Endogenous Fluctuations in Endogenous Growth Models

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Objectives:

Develop monetary endogenous growth overlapping generations models with inflation targeting to analyze growth dynamics:

- Multiple Equilibria;

-Indeterminancy;

-Endogenous fluctuations;

-Topological Chaos.

Motivation and Contribution:

–[Murota, Ryu-Ichiro, Monetary Expansion and Productive Public Expenditure in a Cash-in-Advance Economy. Japanese Economic Review, Vol. 58, No. 2, pp. 255-272, June 2007];
– Ireland and Zimmermann;
– Chetty and Ratha (1994);
– Gupta (2011)- endogenous fluctuations;

– Back to Ireland ⇒ Michel (1993)[story and similarity: $y_t = A_t k_t^{\alpha} n_t^{(1-\alpha)}$; $A_t = a k_{t-1}^{(1-\alpha)}$ with Gupta (2011)], Futagami and Mino (1995), Griener (1996), Greiner and Semmler (1996), Shigoka (1997), Evans et al. (1998), Matsuyama (1999), Horii (2001), Kitagawa and Shibata (2001, 2005), Gupta and Vermeulen (2010) [unintentional];

– Extension of Gupta (2011).

Economic Environment: Four agents: (a) Consumers; (b) Financial Intermediaries/Banks; (c) Firms, and; (d) The government/monetary authority

(I) **Consumer's Problem:**

$$\max U(c_{t+1})$$

s.to.

$$\begin{aligned} p_t d_t &= p_t w_t \\ p_{t+1} c_{t+1} &= (1 + i_{dt+1}) p_t d_t \end{aligned}$$

Financial Intermediaries

– Banks behave competitively but are subjected to cash reserve requirements

– Provide a simple pooling function

– For simplicity bank deposits are assumed to be one period contracts

Formally,

$$\Pi_{Bt} = i_{Lt} L_t - i_{dt} D_t$$

$$\begin{aligned} M_t + L_t &\leqslant D_t \\ M_t &\geq \gamma_t D_t \end{aligned}$$

Firms:

All firms are identical and produces a single final good using:

$$y_t = Ak_{t-1}^{\alpha}(n_t g_t)^{(1-\alpha)}$$

where $A>0$.

The representative firm maximizes the discounted stream of profit flows subject to the capital evolution and loan constraints. Formally,

$$\max_{k_t, n_t} \sum_{i=0}^{\infty} \rho^i [p_t y_t - p_t w_t n_t - (1 + i_{Lt}) L_{t-1}]$$

$$\begin{aligned} k_t &\leq (1 - \delta_k) k_{t-1} + i_{kt-1} \\ p_{t-1} i_{kt-1} &\leq L_{t-1} \\ L_{t-1} &\leq (1 - \gamma_t) D_{t-1} \end{aligned}$$

Government:

An infinitely-lived government purchases g_t units of the consumption good;

– Assumed to be useful to the agents;

– Targets Inflation;

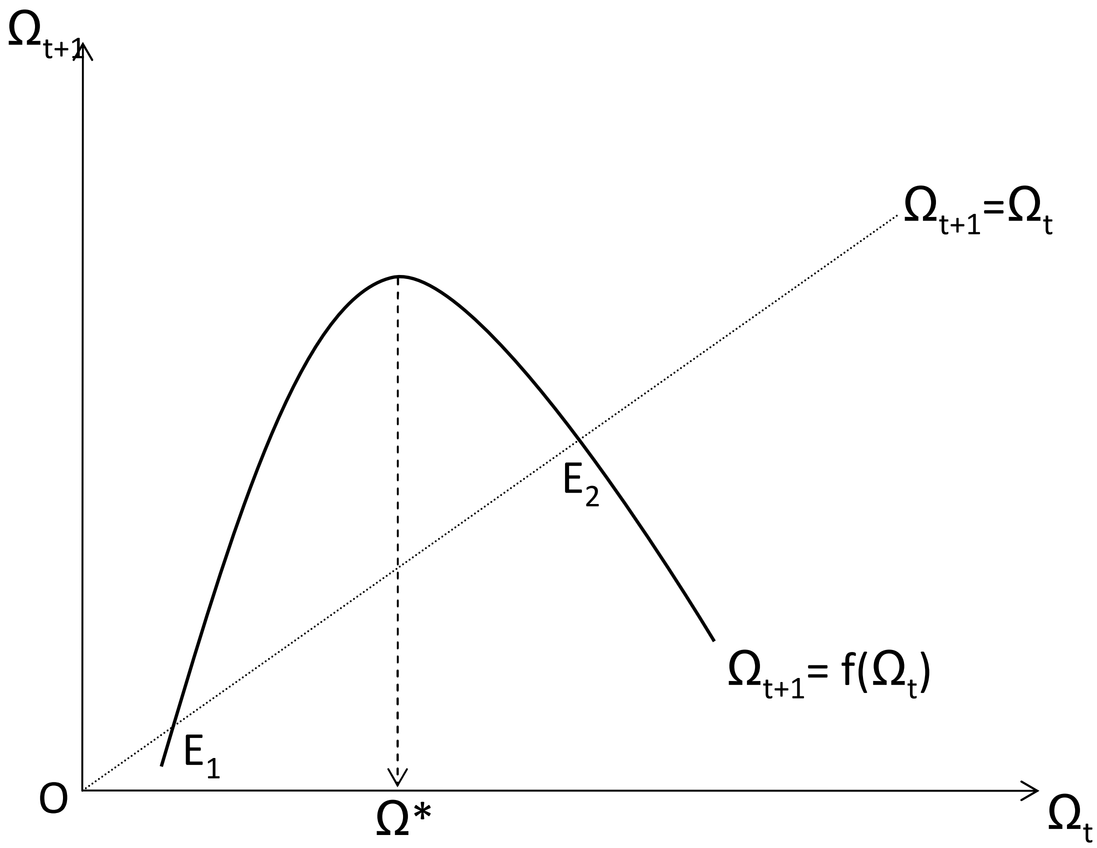
– The government finances these purchases by seigniorage.

$$
\begin{aligned}
g_t &= \frac{M_t - M_{t-1}}{p_t} \\
g_t &= \gamma_t d_t \left(1 - \frac{1}{\Omega_t \widehat{\Pi}}\right)
\end{aligned}
$$

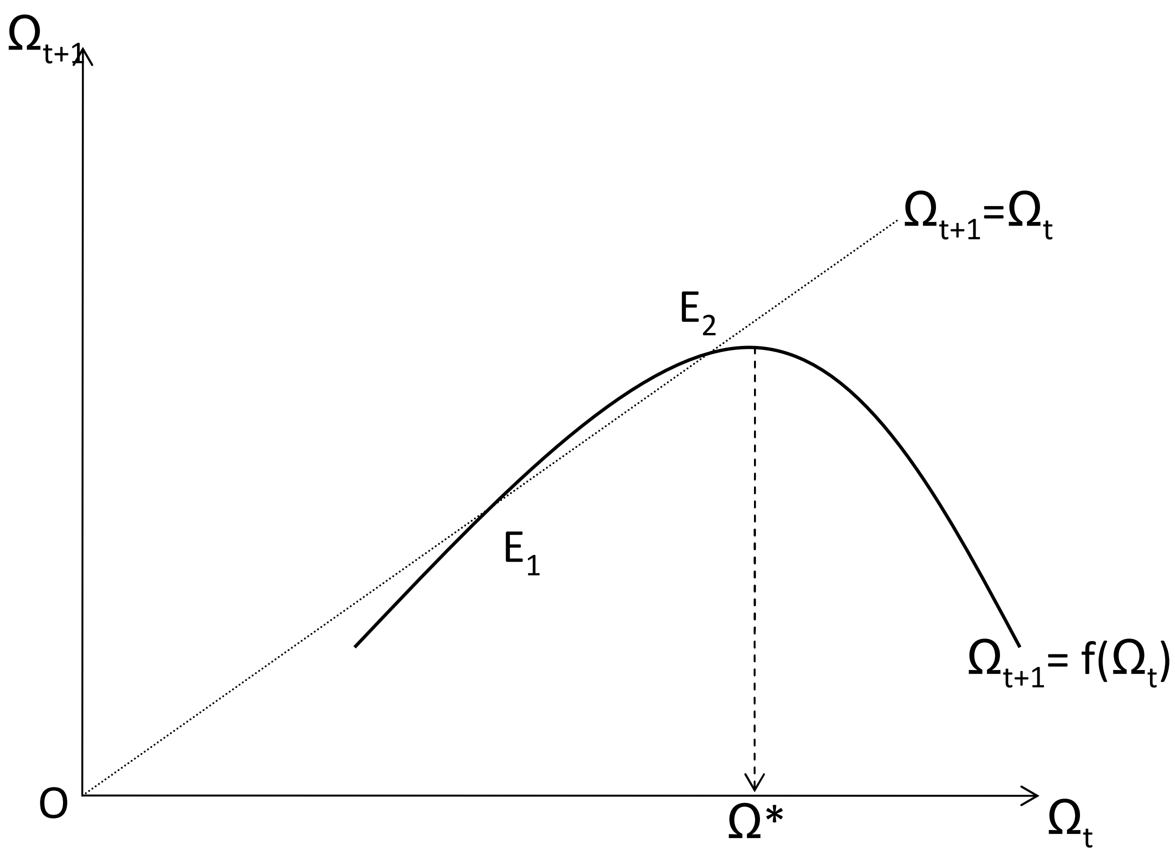
Growth Dynamics:

$$\Omega_{t+1} = (1-\gamma_t)A(1-\alpha)\left(A(1-\alpha)\left[1-\frac{1}{\hat{\Pi}\Omega_t}\right]\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\frac{1}{\Omega_t}$$

Ω_t is not a state variable, so it can jump $\Rightarrow$ a stable (Ω_H) steady-state is indeterminate ($\Rightarrow$ infinitely many RATEX paths to the stable equilibrium from initial k_1).



Multiple Equilibria with Endogenous Fluctuations/Chaos



Multiple Equilibria without Endogenous Fluctuations

Conclusions:

Multiple Equilibria: low-growth (low-welfare) $\Rightarrow$ unstable; high-growth (high-welfare) $\Rightarrow$ stable); Indeterminancy; Endogenous Fluctuations; Chaos;

Policy Related: Low target (indeterminancy under multiple equilibria) or High-Target (fluctuations or chaotic behavior around the high-growth equilibria) $\Omega^* = \frac{\frac{\alpha}{\widehat{\Pi}}+(1-\alpha)}{\alpha}$;

Few Points: (i) No lagged inputs with money growth rule $\Rightarrow$ no growth dynamics; (ii) Lagged inputs without inflation targeting (money growth rule) $\Rightarrow$ endogenous fluctuations; (iii) No Lagged inputs with inflation targeting $\Rightarrow$ convergent (without oscillations) growth dynamics; (iv) an interest rate rule with growth rate (Gillman *et al.*, 2007);

Future Research: Learning.