SOUTH AFRICAN HISTORICAL INTEREST RATE VOLATILITY - EVIDENCE OF REGIME SWITCHING

1 Introduction

The value of a financial asset may be considered a function of the expected variances and means of its rate of return (Engle, 1982). Accordingly, estimates of volatility (the square root of variance) and mean reversion may be relevant to value derivative instruments and other securities. The valuation of financial assets under a variety of potential volatility scenarios is essential for their risk management from the perspective of all economic stakeholders. According to the Bank of England (2016), a severe but plausible period of financial stress must be included in the scenario testing. The stakeholders include the institution itself who seeks to minimize capital holdings, the institution's counterparties who seek to minimise counterparty credit risk, and regulatory supervisors who are concerned with minimising systemic risk. Longstaff & Schwartz, (1992) advocate the use of a GARCH(p,q) process over options-price implied volatility to extract the volatility estimates of an interest rate. GARCH models are able to capture the volatility clustering phenomenon, which is the grouping of periods of high volatility together rather than being equally spaced (Dueker, 1997). Interest rate data sometimes exhibits a mean reversion characteristic, returning to the long run average rate over time (Holilal, 2011). Mean reversion is commonly modelled using an AR(1) process where the next periods change depends linearly on the current level (Venter, 2010)

In the risk management of a portfolio of financial assets, it is important to generate scenarios of what could actually happen, thus using historical data is appropriate to capture features of the process (Venter, 2010). In South Africa, the 3-month Jibar is typically used as the benchmark interest rate for ZAR denominated IRS (West, 2008; Du Preez, 2011; South African Reserve Bank, 2018). The 3-month Jibar is the average mid of the 3-month Negotiable Certificates of Deposit (NCD) rates quoted by several local and foreign banks (excluding the two highest and lowest rates). Thus, it is a variable used to extract estimates of volatility and mean reversion for short term interest rates. This study shows that fitting a GARCH (1,1) to the differenced Jibar 3 Month data results in estimates of highly persistent conditional variance. Persistence in variance is the degree to which the past volatility of a variable explains its current volatility. The 1 Month, 6 Month and 12 Month Jibar interest rates also exhibit highly persistent variance, indicating a trend in South African interest rates in general over the observed period.

According to Gray (1996), GARCH models of the short-term interest rate often imply highly persistent conditional variance due to unaccounted for regime switching. He identifies possible causes for changes in the conditional variance of the Fed funds rate between January 1970 – April 1994 as the change in monetary instrument targeted by the Federal Reserve (Fed), the OPEC oil crisis, the Black Monday 1987 stock market crash and various wars involving the US.

The standard ARCH and GARCH models do not allow for an asymmetric effect in the data. An asymmetric effect occurs when conditional volatility increases when there is negative market information. Various extensions of the GARCH model were created in order to assess the asymmetric effect. This study will investigate the in-sample accuracy of the ARCH, GARCH, E-GARCH, GJR-GARCH and T-GARCH models for up to four regimes and six conditional distributions. Thus, this Chapter tests 120 GARCH-type volatility models to determine which model, distribution and number of states best fit the Jibar 3 month data in order to extract the most accurate estimation of historical volatility. The mean-reverting rate will be captured through an AR(1) process.

2 Theoretical Underpinnings

In the ARCH model of Engle (1982), the conditional variance depends on the lagged squared change in the variable. The GARCH model of Bollerslev (1986) extends the ARCH model to allow conditional variance to depend on its own past values as well. The standard GARCH(p,q) model regresses on lagged terms of squared returns (p) and variance (q).

The empirical findings of Engle, Ng & Rothschild (1990), Gray (1996), Koedijk et al., (1997) Hillebrand (2005), Bauwens, Preminger & Rombouts (2010) and Olweny (2011) report a high degree of persistence in variance for a variety of financial assets, including interest rates, when only a single regime is considered. In a GARCH(1,1) model persistence in variance occurs when the sum of the $\alpha_1 + \beta$ parameters are close to or exceed 1 (see section 4 below for the full equation). The process is variance-covariance unstable if the sum of $\alpha_1 + \beta$ parameters exceed 1 (Bollerslev 1986).

To account for this empirically observed persistence in variance, Engel and Bollerslav introduced the I-GARCH in which shocks to variance do not decay over time. The sum of the ARCH and GARCH parameters are restricted to equal 1. According to Lamoureux & Lastrapes (1990), the I-GARCH model lacks theoretical motivation because it does not allow asset prices to follow a random walk, instead prices are almost completely explained by their past observations. They argue that the presence of structural shifts in the unconditional variance bias the persistence estimates upwards. Thus, structural or regime shifts are mistaken for periods of volatility clustering. A single regime model assumes that the conditional mean and conditional variance remain fixed throughout the sample period. However, the economic mechanism that generates the variable may change over time, for example, changes in monetary policy or a financial crisis.

Using dummy variables to indicate structural shifts, Lamoureux & Lastrapes (1990) report decreases in the persistence of variance. Therefore, they demonstrate that ignoring structural shifts can result in an overestimation of the persistence of variance. Cai (1994) and Hamilton & Susmel (1994) demonstrate the benefits of a Markov switching ARCH model over the dummy variable technique used by Lamoureux & Lastrapes (1990). A Markov chain assumes the current value of a state variable depends on its immediate past value. It allows for frequent switching between states at random times and its transition probabilities determine the persistence of each regime (Hamilton, 2016). The regimes are not directly observed however, probabilistic statements can be made about the time-varying transition probabilities, and thus the relative likelihood of being in each state. Thus, the regimes are probabilistic and determined by the data, no prior classification is necessary as required by the dummy variable method employed by Lamoureux & Lastrapes (1990).

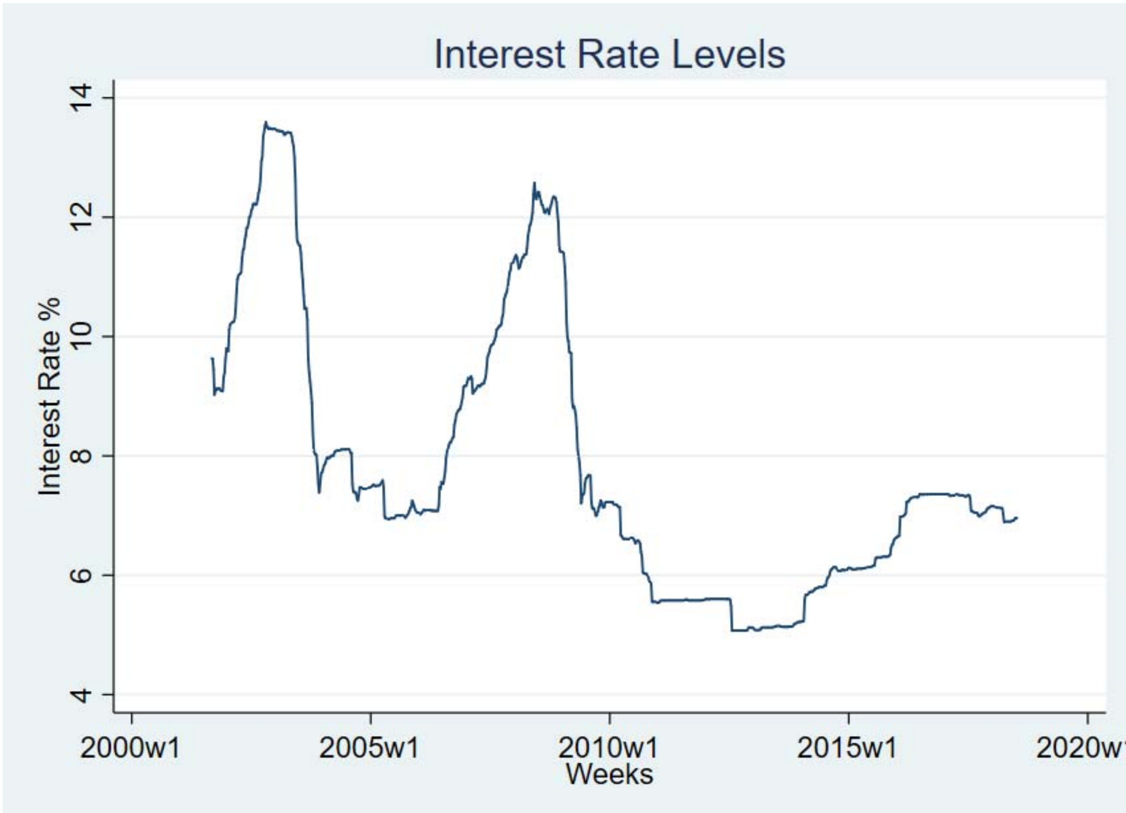
Extending Markov switching to a GARCH model renders the estimation intractable because it requires integration over unobserved regime paths which increases exponentially as the sample size increases. I.e. the conditional variance at time *t* depends on the entire sequence of regimes up to *t-1*. Gray (1996), Dueker (1997), Klaassen (2002) made successive improvements to the estimation and overcame the computational difficulties in estimating

Markov switching GARCH models using an approximating method which collapses the past regime specific conditional variances. Haas, Mittnik & Paolella (2002) further improved the estimation of MS-GARCH models by allowing the GARCH process to evolve independently of those in other states, therefore the model does not face the path-dependency problem.

3 Data

The data analysed consists of 877 weekly observations of 3 Month Jibar in total, spanning from week 36 in 2001 to week 28 in 2018. The data was sourced from Bloomberg. In South Africa, the SARB controls the Repo rate in pursuance of its goal to achieve and maintain price stability within the target band of 3-6%. The Repo rate is the benchmark interest rate used by financial market participants, thus changes in the Repo rate result in changes in market interest rates, including Jibar. A comparison of Figure 1 with Figure 2 presents a clear positive relationship between the Repo rate and Jibar, confirming that Jibar does closely follow movements in the Repo rate.

Figure 1: 3 Month Jibar absolute level



Source: Author.

Figure 2: South African Repo Rate. Period 2001/09/07 - 2018/07/11

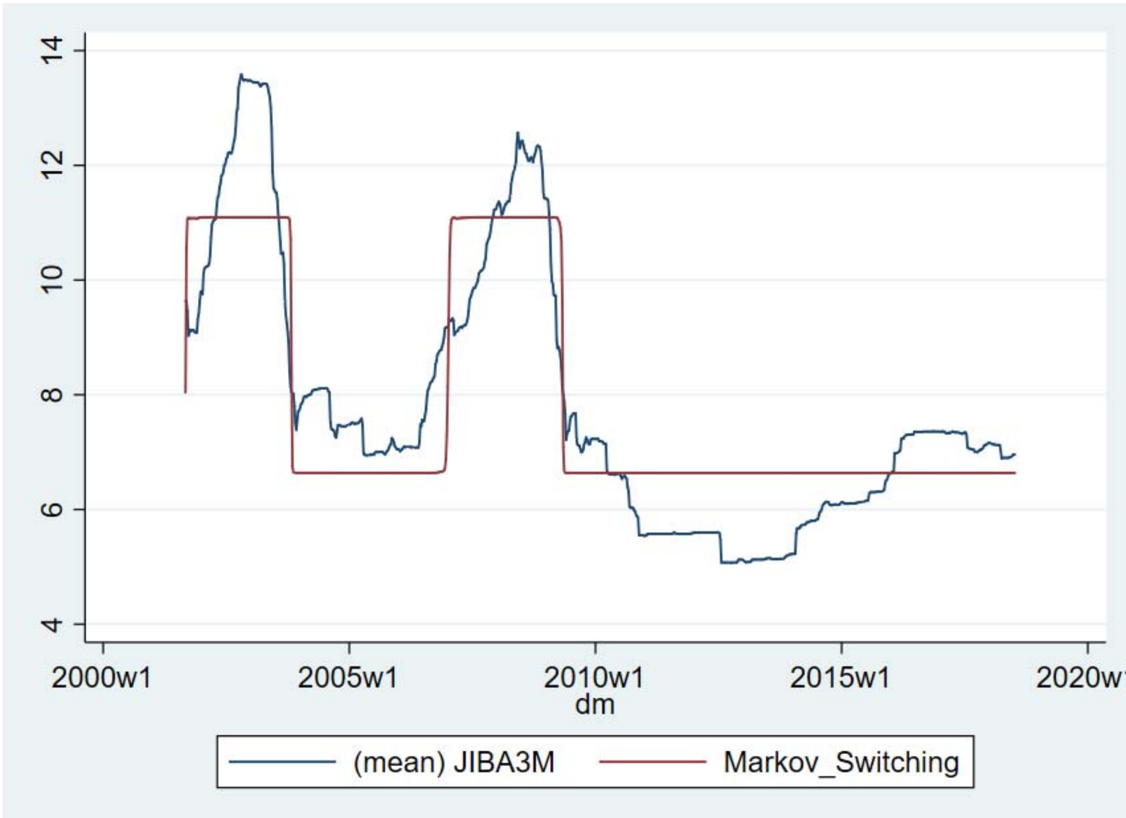
Source: TradingEconomics.com [Accessed on 2019/03/29].

Since the adoption of inflation targeting during 2000 there have been two periods where the inflation rate exceeded the upper target limit of 6% by a sizable amount and for a prolonged time, see figure 3. The first was during 2002 when supply-side shocks such as a depreciation of the rand coupled with increases in global food and oil prices caused inflation rates to soar above the inflation target band (Nell, 2018). The second was in the wake of the global financial crisis of 2007/2008. Although South Africa did not experience a local financial crisis, it was one of the worst affected emerging economies according to the Financial Stability Board. The South African stock market fell by 36.0% between May and December 2008, leading to the loss of almost one million jobs (Financial Stability Board, 2013). A sharp depreciation as foreign investment was withdrawn from the country following the crisis caused inflation to rise.

Figure 3: South African inflation rate. Period 2001/09/07 - 2018/07/11

Source: TradingEconomics.com [Accessed on 2019/03/29].

Figure 4: Evidence of Regime Switching in the 3 Month Jibar interest rate data



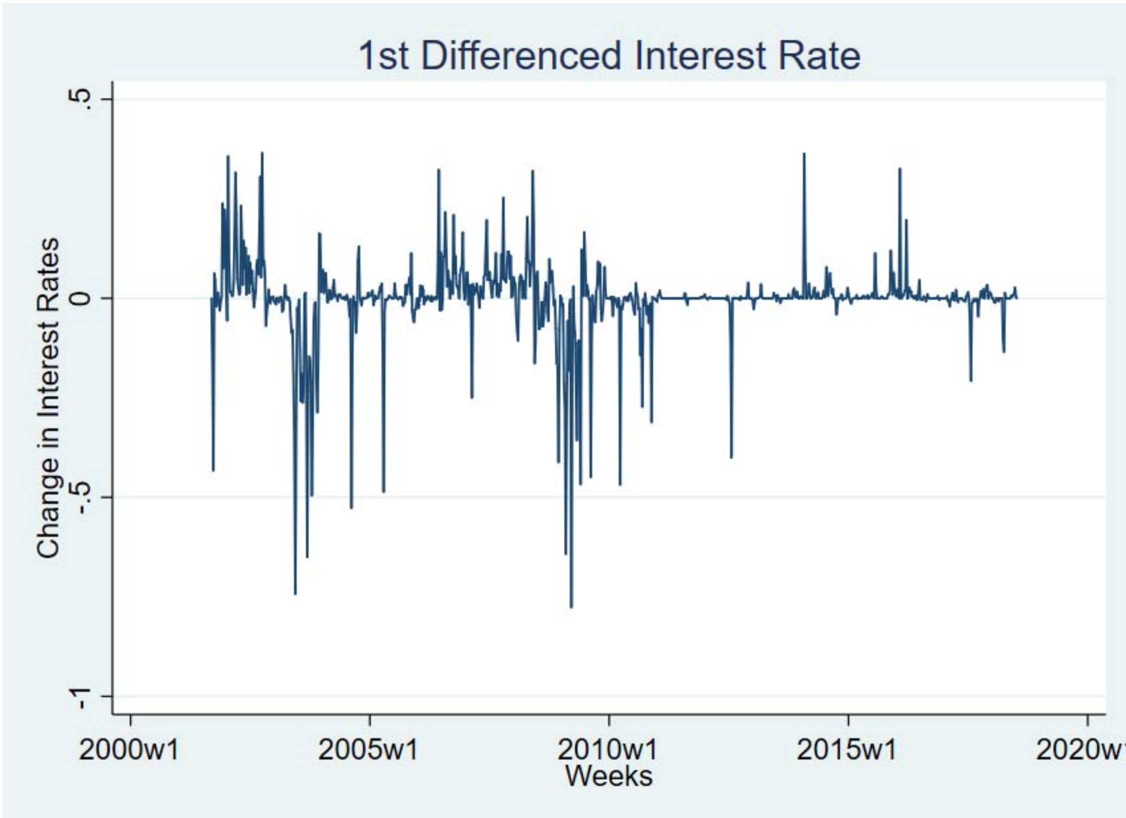
Source: Author.

In response to both of these periods of prolonged high inflation, the SARB employed contractionary monetary policies as can be seen in figure 2. These changes in monetary policy support the presence of regime switching in the data. Figure 4 illustrates these regime shifts from relatively low-interest rates (with an average of 6.62%) to relatively high-interest rates (with an average of 10.86%).

3.1 Characteristics of the data

The Augmented Dickey-Fuller and Phillips Perron unit root tests both indicated that the differenced 3-month Jibar data is stationary, a requirement for the AR and Markov switching GARCH models. The differenced data consists of 876 observations.

Figure 5: Evidence of Regime Switching in the 3 Month Jibar interest rate data



Source: Author.

Following Engle (1982) and Bollerslev (1986), the Lagrange multiplier (LM) test for autoregressive conditional heteroscedasticity (ARCH) is performed to test for the presence of ARCH effects. The LM test detected ARCH effects for the weekly averaged 3 Month Jibar, however, the daily data did not present an ARCH effect. Rejecting the null hypothesis for the weekly 3 Month Jibar data allows for a GARCH model to be fitted.

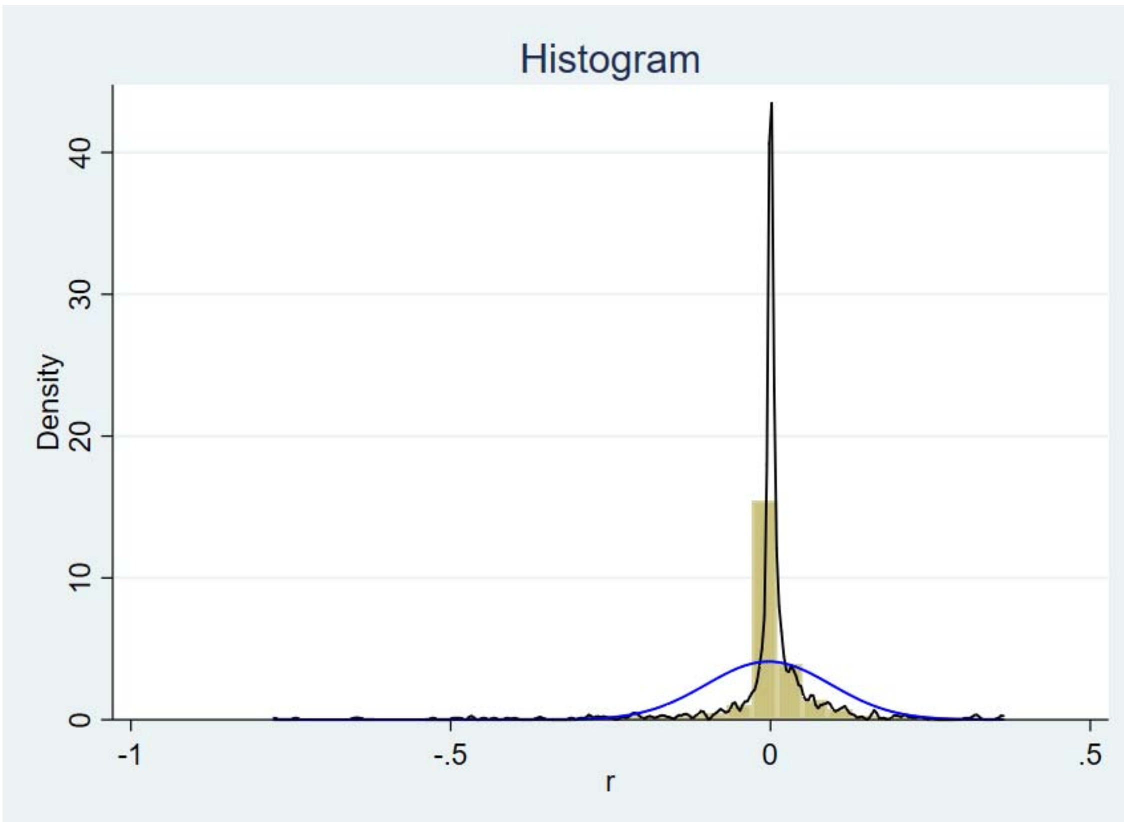
Table 1: Statistics for the first four moments of the differenced weekly averaged 3 Month Jibar

Statistics for the first four moments of the 3 Month Jibar	
Mean	-0,0030525
Standard deviation	0,0973184
Skewness co-efficient	-2,736778
Kurtosis co-efficient	21.3041

Source: Author's calculations.

The skewness and kurtosis coefficients indicate that the conditional distribution is not normally distributed, in addition, the null of normality is rejected by the Jarque-Bera test. The histogram in figure 6 illustrates the highly leptokurtic nature of the data.

Figure 6: Histogram for 3 Month Jibar



Source: Author.

4 Methodology

4.1 Econometric Model for Mean Reversion

A typical model of the behaviour of interest rates is captured by the following AR(1) process:

$$r_t = c + br_{t-1} + e_t$$

Equation 1: Mean equation

where r_t is the differenced interest rate, c is the constant, br_{t-1} is the first order autoregressive process and e_t is the residual.

The mean reversion rate Alpha (α), which can simply be calculated as: $\alpha = 1 - b$

4.2 Econometric Models for Volatility

Four core GARCH type models and an ARCH model are estimated for one to four states using six conditional distributions. A total of 120 models are estimated using the maximum likelihood method and compared using the AIC and BIC criteria for goodness of fit and parsimony.

4.2.1 Conditional volatility models

The data must be serially uncorrelated therefore to model the conditional variance the mean equation for the differenced interest rate is denoted as equation 1 above: $r_t = c + br_{t-1} + e_t$, which is an AR(1) process.

Following Ardia et al. (2018) the general Markov switching GARCH specification employed is:

$$r_t | (s_t = k, I_{t-1}) \sim D(0, \sigma^2_{k,t}, \xi_k)$$

Equation 2: Markov switching GARCH specification

where $D(0, \sigma^2_{k,t}, \xi_k)$ is a continuous distribution with zero mean, $\sigma^2_{k,t}$ is the state and time varying variance and ξ_k captures the additional shape parameters. The integer-valued stochastic state variable s_t is defined on the discrete space {1, …, k}, is assumed to evolve according to a first order ergodic (ie unconditional) homogeneous Markov chain with transition probability matrix $P = \{p_{i,j}\}^k{}_{i,j=1}$ with $p_{i,j} = P[s_t = j | s_{t-1} = i]$. I_{t-1} is the information set observed up to time *t-1*.

$$\text{Transition probability matrix: } P \equiv \begin{bmatrix} p1,1 & \cdots & p1,k \\ \vdots & \ddots & \vdots \\ pk,1 & \cdots & pk,k \end{bmatrix}$$

Following Haas, Mittnik & Paolella (2002) the conditional variance is a function of past observations of the variable as well as a regime-dependent vector of parameters, overcoming the path dependency problem.

All of the GARCH type models follow the above Markov switching specification however the conditional volatility process differs, as discussed in this section. The standardised innovations are allowed to follow one of six conditional distributions, discussed in section 4.2.2.

The ARCH model of Engle (1982) is given by:

$$\sigma_{k,t}^2 = \alpha_{0,k} + \alpha_{1,k} r_{t-1}^2$$

Equation 3: ARCH (1)

where $\sigma_{k,t}^2$ is the variance rate of a variable for period t, α_0 is the constant and r_{t-1}^2 is the squared percentage change in the differenced interest rate. To ensure a positive conditional variance estimate the following must hold: $\alpha_{0,k} > 0$ and $\alpha_{1,k} \geq 0$. Covariance-stationarity in each regime requires $\alpha_{1,k} < 1$

Bollerslev's 1986 GARCH(1,1) model expands on the ARCH model to include the previous period's variance ($\sigma_{k,t-1}^2$).

$$\sigma_{k,t}^2 = \alpha_{0,k} + \alpha_{1,k} r_{t-1}^2 + \beta_k \sigma_{k,t-1}^2$$

Equation 4: GARCH (1,1)

To ensure positivity the following must hold: $\alpha_{0,k} > 0$, $\alpha_{1,k} > 0$ and $\beta_k \geq 0$. The persistence of the volatility is equal to $\alpha_{1,k} + \beta_k$. Covariance-stationarity in each regime requires $\alpha_{1,k} + \beta_k < 1$

Nelson's (1991) Exponential GARCH (E-GARCH) specifies the conditional variance in a logarithmic form allowing big shocks to have a greater impact on variance than GARCH.

$$ln\left(\sigma_{k,t}^2\right) = \alpha_{0,k} + \alpha_{1,k}\left(\left|\eta_{k,t-1}\right| - E\left[\left|\eta_{k,t-1}\right|\right]\right) + \alpha_{2,k} r_{t-1} + \beta_k \ln(\sigma_{k,t-1}^2)$$

Equation 5: E- GARCH (1,1)

This specification accounts for the asymmetric or leverage effect where past negative observations have a larger influence on the conditional volatility than past positive observations; $\alpha_{1,k}$ captures the sign of the asymmetric effect, and $\alpha_{2,k}$ captures the size of the asymmetric effect. Positivity of the conditional variance is ensured by specifying the log of the variance, thus coeffients can have negative values. The persistence is captured by β_k. Covariance-stationarity in each regime requires $\beta_k < 1$. The expectation of the standardised residuals $E\left[\left|\eta_{k,t-1}\right|\right]$ is taken with respect to the conditional distribution.

The GJR-GARCH of Glosten, Jagannathan & Runkle (1993) reflects the asymmetric nature of responses by defining the conditional variance as a linear piecewise function.

$$\sigma_{k,t}^2 = \alpha_{0,k} + (\alpha_{1,k} + \alpha_{2,k} \Vert \{r_{t-1} < 0\}) r_{t-1}^2 + \beta_k \sigma_{t-1}^2$$

Equation 6: GJR-GARCH (1,1)

where $\alpha_{1,k}$ captures the sign effect, and $\alpha_{2,k}$ indicates the degree of asymmetry. The persistence is captured by β_k. To ensure positivity the following must hold: $\alpha_{0,k} > 0$, $\alpha_{1,k} > 0$, $\alpha_{2,k} \geq 0$ and $\beta_k \geq 0$. The persistence is captured by $\alpha_{1,k} + \alpha_{2,k} E\left[\left|\eta_{k,t}^2 \Vert \{\eta_{k,t} < 0\}\right|\right] + \beta_k$ Covariance-stationarity in each regime requires $\alpha_{1,k} + \alpha_{2,k} E\left[\left|\eta_{k,t}^2 \Vert \{\eta_{k,t} < 0\}\right|\right] + \beta_k < 1$

Zakoian's 1994 Threshold GARCH (T-GARCH). The conditional volatility is the dependent variable instead of the conditional variance.

$$\sigma_{k,t} = \alpha_0 + \alpha_{1,k} r_{t-1} \Vert \{r_{t-1} \geq 0\} + \alpha_{2,k} r_{t-1} \Vert \{r_{t-1} < 0\} + \beta_k \sigma_{t-1}$$

Equation 7: T-GARCH (1,1)

To ensure positivity the following must hold: $\alpha_{0,k} > 0$, $\alpha_{1,k} > 0$, $\alpha_{2,k} > 0$ and $\beta_k \geq 0$. The volatility persistence is captured by $\alpha_{1,k}^2 + \beta_k^2 - 2\beta_k(\alpha_{1,k} + \alpha_{2,k}) E[\eta_{t,k} \Vert \{\eta_{t,k} < 0\}] - (\alpha_{1,k}^2 - \alpha_{2,k}^2) E[\eta_{t,k}^2 \Vert \{\eta_{t,k} < 0\}] + \beta_k$. Covariance-stationarity in each regime requires $\alpha_{1,k}^2 + \beta_k^2 - 2\beta_k(\alpha_{1,k} + \alpha_{2,k}) E[\eta_{t,k} \Vert \{\eta_{t,k} < 0\}] - (\alpha_{1,k}^2 - \alpha_{2,k}^2) E[\eta_{t,k}^2 \Vert \{\eta_{t,k} < 0\}] + \beta_k < 1$

The effect of the previous period's differenced interest rate on the conditional variance depends on its sign. It is α_1 when r_{t-1} is positive, and when negative it is $\alpha_1 + \alpha_2$. Therefore α_2 is positive when there is a greater response to bad news than good.

According to Gray (1996), the estimation of regime-switching has typically been done through Maximum Likelihood, this method is applied in this Chapter to estimate the volatility models.

4.2.2 Conditional distributions

The standardised innovations of the conditional distribution for the ARCH, GARCH, E-GARCH, GJR-GARCH and T-GARCH all follow the following form:

$$\eta_{t,k} \equiv \frac{r_t}{\sigma_{k,t}} \sim iid\ D(0, 1, \xi_k)$$

Equation 8: Standardised innovations of the Markov switching volatility models

Each distribution is standardised to have a zero mean and a unit variance as well as a skewness parameter ξ_k. When $\xi_k = 1$ the distribution is symmetric however when $\xi_k \neq 1$ the distribution is skewed.

The probability density function (PDF) of the standard normal conditional distribution is represented by:

$$f_{\mathrm{N}}(\eta) \equiv \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\eta^2} \ , \eta \in \mathbb{R}$$

Equation 9: Normal distribution

The PDF of the standardised Student's t conditional distribution is represented by:

$$f_S(\eta; \upsilon) \equiv \frac{\Gamma \frac{\upsilon+1}{2}}{\sqrt{(\upsilon-2)\pi\Gamma\frac{\upsilon}{2}}} \left(1 + \frac{\eta^2}{(\upsilon-2)}\right)^{-\frac{\upsilon+1}{2}} , \eta \in \mathbb{R}$$

Equation 10: Student's distribution

where υ is constrained to exceed 2 in order to ensure that the second order moment exists. The kurtosis of the distribution is higher for a lower υ. $\Gamma(\cdot)$ is the Gamma function.

The PDF of the standardised generalised error distribution (GED) is represented by:

$$f_{GED}(\eta; \upsilon) \equiv \frac{\upsilon e^{-\frac{1}{2}|\frac{\eta}{\lambda}|^{\upsilon}}}{\sqrt{\lambda 2^{(1+\frac{1}{\upsilon})} \Gamma \frac{1}{\nu}}} , \qquad \lambda \equiv \left(\frac{\Gamma(\frac{1}{\upsilon})}{4^{1/\upsilon}\Gamma(3/\upsilon)}\right)^{\frac{1}{2}} , \eta \in \mathbb{R}$$

Equation 11: GED distribution

where $\upsilon > 0$ is the shape parameter.

The MSGARCH packaged in R by Ardia et al., (2018) also allows for skewed normal, skewed student's t and skewed GED conditional distributions.

4.2.3 Model evaluation criteria

Following Chu et al. (2017), in-sample criteria such as the Akaike information criterion (AIC) and Bayesian information criterion (BIC) are used to select the most appropriate model in terms of the trade-off between goodness of fit and parsimony. The model which represents the best in-sample fit of the data has the smallest AIC and BIC.

Akaike's (1974) AIC is represented by:

AIC = $2\chi - 2\ln\mathrm{L}\ (\widehat{\Theta})$

Equation 12: AIC

where χ is defined as the number as unknown parameters, Θ is the vector of unknown parameters and $\widehat{\Theta}$ their maximum likelihood estimates.

The BIC of Schwartz (1978) is represented by:

BIC = $\chi \ln n - 2\ln\mathrm{L}\ (\widehat{\Theta})$

Equation 13: BIC

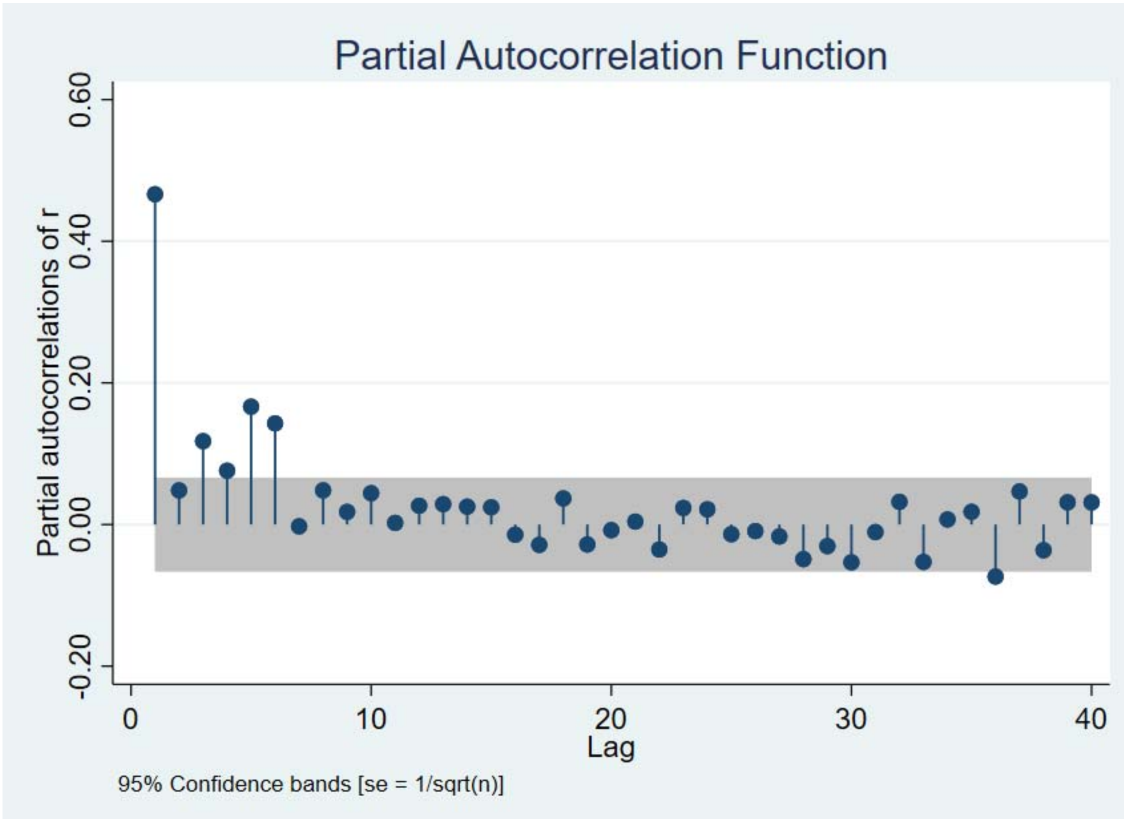
where n is the number of observations.

5 Empirical Analysis

5.1 Mean Reversion

Figure 7 confirms the presence of serial correlation in the first-differenced data. The autoregressive effects are removed using an AR(1) process.

Figure 7: Partial Autocorrelation Function for the first-differenced 3 Month Jibar



Source: Author.

Table 2: Results of the autoregressive model

Coefficients	**AR(1)**
c	0
b	0.4664
α	0.5336
Log-likelihood	905.56
AIC	1807.11
BIC	1797.57

Source: Author's calculations.

The Pearson correlation coefficient of r_t and r_{t-1} is -0.03587804, suggesting that r_t is infact mean reverting. The mean reversion rate is 0.5336.

5.2 Markov Switching GARCH

As mentioned above, this study investigates the in-sample accuracy of the ARCH, GARCH, E-GARCH, GJR-GARCH and T-GARCH models for up to four regimes and six conditional distributions. A total of 120 models were estimated with the results presented in Appendix A.

The best overall in-sample performing model according to both AIC and BIC is the 4 State T-GARCH with Student's t distribution and the second best overall model is the 2 State T-GARCH with Student's t distribution. Table 3 summarises the two overall best performing models as well as the best performing model for each GARCH-type model. The AIC and BIC selection for each top performing type of model was consistent except for the GARCH model. The single state models are to a large extent outperformed by the multiple state models. The results suggest that there is indeed evidence of regime switching in the historical interest rate volatility.

Table 3: Best in-sample performing a combination of state and distribution for each of the GARCH-type models

Model			AIC	BIC
4 State	T-GARCH	Student's t	-8153.5823	-8000.7706
2 State	T-GARCH	Student's t	-7230.7734	-7173.469
2 State	E-GARCH	Student's t	-6317.0747	-6259.7703
4 State	GARCH	GED	-6002.6079	-5868.8977
4 State	GARCH	Skewed Student's t	-6005.3204	-5852.5087
2 State	E-GARCH	Student's t	-6317.0747	-6259.7703
2 State	GJR-GARCH	Skewed GED	-5885.9502	-5819.095
4 State	ARCH	Student's t	-5731.6621	-5617.0533

Source: Author's calculations.

The Normal and Skewed Normal distribution results suggest the worst fit thus confirming a fat-tailed distribution is preferred. This is in accordance with expectations given the highly leptokurtic nature of the data. The kurtosis coefficient reported in section 3.1 above is 21.3041, while the kurtosis coefficient of a normal distribution is 3. For all five models types, the selection criteria favour the Student's t, Skewed Student's t, GED and Skewed GED distributions, confirming the presence of fat tails. The majority of the top performing models favour an un-skewed distribution.

Table 4 presents the estimates from the selected model as well as a simple 1 state GARCH with the same distribution.

Table 3: Performance of 4 State T-GARCH vs 1 State GARCH

Coefficients	4 State T-GARCH Student's t	1 State GARCH Student's t
$\alpha_{0,1}$	0.0000001 (2.00E-15)	0.0000001 (2.00E-15)
$\alpha_{1,1}$	0.9997895 (6.63E-24)	0.5222659 (8.54E-21)
$\alpha_{2,1}$	0.0001004 (4.09E-26)	-
β_1	0.0104750 (2.72E-21)	0.4775062 (2.38E-25)
υ_1	2.1000310 (7.31E-26)	2.5917760 (3.81E-21)
$\alpha_{0,2}$	0.0000012 (2.27E-29)	-
$\alpha_{1,2}$	0.8569539 (4.99E-22)	-
$\alpha_{2,2}$	0.1302990 (5.27E-24)	-
β_2	0.5188921 (2.43E-22)	-
υ_2	2.1000310 (6.57E-26)	-
$\alpha_{0,3}$	0.0179356 (7.21E-21)	-
$\alpha_{1,3}$	0.7716386 (4.79E-21)	-
$\alpha_{2,3}$	0.1289264 (6.97E-22)	-
β_3	0.5863053 (1.49E-22)	-
υ_3	2.1000310 (1.80E-25)	-
$\alpha_{0,4}$	0.0507341 (3.54E-21)	-
$\alpha_{1,4}$	0.4584096 (7.80E-21)	-
$\alpha_{2,4}$	0.3530546 (1.53E-22)	-
β_4	0.6429577 (2.83E-23)	-
υ_4	2.1000310 (3.61E-26)	-

σ_1Mean	0.009105	0.0338402
σ_1Maximum	0.1702	0.2661557
σ_2Mean	0.01886	-
σ_2Maximum	0.2024	-
σ_3Mean	0.06345	-
σ_3Maximum	0.24165	-
σ_4Mean	0.1642	-
σ_4Maximum	0.3230	-
Log-likelihood	4108.7912	2535.9746
AIC	-8153.5823	-5063.9492
BIC	-8000.7706	-5044.8477

Source: Author's calculations. Standard errors are in brackets. All of the coefficients are significant at the 1% level.

The estimated volatility from the simple 1 State GARCH is in effect an average of the four volatility estimates from the 4 State T-GARCH, failing to represent the lowest and highest volatility regimes. Therefore the 4 State T-GARCH is better at capturing the volatility clustering in the data.

The persistence of the simple 1 State GARCH is calculated by summing $\alpha_1 + \beta$ which equals 0.999. The persistence for each state of a T-GARCH is calculated as follows:

$$\alpha_{1,k}^2 + \beta_k^2 - 2\beta_k(\alpha_{1,k} + \alpha_{2,k})E\left[\eta_{t,k} \, \| \, \{\eta_{t,k} < 0\}\right] - (\alpha_{1,k}^2 - \alpha_{2,k}^2)E\left[\eta_{t,k}^2 \, \| \, \{\eta_{t,k} < 0\}\right] + \beta_k$$

where, $E\left[\eta_{t,k} \, \| \, \{\eta_{t,k} < 0\}\right]$ = 0.5 for an unskewed distribution (see Ardia et al., 2018)

Table 4: The persistence in volatility per state of the 4 State T-GARCH Student's t

Volatility persistence of each state of the 4 State T-GARCH Student's t	
State 1	0.739320174
State 2	0.151004597
State 3	0.266472973
State 4	0.080423857

Source: Author's calculations.

For all four of the states, the persistence is lower than the persistence of the 1 state GARCH indicating that the high persistence in the single-regime model is indeed spurious.

State 1 is the low volatility state with a high volatility persistence indicating that in this tranquil market state, volatility reacts slowly to recent data. The asymmetry parameter in this state is extremely small indicating almost no asymmetric volatility reaction to past negative news.

States 2 and 3 are both moderate market volatility states with a similar volatility reaction to past negative news. State 3 is more persistent than state 2 and exhibits higher mean and maximum volatility within the state.

State 4 is characterized by the highest volatility and a very strong volatility reaction to past negative news. This period of extreme market stress also exhibits a low volatility persistence indicating that more weighting is given to more recent data as well as a quick reversion to the mean.

Recall from section 4.2 that the shape parameter υ is constrained to exceed 2 for the Student's t distribution in order to ensure that the second order moment exists and the kurtosis of the distribution is higher for a lower υ. The υ of all the states is 2.1 once again confirming that the data is highly leptokurtic.

Table 5: Transition probability matrix for the 4 State T-GARCH Student's t

	t+1|k=1	t+1|k=2	t+1|k=3	t+1|k=4
t|k=1	0.52329418	0.01834748	0.4529629	0.00539542
t|k=2	0.51544280	0.18835601	0.0845859	0.2116153
t|k=3	0.20857706	0.03624498	0.7463898	0.00878816
t|k=4	0.03699331	0.03481171	0.1621211	0.76607385

Source: Author's calculations.

The probability matrix represents the probability of moving from one state to another. For example, the probability of transitioning from State 1 in time *t* to State 4 in time *t+1* is 0.00539542 or 0.5%, thus it is highly unlikely to jump from the 1st State directly to the 4th State in one time period. While in States 1, 3 and 4 in time *t* it is more likely to remain in the initial state in *t+1* than it is to change states. The exception is State 2 which is more likely to transition back to State 1 in *t+1*. This is supported by the low volatility persistence of 0.15 for State 2 reported in Table 5. Interestingly, the high volatility State 4 is characterised by an extremely low volatility persistence of 0.08 and a high probability of remaining in State 4 at *t+1*. Thus indicating that the immediate impact of a shock is high and that the main source of volatility clustering in State 4 is due to the persistence of the regime itself.

5 Conclusion

The study finds that there is indeed regime switching evident in South African historical interest rate volatility which is of consequence for the valuing of financial assets. The four states identified represent tranquil, normal and extreme market volatility conditions. The data was also found to have an asymmetric effect to negative information. The asymmetric effect increases as the volatility conditions increase, suggesting that the bigger the negative information shock the higher the volatility response.

The high volatility regime is characterised by low volatility persistence while the low volatility regime is characterised by high volatility persistence. An interesting finding is that the volatility clustering in State 4 is due to the regime's persistence.

Recommendations for further research include using out of sample forecasts to compare the goodness of fit. Although Wu (2010) finds that the maximum likelihood estimate for the T-GARCH is unbiased and normally distributed for modest sample sizes given stationarity, a further area of research is the comparison of other estimation techniques such as the Bayesian Markov Chain Monte Carlo (MCMC) estimation technique. At the time of writing, proposals on the reform of the Jibar calculation as well as proposals for potential new benchmarks are being considered by South African authorities, the potential impact on derivatives pricing of a new benchmark interest rate is therfore an additional area for future research.

References

Akaike, H. 1974. A new look at the statistical model identification. In *Selected Papers of Hirotugu Akaike*. New York, NY: Springer. 215–222.

Ardia, D., Bluteau, K., Boudt, K., Catania, L., Ghalanos, A., Peterson, B. & Trottier, D.-A. 2018. MSGARCH: Markov-Switching GARCH Models in R. *R package version 2.3, URL https://cran.r-project.org/package=MSGARCH.*

Bank of England. 2016. *The Prudential Regulation Authority's approach to banking supervision*. Available: http://www.bankofengland.co.uk/publications/Documents/praapproach/bankingappr1603.pdf.

Bauwens, L., Preminger, A. & Rombouts, J.V.K. 2010. Theory and inference for a Markov switching GARCH model. *The Econometrics Journal*. 13(2):218–244. DOI: 10.1111/j.1368-423X.2009.00307.x.

Bollerslev, T. 1986. Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics*. 31:307–327. DOI: 10.1016/0304-4076(86)90063-1.

Cai, J. 1994. A markov model of switching-regime arch. *Journal of Business and Economic Statistics*. 12(3):309–316. DOI: 10.1080/07350015.1994.10524546.

Chu, J., Chan, S., Nadarajah, S. & Osterrieder, J. 2017. GARCH modelling of Cryptocurrencies. *Risk and Financial Management*. 17(10):1–15. DOI: 10.3390/jrfm10040017.

Dueker, M.J. 1997. Switching in GARCH processes and mean-reverting stock-market volatility. *Journal of Business & Economic Statistics*. 15(1):26–34.

Engle, R.F. 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*. 50(4):987–1007.

Engle, R.F., Ng, V.K. & Rothschild, M. 1990. Asset pricing with a Factor-ARCH covariance structure. *Journal of Econometrics*. 45(1–2):213–237. DOI: 10.1016/0304-4076(90)90099-F.

Financial Stability Board. 2013. *Peer review of South Africa*. Financial Stability Board. Available: http://www.fsb.org/2013/02/r_130205/ [accessed 14-04-2014].

Glosten, L.R., Jagannathan, R. & Runkle, D.E. 1993. On the relation between the expected value and the volatility of the nominal excess return on stocks. *The Journal of Finance*. 48(5):1779–1801.

Gray, S.. 1996. Modeling the conditional distribution of interest rates as a regime-switching process. *Journal of Financial Economics*. 42:27–62. DOI: 10.1055/s-0029-1215315.

Haas, M., Mittnik, S. & Paolella, M. 2002. A New Approach to Markov-Switching GARCH Models. *Journal of Financial Econometrics*. 2(4):493–530. DOI: 10.1093/jjfinec/nbh020.

Hamilton, J.D. 2016. Macroeconomic Regimes and Regime Shifts. In *Handbook of Macroeconomics*. V. 2. J.B. Taylor & H. Uhlig, Eds. Elsevier B.V. 163–201. Available: https://www.sciencedirect.com/handbook/handbook-of-macroeconomics/vol/2/suppl/C.

Hamilton, D. & Susmel, R. 1994. and Changes in Regime. *Journal of Econometrics*. 64:307–333.

Hillebrand, E. 2005. Neglecting parameter changes in GARCH models. *Journal of Econometrics*. 121(1–2):121–138.

Holilal, A. 2011. Choice of one factor interest rate term structure models for pricing and hedging Bermudan swaptions. University of Cape Town.

Klaassen, F. 2002. *Improving GARCH volatility forecasts with regime-switching GARCH*. Berlin Heidelberg. DOI: 10.1007/s001810100100.

Koedijk, K.G., Nissen, F.G.J.A., Schotman, P.C. & Wolff, C.C.P. 1997. The Dynamics of short-term interest rate volatility reconsidered. *European Finance Review*. 1(1):105–130. DOI: 10.1023/A:1009714314989.

Lamoureux, C.G. & Lastrapes, W.D. 1990. Persistence in variance, structural change, and the GARCH Model. *Journal of Business and Economic Statistics*. 8(2):225–234.

Longstaff, F.A. & Schwartz, E.S. 1992. Interest rate volatility and the term structure: A two-factor general equilibrium model. *The Journal of Finance*. 47(4):1259–1282. DOI: 10.1111/j.1540-6261.1992.tb04657.x.

Nell, K.S. 2018. Re-examining the role of structural change and nonlinearities in a Philips curve model for South Africa. *South African Journal of Economics*. 86(2):173–196.

Nelson, D.B. 1991. Conditional heteroskedasticity in asset returns: A new approach. *Econometrica*. 59(2):347–370.

Olweny, T. 2011. Modelling Volatility of Short-term Interest Rates in Kenya. *International Journal of Business and Social Science*. 2(7):289–303.

Du Preez, P.F. 2011. An investigation into popular methods for constructing yield curves by. The University of Pretoria.

Schwartz, G. 1978. Estimating the dimension of a model. *The Annals of Statistics*. 6(2):461–464.

South African Reserve Bank. 2018. Consultation paper on selected interest rate benchmarks in South Africa. Pretoria: South African Reserve Bank.

Venter, G.G. 2010. *Advances in Modeling of Financial Series*.

West, G. 2008. Interest rate derivatives in the South African market based on the prime rate. *Journal for Studies in Economics and Econometrics*. 32(1):77–87.

Wu, J. 2010. Threshold GARCH Model : Theory and Application. In *The University of Western Ontario*. 1–42.

Zakoian, J.-M. 1994. Threshold heteroskedastic models. *Journal of Economic Dynamics and control*. 18(5):931–955.

Appendix A: Results of in sample model testing

Model			AIC	BIC
1 State	ARCH	Normal	-3323.727	-3314.1763
2 State	ARCH	Normal	-4456.8897	-4428.2375
3 State	ARCH	Normal	-4454.2869	-4396.9825
4 State	ARCH	Normal	-4043.1072	-3947.5999
1 State	ARCH	Student's t	-4669.7947	-4655.4686
2 State	ARCH	Student's t	-5435.7283	-5397.5253
3 State	ARCH	Student's t	-5550.0003	-5478.3698
4 State	ARCH	Student's t	-5731.6621	-5617.0533
1 State	ARCH	GED	-4946.9775	-4932.6514
2 State	ARCH	GED	-5657.0886	-5618.8856
3 State	ARCH	GED	-5585.2236	-5513.5931
4 State	ARCH	GED	-5536.548	-5421.9392

1 State	ARCH	Skewed Normal	-3357.9975	-3343.6714
2 State	ARCH	Skewed Normal	-3685.7232	-3647.5203
3 State	ARCH	Skewed Normal	-3983.5187	-3911.8882
4 State	ARCH	Skewed Normal	-4080.6067	-3965.9979
1 State	ARCH	Skewed Student's t	-4668.8176	-4649.7162
2 State	ARCH	Skewed Student's t	-5163.0647	-5115.311
3 State	ARCH	Skewed Student's t	-5302.4147	-5216.4581
4 State	ARCH	Skewed Student's t	-5256.0476	-5122.3373
1 State	ARCH	Skewed GED	-4943.7662	-4924.6647
2 State	ARCH	Skewed GED	-5616.5612	-5568.8075
3 State	ARCH	Skewed GED	-5451.1697	-5365.2131
4 State	ARCH	Skewed GED	-5314.4201	-5180.7098
1 State	GARCH	Normal	-3429.5019	-3415.1758
2 State	GARCH	Normal	-4869.6446	-4831.4417
3 State	GARCH	Normal	-4664.1143	-4592.4838
4 State	GARCH	Normal	-3989.9904	-3875.3816
1 State	GARCH	Student's t	-5063.9492	-5044.8477
2 State	GARCH	Student's t	-5634.8962	-5587.1426
3 State	GARCH	Student's t	-5685.4722	-5599.5156
4 State	GARCH	Student's t	-4775.6281	-4641.9179
1 State	GARCH	GED	-4944.9751	-4925.8737
2 State	GARCH	GED	-5804.5938	-5756.8401
3 State	GARCH	GED	-5788.0663	-5702.1097
4 State	GARCH	GED	-6002.6079	-5868.8977
1 State	GARCH	Skewed Normal	-3451.2373	-3432.1358
2 State	GARCH	Skewed Normal	-4683.5004	-4635.7468

3 State	GARCH	Skewed Normal	-4726.4295	-4640.4729
4 State	GARCH	Skewed Normal	-4232.8412	-4099.131
1 State	GARCH	Skewed Student’s t	-4830.9698	-4807.093
2 State	GARCH	Skewed Student’s t	-5862.3356	-5805.0312
3 State	GARCH	Skewed Student’s t	-5336.6339	-5236.3512
4 State	GARCH	Skewed Student’s t	-6005.3204	-5852.5087
1 State	GARCH	Skewed GED	-5145.8819	-5122.0051
2 State	GARCH	Skewed GED	-5304.5827	-5247.2783
3 State	GARCH	Skewed GED	-5407.0603	-5306.7776
4 State	GARCH	Skewed GED	-5860.4319	-5707.6202
1 State	E-GARCH	Normal	-3491.8357	-3472.7342
2 State	E-GARCH	Normal	-4405.7948	-4358.0411
3 State	E-GARCH	Normal	-4317.834	-4231.8774
4 State	E-GARCH	Normal	-4113.0419	-3979.3317
1 State	E-GARCH	Student’s t	-4875.456	-4851.5791
2 State	E-GARCH	Student’s t	-6317.0747	-6259.7703
3 State	E-GARCH	Student’s t	-5615.7922	-5515.5095
4 State	E-GARCH	Student’s t	-5581.8848	-5429.0731
1 State	E-GARCH	GED	-4998.8754	-4974.9986
2 State	E-GARCH	GED	-6206.5154	-6149.211
3 State	E-GARCH	GED	-5409.6772	-5309.3945
4 State	E-GARCH	GED	-5305.8768	-5153.0651
1 State	E-GARCH	Skewed Normal	-3494.3544	-3470.4776
2 State	E-GARCH	Skewed Normal	-4332.6692	-4275.3648
3 State	E-GARCH	Skewed Normal	-4216.2828	-4116.0001
4 State	E-GARCH	Skewed Normal	-4408.5682	-4255.7565

1 State	E-GARCH	Skewed Student's t	-4880.3785	-4851.7263
2 State	E-GARCH	Skewed Student's t	-6282.7303	-6215.8752
3 State	E-GARCH	Skewed Student's t	-5446.2887	-5331.6799
4 State	E-GARCH	Skewed Student's t	-5427.3738	-5255.4606
1 State	E-GARCH	Skewed GED	-4995.4721	-4966.8199
2 State	E-GARCH	Skewed GED	-5772.323	-5705.4679
3 State	E-GARCH	Skewed GED	-5406.3235	-5291.7148
4 State	E-GARCH	Skewed GED	-5322.0586	-5150.1454
1 State	GJR-GARCH	Normal	-3318.5438	-3299.4423
2 State	GJR-GARCH	Normal	-4824.916	-4777.1624
3 State	GJR-GARCH	Normal	-4656.3639	-4570.4074
4 State	GJR-GARCH	Normal	-4704.3109	-4570.6006
1 State	GJR-GARCH	Student's t	-5060.3614	-5036.4846
2 State	GJR-GARCH	Student's t	-5745.3222	-5688.0178
3 State	GJR-GARCH	Student's t	-5544.9469	-5444.6642
4 State	GJR-GARCH	Student's t	-5089.7412	-4936.9295
1 State	GJR-GARCH	GED	-5146.0264	-5122.1496
2 State	GJR-GARCH	GED	-5868.1286	-5810.8242
3 State	GJR-GARCH	GED	-5768.7268	-5668.4441
4 State	GJR-GARCH	GED	-5804.3064	-5651.4947
1 State	GJR-GARCH	Skewed Normal	-3449.247	-3425.3702
2 State	GJR-GARCH	Skewed Normal	-4713.1894	-4655.885
3 State	GJR-GARCH	Skewed Normal	-3480.6027	-3380.32
4 State	GJR-GARCH	Skewed Normal	-4792.336	-4639.5243
1 State	GJR-GARCH	Skewed Student's t	-4980.0462	-4951.394
2 State	GJR-GARCH	Skewed Student's t	-5605.3549	-5538.4997

3 State	GJR-GARCH	Skewed Student's t	-5155.334	-5040.7252
4 State	GJR-GARCH	Skewed Student's t	-4750.1728	-4578.2597
1 State	GJR-GARCH	Skewed GED	-5142.4531	-5113.8009
2 State	GJR-GARCH	Skewed GED	-5885.9502	-5819.095
3 State	GJR-GARCH	Skewed GED	-5657.2097	-5542.601
4 State	GJR-GARCH	Skewed GED	-5530.9493	-5359.0362
1 State	T-GARCH	Normal	-3512.2893	-3493.1878
2 State	T-GARCH	Normal	-4672.6212	-4624.8676
3 State	T-GARCH	Normal	-4562.1034	-4476.1468
4 State	T-GARCH	Normal	-4353.6031	-4219.8929
1 State	T-GARCH	Student's t	-5045.2892	-5021.4123
2 State	T-GARCH	Student's t	-7230.7734	-7173.469
3 State	T-GARCH	Student's t	-5771.0687	-5670.786
4 State	T-GARCH	Student's t	-8153.5823	-8000.7706
1 State	T-GARCH	GED	-5131.1014	-5107.2246
2 State	T-GARCH	GED	-5397.1094	-5339.805
3 State	T-GARCH	GED	-6234.9244	-6134.6417
4 State	T-GARCH	GED	-5805.3561	-5652.5444
1 State	T-GARCH	Skewed Normal	-3516.935	-3493.0582
2 State	T-GARCH	Skewed Normal	-4854.2182	-4796.9138
3 State	T-GARCH	Skewed Normal	-4530.6532	-4430.3706
4 State	T-GARCH	Skewed Normal	-3947.9316	-3795.1199
1 State	T-GARCH	Skewed Student's t	-5048.502	-5019.8498
2 State	T-GARCH	Skewed Student's t	-7228.827	-7161.9718
3 State	T-GARCH	Skewed Student's t	-6552.7525	-6438.1437
4 State	T-GARCH	Skewed Student's t	-7009.412	-6837.4988

1 State	T-GARCH	Skewed GED	-5136.1692	-5107.517
2 State	T-GARCH	Skewed GED	-6220.4927	-6153.6376
3 State	T-GARCH	Skewed GED	-6160.9199	-6046.3111
4 State	T-GARCH	Skewed GED	-6189.0816	-6017.1684

Source: Author's calculations.

**The models were estimated using the MSGARCH package in R (Ardia et al., 2018), all output results are available on request.*