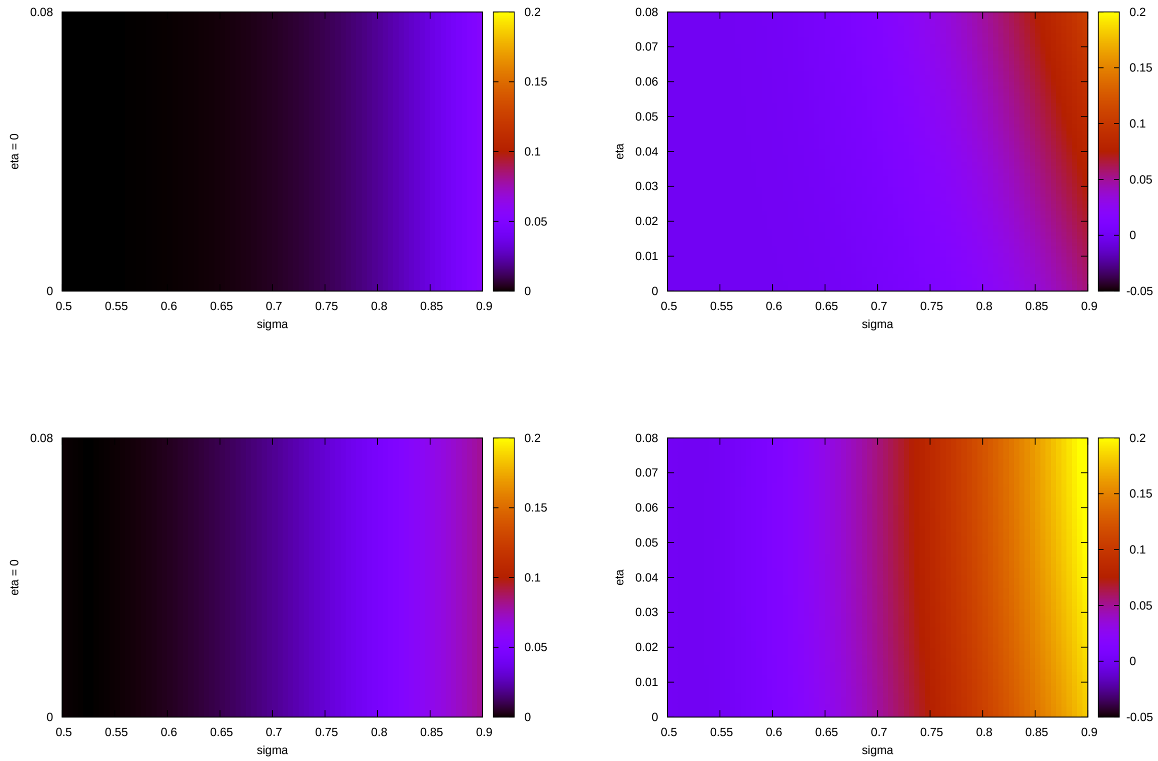

Figure 7: Systemic risk in each of the four cases: $\bar{a}_D^{(1)}$ (top left), $\bar{a}_D^{(2)}$ (top right), $\bar{a}_D^{(3)}$ (bottom left), $\bar{a}_D^{(4)}$ (bottom right) for $\sigma \in [0.5, 0.9]$ and $\eta \in [0.0, 0.08]$.

and common shocks. This is driven by the large common shock and we expect a different effect once a generalized common shock with positive but less than full correlation $\rho < 1$ is considered. Our calculation, however, shows that there is non-negligible systemic risk associated with common exposures, as they yield large informational spillovers. Also note that the probability of systemic crisis is especially high when both the risky asset's volatility is high (financial crisis) and there are large regional liquidity shocks (high extent of interconnectedness).

Figure (8) depicts the absolute systemic interaction risk Δ . It can be seen, that the systemic interaction risk, the interaction effect of several forms of systemic risk, is much larger in times of financial crises (high asset return volatility) and in times of small bank interconnectedness. In tranquil times (low long-asset return volatility), the systemic interaction risk is relatively small and can even be negative for financially stable economies with a high degree of interconnectedness. This highlights the pro-cyclical behaviour of the systemic interaction risk term.

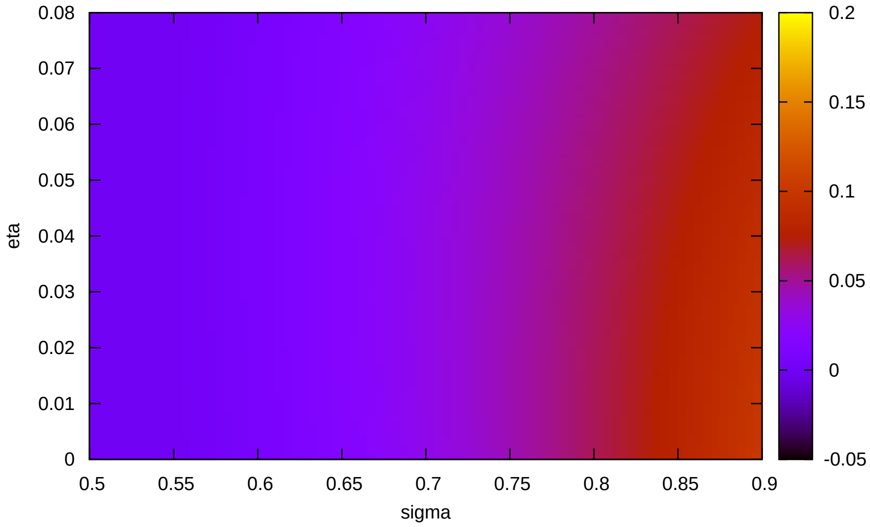

Figure 8: Absolute systemic interaction risk Δ for $\sigma \in [0.5, 0.9]$ and $\eta \in [0.0, 0.08]$.

5 Conclusion

The financial crisis has highlighted the necessity for a better understanding of the different forms of systemic risk. The existing literature focusses largely on contagion effects via interbank connections and only recently analyses common shocks and informational spillovers. However,the different forms of systemic risk have been studied in isolation only and a unified framework of systemic risk was still missing. This papers closes this gap by developing a model of a banking system that allows for the simultaneous analysis of interbank contagion, common shocks, and informational spillovers. This theoretical framework allows us to study the contribution of the various forms of systemic risk to financial (in-)stability. We furthermore show that the size of the interaction effect of the different forms of systemic risk, the systemic interaction risk, depends on the volatility of the long asset and the regional liquidity shock. While low asset return volatility implies small systemic interaction risk, high asset return volatility, as in times of financial crises, leads to high systemic interaction risk.

This highlights the importance of a unified systemic risk framework in the analysis of regulation proposals that aim at strengthening financial stability. A number of policy conclusions can be drawn from our analysis. First, prudential regulation has to take all forms of systemic risk into account in order to be effective. The different forms of systemic risk act pro-cyclical as the interaction term reduces the overall systemic risk in normal times (emphasizing the insurance character of interbank networks), while it significantly contributes to overall systemic risk in times of distress. Regulation proposals that take only individual forms of systemic risk into account will necessarily underestimate the overall systemic risk and hence be less effective. Second, given the large overall effect if all forms of systemic risk are considered ($a_D^{(4)}$ in the notation above), the overall capital adequacy requirements may need to be adjusted substantially. While the new Basel III capital requirements strengthen the quality and quantity of regulatory capital, the risk weights used to calculate the amount of required capital are almost unchanged. This incentivizes banks to hold financial assets and effectively increases the interconnectedness in the financial system. Our results show that it is precisely this situation where the systemic interaction effect is most severe. And third, systemic risks emerging from common shocks and informational spillovers have to be adequately regulated. There are currently no incentives for banks to diversify their portfolio, which can lead to high correlations amonst banks' portfolios. Common shocks, however are not subordinate to contagion effects and thus have to be taken into account. One way of incentivizing banks to diversify their portfolios would be to employ dynamic asset value correlations in Basel III. A macroprudential supervisory authority could calculate the asset value correlations for certain classes of assets and disseminate them to banks who would have to hold more regulatory capital for higher correlated assets. This proposal is outlined in chapter (**??**) in more detail.

There are several promising avenues for future research. First, a natural next step would be the analysis of design of optimal regulatory policy. In particular, Basel III suggests the use of capital requirements, leverage ratio, and liquidity ratios. It would be interesting to study the role of these tools in the context of the unified model of systemic risk. Second, we are interested in further exploring the role of shadow banks within the proposed banking model. This should include the non-trivial trade-off of enhanced liquidity provision and risk-sharing in tranquil times and amplification of systemic risks during a crisis.

A Appendix

A.1 Figures

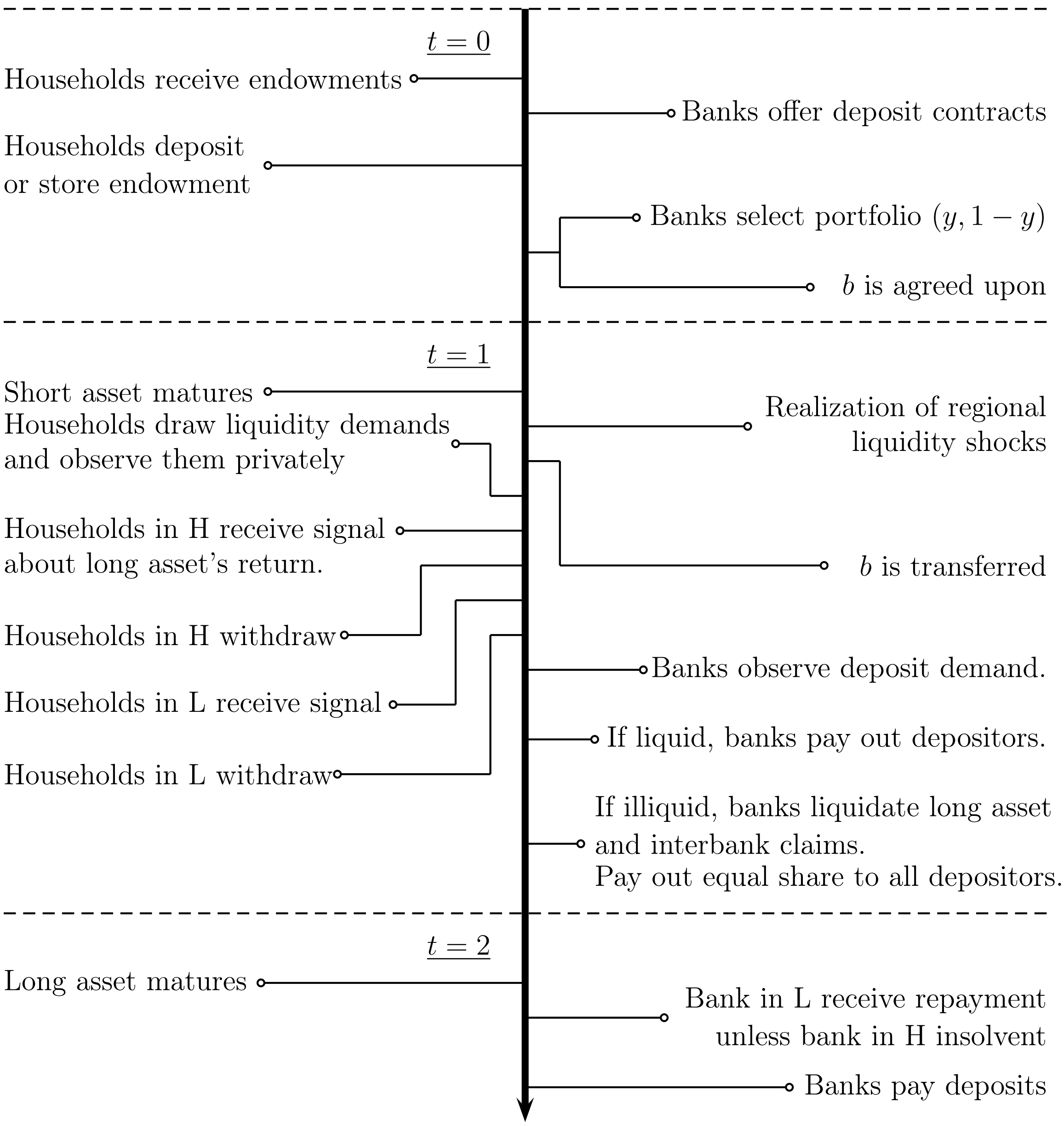

Figure 9: Timeline of the model

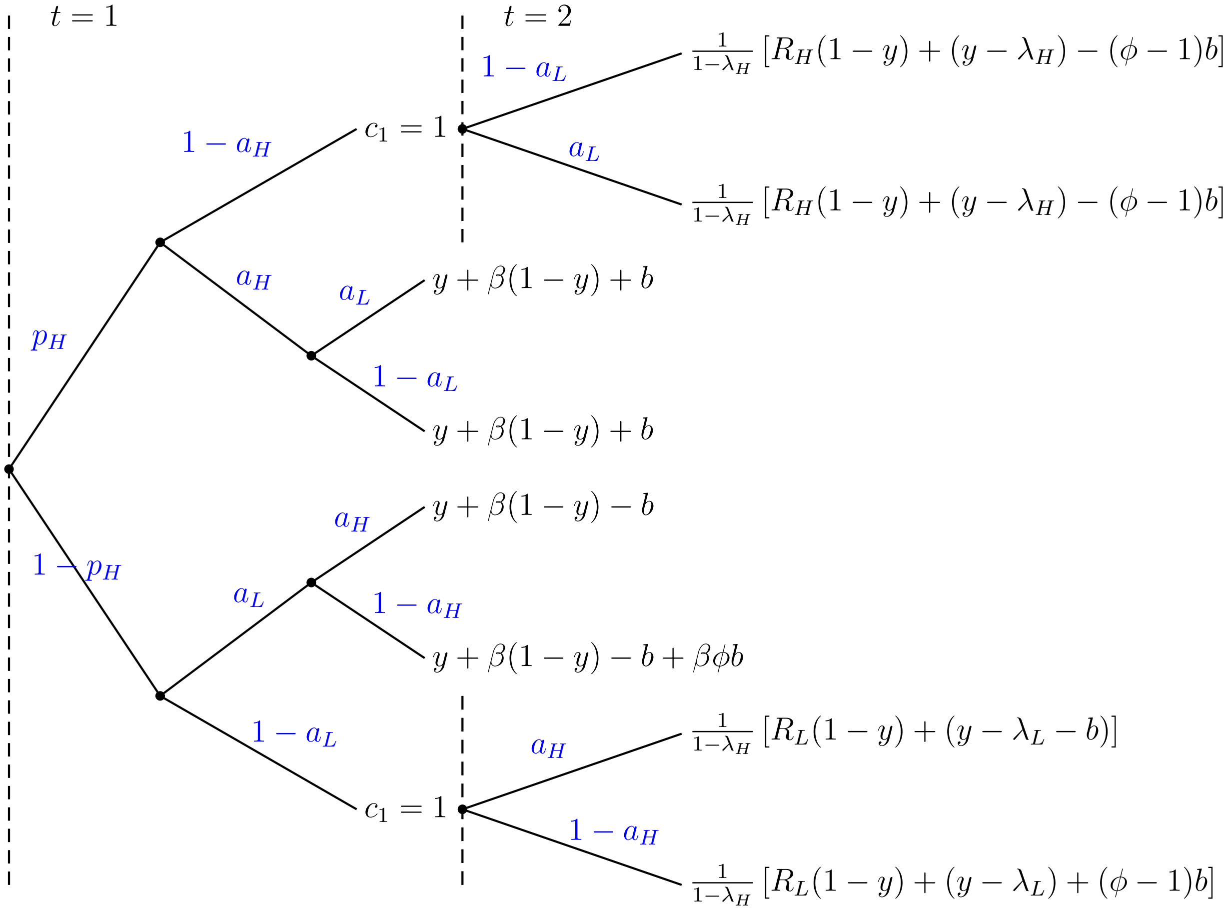

Figure 10: Payoff structure of the model.

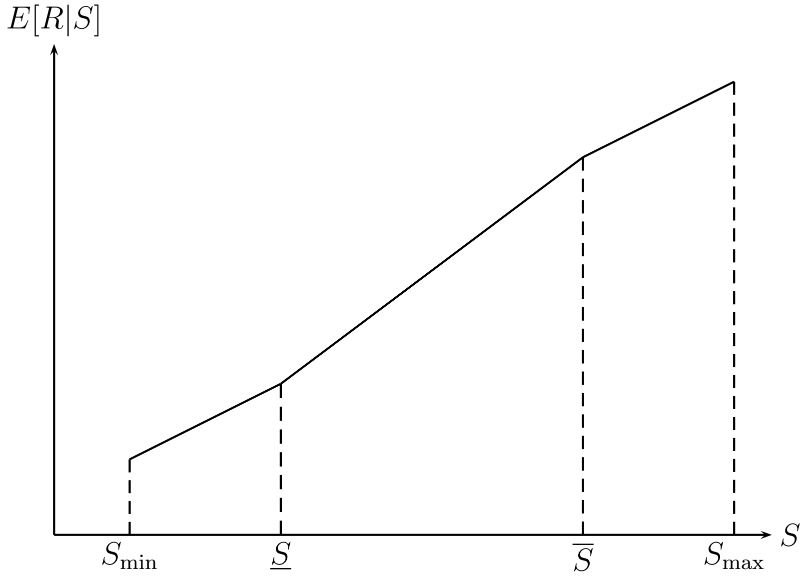

Figure 11: Conditional expectation $E[R|S]$ as a function of S.

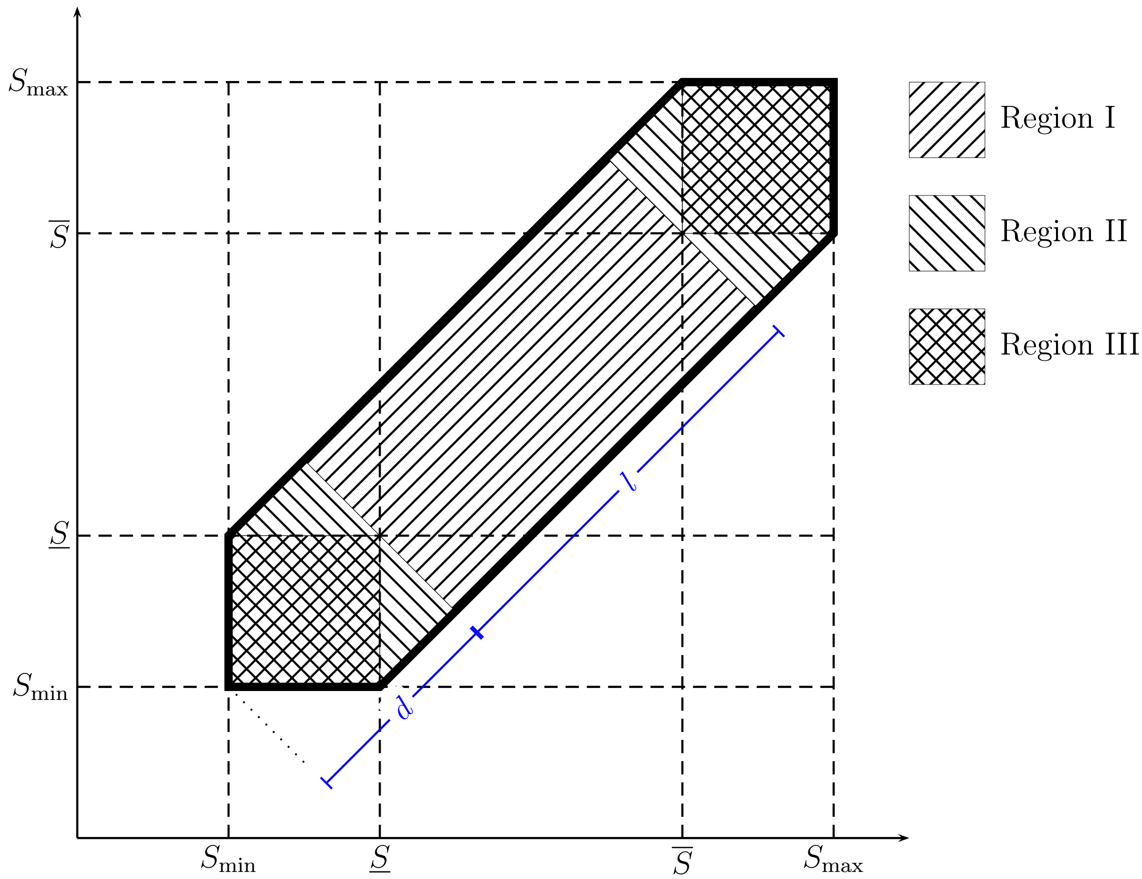

Figure 12: Support of $g(S_H, S_L)$ with partitioning into three regions.

A.2 Proofs

A.2.1 Distribution of the signal S

A convolution for random variable $Z \equiv X + Y$ is defined as:

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(z-y) f_Y(y) dy \tag{34}$$

To rewrite our signal extraction problem accordingly, let $R' \equiv \gamma\, R$ and $E' \equiv (1-\gamma)\, E$ where γ and $(1-\gamma)$ are weights.[10] Then,

$$\begin{aligned} R' &\sim U[\gamma(\mu-\sigma), \gamma(\mu+\sigma)] && (35)\\ E' &\sim U[-(1-\gamma)\chi, (1-\gamma)\chi] && (36)\\ S &= R' + E' && (37) \end{aligned}$$

Applying the convolution theorem, the density of the signal reads as:

$$\begin{aligned} f_S(S) &= \int_{-\infty}^{\infty} f_{R'}(S-E') \cdot f_{E'}(E') dE' \\ &= \int_{-(1-\gamma)\chi}^{(1-\gamma)\chi} f_{R'}(S-E') \cdot \frac{1}{2(1-\gamma)\chi} dE' \\ &= \frac{1}{2\chi(1-\gamma)} \int_{-(1-\gamma)\chi}^{(1-\gamma)\chi} f_{R'}(S-E') dE' \end{aligned}$$

It is useful to distinguish three cases throughout. The idea is that in the case II, the intermediate case, the noise has full support. In other words, the signal is not sufficiently bad or good to constraint the noise's support.

Case I: $S \in [S_{\min}, \underline{S}] = [\underline{R}' + \underline{E}', \underline{R}' + \overline{E}']$.

$$f_S(S) = \frac{1}{2(1-\gamma)\chi} \int_{-(1-\gamma)\chi}^{UB} f_{R'}(S-E') dE' \tag{38}$$

where the upper bound UB is a function of S. If $S = S_{\min}$ then $UB = \underline{E}' = -(1-\gamma)\chi$, and if $S = \underline{S}$ then $UB = \overline{E}' = +(1-\gamma)\chi$. Given the linearity of the

[10] This explicitely allows for the special case of $\gamma = 1$ and $(1-\gamma) = 1$, as the two weights are independent.

setup, we conjecture that $UB(S) = \kappa_0 + \kappa_1 S$. From the conditions

$$
\begin{aligned}
UB(S_{\min} = \gamma(\mu-\sigma) + (1-\gamma)(-\chi)) &= -(1-\gamma)\chi && (39)\\
UB(\underline{S} = \gamma(\mu-\sigma) + (1-\gamma)(\chi)) &= +(1-\gamma)\chi && (40)
\end{aligned}
$$

we obtain $\kappa_0 = -\gamma(\mu-\sigma)$ and $\kappa_1 = 1$. Thus, the upper bound is given by $UB(S) = -\gamma(\mu-\sigma) + S$. This gives:

$$
\begin{aligned}
f_S(S) &= \frac{1}{2(1-\gamma)\chi}\int_{-(1-\gamma)\chi}^{S-\gamma(\mu-\sigma)} f_{R'}(S-E')dE' && (41)\\
&= \frac{S-\gamma(\mu-\sigma)+(1-\gamma)\chi}{4(1-\gamma)\chi\gamma\sigma} && (42)\\
&= \frac{S-S_{\min}}{4(1-\gamma)\chi\gamma\sigma} \text{ for } S\in[S_{\min},\underline{S}] && (43)
\end{aligned}
$$

Case II: $S \in [\underline{S}, \overline{S}] = [\underline{R}' + \overline{E}', \overline{R}' + \underline{E}']$. Then, E has full support.

$$
\begin{aligned}
f_S(S) &= \frac{1}{2\chi(1-\gamma)}\int_{-(1-\gamma)\chi}^{(1-\gamma)\chi} f_{R'}(S-E')dE' && (44)\\
&= \frac{1}{2\gamma\sigma} \text{ for } S\in[\underline{S},\overline{S}] && (45)
\end{aligned}
$$

Case III: $S \in [\overline{S}, S_{\max}] = [\overline{R}' + \underline{E}', \overline{R}' + \overline{E}']$. This case is treated analogous to case I. Then:

$$
f_S(S) = \frac{1}{2(1-\gamma)\chi}\int_{LB}^{(1-\gamma)\chi} f_{R'}(S-E')dE' \qquad (46)
$$

where the lower bound again is a function of S. Again we conjecture that $LB = \kappa_0' + \kappa_1' S$ and obtain $\kappa_1' = 1$, $\kappa_0' = -\gamma(\mu-\sigma)$ and hence $LB = -\gamma(\mu-\sigma) + S$. This gives:

$$
\begin{aligned}
f_S(S) &= \frac{(1-\gamma)\chi+\gamma(\mu-\sigma)-S}{4(1-\gamma)\chi\gamma\sigma} && (47)\\
&= \frac{S_{\max}-S}{4(1-\gamma)\chi\gamma\sigma} \text{ for } S\in[\overline{S},S_{\max}] && (48)
\end{aligned}
$$

A.2.2 Conditional Expectation

The calculation of the conditional expectation also uses the partitioning support of the signal S support, giving rise to three three cases.

Case I: $S \in [S_{\min}, \underline{S}]$ Even if the lowest possible value for R is attained, receiving such a bad signal implies that not all realizations E are consistent with it. Hence, we have $E_{LB} = -\chi$, and $E_{UB} = (S - (\mu - \sigma)/(1-\gamma)$. Note that $E_{UB} \to -\chi$ if $S \to S_{\min}$ and $E_{UB} \to +\chi$ if $S \to \underline{S}$. This leads to:

$$\begin{aligned}
E[R|S] &= E[R|R = \frac{S}{\gamma} - \frac{1-\gamma}{\gamma}E] && (49)\\
&= E[R|\frac{S}{\gamma} - \frac{1-\gamma}{\gamma}E_{UB} \leq R \leq \frac{S}{\gamma} - \frac{1-\gamma}{\gamma}E_{LB}] && (50)\\
&= E[R|R_{\min} \leq R \leq \frac{S}{\gamma} + \frac{1-\gamma}{\gamma}\chi] && (51)\\
&= \frac{1}{2}\left[R_{\min} + \frac{S}{\gamma} + \frac{1-\gamma}{\gamma}\chi\right] \text{ for } S \in [S_{\min}, \underline{S}] && (52)
\end{aligned}$$

Case II: $S \in [\underline{S}, \overline{S}]$ E has now full support: $E_{LB} = -\chi$, $E_{UB} = \chi$. Then:

$$\begin{aligned}
E[R|S] &= E[R|\frac{S}{\gamma} - \frac{1-\gamma}{\gamma}\chi \leq R \leq \frac{S}{\gamma} + \frac{1-\gamma}{\gamma}\chi] && (53)\\
&= \frac{S}{\gamma} \text{ for } S \in [\underline{S}, \overline{S}] && (54)
\end{aligned}$$

Case III: $S \in [\overline{S}, S_{\max}]$ Similar to case I again. $E_{LB} = \frac{S-\gamma(\mu+\sigma)}{1-\gamma}$, $E_{UB} = \chi$. Then:

$$\begin{aligned}
E[R|S] &= E[R|\frac{S-(1-\gamma)\chi}{\gamma} \leq R \leq R_{\max}] && (55)\\
&= \frac{1}{2}\left[R_{\max} + \frac{S}{\gamma} - \frac{1-\gamma}{\gamma}\chi\right] \text{ for } S \in [\overline{S}, S_{\max}] && (56)
\end{aligned}$$

A.2.3 Joint Density $g(S_H, S_L)$

The support of $g(S_H, S_L)$ is shown in Figure (12) and partitioned into three regions. Region (I) is described by the length $l = \sqrt{2}(S_{\max} - \overline{S})$ and the width $b = 2\sqrt{2}(1-\gamma)\chi$; Region (II) is given by the two areas $S_L \in [S_{\min}, \underline{S}], S_H \in [\underline{S}, \underline{S} + (1-\gamma)\chi]$ (II-A) and $S_L \in [\underline{S}, \underline{S} + (1-\gamma)\chi], S_H \in [S_{\min}, \underline{S}]$ (II-B); Region (III) is given by the two areas $S_L \in [S_H, \hat{S}], S_H \in [S_{\min}, \hat{S}]$ (III-A) and $S_L \in [S_{\min}, S_H], S_H \in [S_{\min}, \hat{S}]$ (III-B).

We focus on the case $S_k \leq \underline{S}$. If $\chi = (1-\gamma)/\gamma\sigma$, then half of the probability lies in (i) $S_H \leq \underline{S}$, $S_L \leq \underline{S}$; (ii) $S_H \in [\underline{S}, \overline{S}]$, $S_L \leq \underline{S}$; (iii) $S_L \in [\underline{S}, \overline{S}]$, $S_H \leq \underline{S}$. We solve the two-dimensional density $g(S_H, S_L)$ explicitely and find the following geometric figures in the three regions (I)-(III): Region I - prism; Region II - pyramid with triangular base; Region III - pyramid with a quadratic base. In order to determine the two-dimensional distribution $g(S_H, S_L)$ we proceed in two steps. First, we obtain the height h at $S_H = S_L = \underline{S}$ by using geometric methods. The volume of the distribution is normalized to unity: $V \stackrel{!}{=} 1 = I + 4II + 2III$. Then we determine $g(S_H, S_L)$ for the three regions shown in Figure (12).

Region I. First, we partition the support as shown in Figure (12). Then, $D = l + 2d$ and we can write it as $D = \sqrt{2}(S_{\max} - S_{\min}) = \sqrt{2}[\gamma(\mu+\sigma) + (1-\gamma)\chi - \gamma(\mu-\sigma) - (1-\gamma)\chi] = 2\sqrt{2}(\gamma\sigma + (1-\gamma)\chi)$. The length l of Region III is given as $l = \sqrt{2}(S_{\max} - \overline{S}) = 2\sqrt{2}(1-\gamma)\chi$. From this we obtain $l = 2\sqrt{2}(\gamma\sigma - (1-\gamma)\chi) \geq 0$ as $\sigma \geq (\frac{1-\gamma}{\gamma})\chi$. The width b of the prism is given as $b = 2\sqrt{2}(1-\gamma)\chi$ and the base thus is $A_{base} = \frac{1}{2}hb = h(1-\gamma)\chi$ and the volume V_{prism} is given as $V_{prism} = A_{base} l = h(1-\gamma)\chi \cdot 4(\gamma\sigma - (1-\gamma)\chi$.

Region II. The volume of a pyramid with triangular base is given as $V_{pyr,3} = \frac{1}{3}hA_{base}$ with $A_{base} = \frac{1}{2}(1-\gamma)\chi \cdot 2(1-\gamma)\chi = (1-\gamma)^2\chi^2$.

Region III. The volume of a pyramid with squared base is determined by $A_{base} = [2(1-\gamma)\chi]^2 = 4(1-\gamma)^2\chi^2$ to be $V_{pyr,4} = \frac{4}{3}h(1-\gamma)^2\chi^2$.

From the total volume $V_{total} = V_{prism} + 2V_{pyr,4} + 4V_{pyr,3} \stackrel{!}{=} 1$ we obtain for the height h:

$$h = \frac{1}{4\gamma(1-\gamma)\chi\sigma} \tag{57}$$

We are now interested in the two-dimensional density $g(S_H, S_L)$ in the region $S_H, S_L \in [S_{\min}, \underline{S}]$, which is Region III in Figure (12) and has the shape of a pyramid with squared base. The apex of the pyramid is at the top right corner (point C) of the base and has height h. This effectively partitions the base into two triangular regions A and B, as shown in Figure (5). The points D, C, E of the partitioned base and the apex H are given as:

$$C = \begin{pmatrix} \underline{S} \\ \underline{S} \\ 0 \end{pmatrix}, \quad D = \begin{pmatrix} S_{\min} \\ S_{\min} \\ 0 \end{pmatrix}, \quad E = \begin{pmatrix} S_{\min} \\ \underline{S} \\ 0 \end{pmatrix}, \quad H = \begin{pmatrix} \underline{S} \\ \underline{S} \\ h \end{pmatrix} \tag{58}$$

We now can identify the plane that is determined by the points E, D and H:

$$\epsilon_0 = \begin{pmatrix} S_{\min} \\ \underline{S} \\ 0 \end{pmatrix} + \delta_0 \begin{pmatrix} 0 \\ S_{\min} - \underline{S} \\ 0 \end{pmatrix} + \delta_1 \begin{pmatrix} \underline{S} - S_{\min} \\ 0 \\ h \end{pmatrix} = \begin{pmatrix} S_{\min} + \delta_1(\underline{S} - S_{\min}) \\ \underline{S} - \delta_0(\underline{S} - S_{\min}) \\ \delta_1 h \end{pmatrix} \tag{59}$$

and interject it with the line k_0 that goes through the point $G = (S_H, S_L)$, $k_0 = (S_H, S_L, t)^t$ where $t = g(S_H, S_L)$.

We obtain for t:

$$t = h\frac{S_H - S_{\min}}{\underline{S} - S_{\min}} = \frac{1}{\kappa}(S_H - S_{\min}) \tag{60}$$

where $\kappa \equiv 4(1-\gamma)\gamma^2\chi\pi$ and $\pi \equiv 2\sigma\frac{1-\gamma}{\gamma}\chi$. Analogously, we obtain for $S_L \in [S_{\min}, \underline{S}]$, $S_H \in [S_L, \underline{S}]$ (Region B):

$$g(S_H, S_L) = \frac{1}{\kappa}(S_L - S_{\min}) \tag{61}$$

Now, we consider Region II in Figure (12) and repeat the above calculation. Therefore, $S_L \in [S_{\min}, \underline{S}]$, $S_H \in [\underline{S}, \underline{S}+(1-\gamma)\chi]$. The system of three equations has more interaction now, as δ_1 depends on δ_0 as well and $t = g(S_H, S_L)$ thus depends on both S_H and S_L. Consider the points:

$$C = \begin{pmatrix} \underline{S} \\ \underline{S} \\ 0 \end{pmatrix} \quad , \quad D = \begin{pmatrix} \underline{S} + (1-\gamma)\chi \\ \underline{S} - (1-\gamma)\chi \\ 0 \end{pmatrix} \quad , \quad E = \begin{pmatrix} \underline{S} \\ S_{\min} \\ 0 \end{pmatrix} \quad , \quad H = \begin{pmatrix} \underline{S} \\ \underline{S} \\ h \end{pmatrix} \tag{62}$$

and define a plane $\epsilon_1 = \overrightarrow{OD} + \delta_0\overrightarrow{DE} + \delta_1\overrightarrow{DH}$:

$$\epsilon_1 = \begin{pmatrix} S_{\min} + (1-\gamma)\chi[3 - \delta_0 - \delta_1] \\ S_{\min} + (1-\gamma)\chi[1 - \delta_0 + \delta_1] \\ \delta_1 h \end{pmatrix} \tag{63}$$

which yields

$$g(S_H, S_L) = h\frac{(S_L - S_H) + (\underline{S} + S_{\min})}{\underline{S} - S_{\min}} \leq h \tag{64}$$

Likewise, one obtains for $S_L \in [\underline{S}, \underline{S} + (1-\gamma)\chi]$, $S_H \in [S_{\min}, \underline{S}]$:

$$g(S_H, S_L) = h\frac{(S_H - S_L) + (\underline{S} - S_{\min})}{\underline{S} - S_{\min}} \leq h \tag{65}$$

A.2.4 Conditional Expectation $E[R|S_H, S_L]$

The conditional expectation has the same mathematical structure as $g(S_H, S_L)$ for $S_k \in [S_{\min}, \underline{S}]$, $k \in \{H, L\}$ (Region III). We thus use the same geometric approach as before, with the height is now denoted as m instead of h and the reference point O being $R_{\min}$. The height m is obtained by observing that for $S_H = S_L = \overline{S}$: $E[R|S_H, S_L] = E[R|\overline{S}]$. In particular: $E[R|\underline{S}, \underline{S}] = \frac{\underline{S}}{\gamma} = R_{\min} + \frac{1-\gamma}{\gamma}\chi \equiv m$. The four points we now use to obtain the equations of the planes describing $E[R|S_H, S_L]$ are:

$$E = \begin{pmatrix} S_{\min} \\ S_{\min} \\ R_{\min} \end{pmatrix} \quad , \quad C = \begin{pmatrix} \underline{S} \\ S_{\min} \\ R_{\min} \end{pmatrix} \quad , \quad D = \begin{pmatrix} \underline{S} \\ \underline{S} \\ R_{\min} \end{pmatrix} \quad , \quad H = \begin{pmatrix} \underline{S} \\ \underline{S} \\ m \end{pmatrix} \tag{66}$$

from which we obtain for the plane $\epsilon_2 : \overrightarrow{OC} + \delta_0 \overrightarrow{OE} + \delta_1 \overrightarrow{CH}$ and the line $k_2 : (S_H, S_L, t)^t$:

$$t = R_{\min} + \frac{1}{2\gamma}(S_L - S_{\min}) = E[R|S_H, S_L] \; \forall S_H \in [S_{\min}, S_L] \tag{67}$$

Analogously we obtain for $S_H \in [S_{\min}, \underline{S}]$, $S_L \in [S_H, \underline{S}]$:

$$E[R|S_H, S_L] = R_{\min} + \frac{1}{\gamma}(S_H - S_{\min}) \; \forall S_L \in [S_H, \underline{S}] \tag{68}$$

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