Euro Area Quantitative Easing in a Portfolio Balance Model with Heterogeneous Agents and Assets

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Abstract

We develop a dynamic portfolio balance model to study the effects of Quantitative Easing (QE) on the returns of international financial assets through the portfolio balance channel. Our two-country agent-based model features heterogeneity in assets and in investor preferences. Both are crucial for a meaningful model-based impact assessment of QE because preferences for asset maturity, asset class (bonds, equities and currencies) and whether an asset is issued at home or abroad can influence the substitutability of assets, and hence the portfolio balance effect of central bank asset purchases. We implement a novel pricing mechanism that allows us to approach market clearing prices. This allows us to take advantage of the flexibility of the agent-based methodology, while keeping the model comparable to more standard equilibrium-based portfolio balance models.

We calibrate the two countries in our model to the Eurozone and a representative sample of rest-of-the-world countries in order to estimate the international impact of the ECB's asset purchase program announced in January 2015. Specifically, we compile a data set on asset holdings of international banks and investment funds and use it to calibrate the preferences of agents and the characteristics of assets. When simulating our model, we find a negative impact of central bank asset purchases on both domestic and foreign returns. While the effects of QE on domestic yields and the exchange rate are rather modest and smaller than commonly assumed in the literature, domestic stock prices increase substantially.

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Contents

1 Introduction

Ben Bernanke once famously said "(...)the problem with QE is it works in practice, but it doesn't work in theory."[1] Quantitative Easing (QE) is a modern central bank policy that involves large asset purchases in exchange for newly created reserves. It has been used extensively in advanced economies in the post crisis area. Despite this, however, our understanding of the underlying transmission channels of QE is inadequate. In particular, the economic theory literature is lacking models that convincingly describe the relationship between QE, financial markets and the economy. We seek to contribute to this research gap and present a dynamic heterogenous agent model to study spillover effects by means of a two-country set-up. Our focus is the effect of QE on *financial asset returns and prices* through the portfolio balancing channel.

Our framework allows for heterogeneity in assets and in agents, which is the main difference between our model and other portfolio models in the literature. Since the effect of portfolio balancing on returns is essentially a function of the substitutability of assets, it is absolutely crucial to model differences in properties of asset classes and to account for idiosyncratic preferences of agents. At the heart of the model are portfolio optimising investors who allocate capital across a domestic and a foreign market. We then conduct a policy experiment in which the central bank of the domestic market conducts asset purchases in their market ranging from EUR200bn to EUR2.6tn. We contribute methodologically by including heterogeneous agents and assets, which we believe is crucial for a meaningful assessment of QE within the portfolio balance channel. To our knowledge, we are the first to demonstrate the effect of QE while accounting for:

- Endogenous outcome variables. QE-induced asset price and exchange rate effects are the result of agents' portfolio optimisation given the interaction of risk and return variables, availability of assets and expectations formation.
- Dynamic covariance structure. Our model allows for the interactions between variables over time. Agents take into account a time-varing variance-covariance structure of returns.
- Consideration of preferred habitat preferences. We introduce asset specific preferred habitat parameters to account for heterogenous investors' preferences for asset maturity, issuer nationality, default risk and inflation risk. These preferred habitat parameters are akin to the risk aversion parameter from the standard mean variance portfolio selection problem Markowitz (1952), however they conflate all the aforementioned preferences above.
- Computational tractability. A shortfall of many agent based models is their lack of tractability. We use a price a setting algorithm that finds daily equilibrium prices.
- Parsimonious modelling of maturity. We introduce heterogeneity in asset maturity by introducing constant repayment rates and a maturity parameter. In this manner, we can model equity portfolios (that never mature) and bond maturities with different repayment characteristics in a parsimonious manner.
- Empirical data. For the calibration of the model, we compile a global dataset of investment funds' and banks' balance sheets. We match the maturity profile of funds by applying our modelling approach to a holding sample of 15374 open-ended investment funds. Furthermore, we calibrate our model to reflect preferred habitat preferences as revealed in investor holding positions.
- Result replication. Our framework gives us the advantage of conducting policy experiments in a 'lab environment'. We can replicate results and vary conditions (for example what happens if the business cycle features higher defaults and hence, higher default risk) to investigate determinants of asset price effects.
- Quantification. Our policy experiment leads us to believe that asset price effects from portfolio balancing are smaller than what is commonly assumed in the literature. Breckenfelder et al. (2016) conduct a review of empirical studies on the ECB's APP and find a reduction in the domestic government bond yield of 37 - 88 bps. Our findings can be summarised as follows:

[1] transcript of Brooking Institution conference, page 14, https://www.brookings.edu/wp-content/uploads/2014/01/20140116_bernanke_remarks_transcript.pdf

EZ bonds: -16 bps in yields, +0.9% in price

EZ equities: -50 bps in yields, +6.6% in price

ROW bonds: -1 bps in yields, +0% in price

ROW equities: -3 bps in yields, +0.4 in price

EUR/ROW exchange rate: -0.4%

One should note that the effect we measure is in respect of market EZ bond portfolios, market EZ equity portfolios etc. These results, although contrary to the results from most event studies, are in line with recent empirical results presented by Koijen et al. (2016).

The remainder of the paper is organised as follows. Section 2 will give an overview of the theoretical and empirical literature that looks at the relationship between Quantitative Easing and financial asset returns. A special focus will be laid on the portfolio balance channel as underlying reason for asset price movements. The subsequent section is devoted to giving a brief overview of a new set of economic models known as agent based models. It is argued why those models are suited for investigating the portfolio balance effect. Section 3 develops the model framework, while section 4 presents the calibration and simulation of Euro area purchase programs. The results will be discussed in section 5 and section 6 presents conclusions and policy implications resulting from the findings of the paper.

2 Literature

After having exhausted the scope of conventional monetary policy by driving short term interest rates towards their zero lower bound, several central banks resorted to asset purchase programs in the wake of the financial crisis. Central bankers anticipated that a portfolio balancing effect would justify large scale asset purchasing programs (see e.g Bernanke et al., 2004; Dale, 2010). The effect appears in a class of models, so called portfolio balance models, that can be traced back to Tobin (1958). By letting risk averse households optimize the composition of a portfolio comprising riskless but low yielding cash and risky but higher yielding bonds, Tobin's model provided a microfoundation for the liquidity preference, which was a standard component of macroeconomic models at the time. In equilibrium, lower yields on bonds would elicit a rebalancing of the portfolio towards cash and thereby explain the inverse relation between the demand for cash and the interest rate on bonds from Keynes' liquidity preference theory. In essence, the portfolio balancing channel is the consequence of an assumed imperfect substitutability of assets. Whether this arises merely from households' risk aversion as in Tobin's model, or from investor preferences, e.g. for certain maturities (so called preferred habitats) as is commonly assumed in contemporary portfolio balance models (see e.g. Vayanos and Vila, 2009; Hamilton and Wu, 2012; Greenwood and Vayanos, 2014), seems irrelevant.

Outside of the portfolio balance literature, however, the effectiveness of the portfolio balancing channel is contested. In most standard term structure models of interest rates that are derived from a no arbitrage argument, yields are determined by expected future interest rates and the term premium (Cox et al., 1985). Demand curves for bonds are flat and investors view bonds of different maturities as perfect substitutes. Any additional risk associated with longer term assets is entirely compensated by the term premium. Changes in relative asset supplies do not affect the assets' risk nor the risk aversion of investors, hence no portfolio balancing channel can materialize (Doh, 2010).[2]

Modern macroeconomic theory is also rather bleak regarding the effectiveness of QE. In most New-Keynesian macro models the microfoundations once provided by Tobin have long been replaced. Shifts in the supply of bonds are shown to be irrelevant within the frameworks of Wallace (1981) and Eggertsson and Woodford (2003).[3] Cash flows from assets are state-contingent and priced according to their relation to households' state-contingent income. Under the assumption that a swap of assets

[2] More recent arbitrage-free term structure models have included supply factors in the state variables (see e.g. Li and Wei, 2013).

[3] There are some New-Keynesian models were central bank asset purchases can effect interest rates. Andrés et al. (2004) e.g. incorporate portfolio adjustment costs and heterogeneous preferences for asset of different maturities into a DSGE framework and show that central bank asset purchases can impact yields. Chen et al. (2012a) use a similar approach and show that the effectiveness of QE to stimulate the economy depends on the degree of market segmentation between short term and long-term bonds.

between the central bank and the private sector does not affect expectations of households' income stream, it is only logical that asset prices should also remain unaffected. A Ricardian equivalence type argument is furthermore invoked to show that asset purchases by the central bank does not change households' overall exposure to desired assets (Woodford, 2012).

Empirically, there seems to be little doubt that asset purchasing programs by major central banks had at least some effect on yields directly after they were announced. A widely used approach are event studies that use high frequency data to quantify the changes in bond yields around QE announcement. Kuttner (2018) provides a recent review of empirical findings for US-based purchase programs. The largest effect is associated with the announcement of the initial QE1 program, which caused US Treasury yields to decline by approx. 100 bps, while subsequent QE announcements lead to a decline of between 14 and 40 bps (see Krishnamurthy and Vissing-Jorgensen, 2012; Gagnon et al., 2011a; Ehlers, 2012). For the Eurozone, a few recent papers study the effects of the ECB's asset purchase program (APP) that was announced in January 2015. Event studies quantifying the effect of the APP on euro area 10-year government bonds estimate that yields declined by between 30 bps and 70 bps (see Motto et al., 2015; **?**; De Santis, 2016). Koijen et al. (2016) show the changes in securities' holdings of domestic and foreign investors per asset class, sector and region over the seven quarters following the ECB's announcement. They find that the ECB's purchases of government bonds were mainly accommodated by foreign investors, who provided 70% of the ECB's asset purchases, and to a lesser degree by euro area banks and mutual fund investors. In addition, Koijen et al. (2016) show that insurance companies and pension funds reacted by buying bonds with similar maturities as the ECB, thereby amplifying the reduction in government bond supply.

Notwithstanding the growing evidence of QE's role in impacting asset prices, the channel through which prices are affected and the persistence of effects are controversial. While many empirical studies associate the impact of QE with the change in asset supply associated with the portfolio balance effect (see e.g. Neely, 2015; Joyce et al., 2011; Gagnon et al., 2011b; Stefania and King, 2013)), a few associate the effects with a signaling channel (e.g. Bauer and Rudebusch, 2014; Bauer and Neely, 2012). The signaling channel builds on the assumption that central bank announcements of QE can impact the expectations of market participants, e.g. by conveying a commitment to keeping short term interest rates low for an extended period of time. However, disentangling the signaling effect from the portfolio balancing effect is challenging. One approach is to decompose the bond yield into a component representing the expected average short term interest rates and a component representing the risk premium. Observed changes in the expectation component are associated with the signaling channel, while changes in the risk premium are mostly assumed to be driven by portfolio balance effects. Krishnamurthy and Vissing-Jorgensen (2011) for example, find that the signaling channel plays the primary role in the decline of US-Treasury yields in connection to the US-based QE2 program, while the portfolio balancing channel was responsible for lowering Mortgage Backed Securities (MBS) rates and corporate bond yields in connection to the QE1 program.

Other papers try to separate the two effects by comparing event study results with results implied by a portfolio model. For the UK, Joyce et al. (2011) calibrate a portfolio choice model to determine the impact of the Bank of England's purchases on domestic returns for government and corporate bonds, equities and cash. By estimating a VAR impulse response function, Joyce et al. (2011) conclude that through the portfolio balance channel British QE led to a decline in gilt yields in the range between 30 and 85 bps. Expanding the portfolio model to an international scale, Neely (2015) evaluates the impact of US QE on international long term interest rates and exchange rates. He also back-tests his empirical findings with a portfolio model, lending support to the existence of the portfolio balance channel. Expected foreign returns are adjusted by expected inflation and expected exchange rate changes, assuming that long run purchasing power parity holds. The US Fed's LSAP purchases are introduced by a swap of US government bonds for the risk-free liquidity asset and lead to domestic and international bond yield declines ranging from 35 and 144 bps and a US Dollar depreciation between 3.5% and 7.8%.

An alternative model of portfolio balancing is presented by Christensen and Krogstrup (2017), who argue that the supply-side induced portfolio balance effect can be amplified by the fact that QE leads to an expansion of bank balance sheets when transactions are done with non-bank entities because of additional deposits that are created. The authors refer to this as reserve-side induced portfolio balance effect of QE and present empirical evidence that the Swiss National Bank's asset purchases lowered the yield of the Swiss 10-year bond by 28 bps through this channel.

Beside the difficulty of disentangling the impact channel of QE announcements, identifying the information that drives changes in investor behavior is not straight forward either. Greenlaw et al. (2018) have raised the concern that empirical results derived from event studies may overstate the effect of QE. The authors doubt that only effects of monetary policy announcements rather than other economic news are measured. Event studies assume that any impact within the narrow window around an announcement event is solely associated with market participants reaction to QE announcements. However, when Greenlaw et al. (2018) consider a larger than usual sample of possible events, they find that announcements by the US Fed have not played a major role in changing yields and that the initial impact, if measurable, did not persist.

Three manifestations of those preferences stand out (all three can be accounted for in our model): First, the country where an asset is issued is a strong determinant of who holds that asset. The home bias of funds and banks is a well-established empirical fact in the literature (see e.g. Hau and Rey, 2008; Coeurdacier and Rey, 2013, for evidence of a home bias for funds and banks in different geographical regions). Second, the maturity of bonds is of importance to investors regardless of their ability to trade bonds immediately and at negligible cost. Two factors play a role here: duration risk, which is endogenous in our model, makes long term bonds less attractive to risk averse investors (see e.g. Gagnon et al., 2011a) and some investors simply prefer certain maturities over others.[4] Third, the asset class itself matters to investors. While the derivation of a liquidity (cash) preference was the first application of a portfolio balance model (see Tobin, 1958), preferences for bonds vs. stocks differ strongly for different groups of investors.[5] We design our model framework to allow for the described preferences to materialize: The incorporation of two countries allows for a home bias to prevail and facilitates the study of international spillovers emerging from QE. Furthermore, assets are calibrated to represent cash, portfolios of bonds and portfolios of equities. One of the differentiating characteristics of these assets is their maturity structure, which endogenously leads to a duration risk premium. Assets representing bond portfolios are modeled with an average repayment rate, while equity portfolios never mature. Agents in the model represent Eurozone (domestic country) and rest-of-the-world (foreign country) funds and banks. Their respective asset holdings in the fourth quarter of 2014 (just before the ECB announced its QE program) are used to calibrate agents' preferences in the form of asset-specific risk aversions.

Agent-based models

We develop a dynamic portfolio model in order simulate QE in a realistic setting and analyze its impact on international asset returns via the portfolio balance channel. Prices, the exchange rate, the covariance structure of returns and the balance sheet composition of agents are endogenous to our model. Our framework furthermore allows for heterogeneity in assets and in agents, which is the main difference between our model and other portfolio models in the literature. Since the effect of portfolio balancing on returns is essentially a function of the substitutability of assets, it is absolutely crucial to model differences in properties of asset classes and to account for idiosyncratic preferences of agents. Three manifestations of those preferences stand out (all three can be accounted for in our model): First, the country where an asset is issued is a strong determinant of who holds that asset. The home bias of funds and banks is a well-established empirical fact in the literature (see e.g. Hau and Rey, 2008; Coeurdacier and Rey, 2013, for evidence of a home bias for funds and banks in different geographical regions). Second, the maturity of bonds is of importance to investors regardless of their ability to trade bonds immediately and at negligible cost. Two factors play a role here: duration risk, which is endogenous in our model, makes long term bonds less attractive to risk averse investors (see e.g. Gagnon et al., 2011a) and some investors simply prefer certain maturities over others.[6] Third,

[4]The literature describes the latter source of imperfect asset substitutability with the theory of preferred habitats, which goes back to Culbertson (1957) and Modigliani and Sutch (1967). Preferred habitat models of the term structure such as Vayanos and Vila (2009) are often invoked to explain potential long-term yield reductions as a consequence of QE (see e.g. Hamilton and Wu, 2012; Greenwood and Vayanos, 2014; Doh, 2010).

[5]It is, for instance, well known that households in many European countries save mostly through bank deposits, while their US-American counterparts prefer stocks and pension fund shares.

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Our framework has characteristics of an agent-based model. Agent-based modeling is a computational methodology that focuses on the interaction of autonomous agents within a system. The methodology has received growing attention in the last two decades and even more in the wake of the crisis which has demonstrated a glaring need for new economic models and thinking. While still regarded with skepticism by the mainstream, ABMs present an alternative approach to the representative agent paradigm found in general equilibrium models like DSGE models.

In essence, agent-based models incorporate micro-founded behavior of rational or bounded-rational agents, who interact according to a given set of rules. Agents' decision rules and characteristics are often heterogenous, e.g. in respect of their preferences or expectations. The agent-based framework is highly suited to include learning behavior in agents' decision rules. Some examples of advanced learning techniques include neural networks or evolutionary algorithms (Bonabeau, 2002). Adaptive behavior of agents may cause them to evolve and one may observe unanticipated phenomena that emerge dynamically over time, such as asset price bubbles or regional segregation of households with similar demographics (Lux, 1995; Boswijk et al., 2007; Schelling, 1971).
It's important to note that ABMs come in many forms and not all agent-based models include adaption and learning. Applications within economics are particularly prominent for **financial market** models and models of **networks**.
ABM applications to financial markets have been successful in the sense that they could reproduce stylized empirical facts like fat tails in returns, clustered volatility and herding behavior (see e.g. LeBaron, 2001; Chen et al., 2012b)). There are equilibrium and disequilibrium models in the literature. Among the latter group, Beja and Goldman (1980) were the first to model a financial market with a market maker adjusting excess demand stemming from traders with diverging believes about asset prices. Examples of disequilibrium models which build on the distinction between fundamentalists and chartists include Day and Huang (1990) and Chiarella (1992) who show that the involvement of chartists/speculators above a certain threshold destabilizes the market. Farmer and Joshi (2002) present a log-linear price impact function and explain how clustered volatility can arise with the emergence of different trading strategies. In the group of equilibrium models, De Grauwe and Rovira Kaltwasser (2012) model an exchange rate in the presence of fundamentalists and chartists where the proportions of the strategies evolve endogenously over time.

Complexity and realism vs tractibility

ABMs are built from the perspective of looking at the economy as a complex evolving system. It is common to include a set of realistic and flexible set of assumptions regarding agents' behaviors and interactions. Examples for such assumption can be limited information or bounded rationality. The realism in agent-based models is an attempt to achieve empirical understanding and replication (see Tesfatsion, 2006). However, the more one tries to incorporate realistic assumptions into the model, the more complex the system becomes.
This may lead to over-complicated models featuring multiple equilibria or out-of equilibrium solutions. Simulation results are not as clear cut as in closed-form solutions and may have ambiguous comparative statics. This is because ABMs are not required ex-ante to be analytically solvable. Thus,

QE (see e.g. Hamilton and Wu, 2012; Greenwood and Vayanos, 2014; Doh, 2010).

[7] It is, for instance, well known that households in many European countries save mostly through bank deposits, while their US-American counterparts prefer stocks and pension fund shares.

one of the main criticism ABMs face has to do with their lack of tractibility.
We believe it's important to balance advantages gained from incorporating realistic assumptions with a a minimum form of tractibility in the ABM framework. This is why our spillover model is solved numerically through an adaptive algorithm that finds daily market clearing prices. The simulation converges to a long-term equilibrium which represents the 'steady-state' we base our analysis on.

To summarise, agent-based models are an alternative to conventional economic models when studying problems around heterogeneity, complexity and non-linearity. For our research question that studies QE-induced spillover effects on financial markets, we believe it to be best approach to account for heterogeneity in assets and investors. This is absolutely crucial when studying the portfolio balance effect of imperfect asset substitution associated with QE.

3 Model framework

This section presents a dynamic heterogenous agent model that was specifically developed to account for heterogeneity in assets and agents. In its present form, we introduce a two-country set up, but the framework can be extended to feature more countries as well.

The model schematized in Figure 1 comprises the following: country $\mathcal{D}$ populated by investor agents $d \in \{1, 2, ..., n^d\}$ and country $\mathcal{F}$ populated by investor agents $f \in \{1, 2, ..., n^f\}$. Both countries have local financial markets in which investors can trade shares of asset portfolios $D \in \{1, 2, ..., n^D\}$ issued in country $\mathcal{D}$ and $F \in \{1, 2, ..., n^F\}$ issued in country $\mathcal{F}$. Introducing portfolios of assets instead of individual assets reduces computational complexity and facilitates a parsimonious modeling of asset maturity. Capital can flow freely between the two countries at endogenous exchange rates $X^{\mathcal{DF}}$ and $X^{\mathcal{FD}}$, with $X^{\mathcal{FD}} = 1/X^{\mathcal{DF}}$. The exchange rate $X^{\mathcal{DF}}$ defines the amount of units of country $\mathcal{D}$'s currency that can be purchased with one unit of country $\mathcal{F}$'s currency. Thus, an increase in $X^{\mathcal{DF}}$ corresponds to an appreciation of country $\mathcal{F}$'s currency vis-à-vis country $\mathcal{D}$'s currency. A central bank agent in country $\mathcal{D}$ can intervene in financial markets by buying and selling shares in portfolio D. In the following we describe agent behavior from the point of view of an agent from country $\mathcal{D}$, which we will refer to as the domestic country. We will furthermore not use superscripts D and F to indicate a domestic or foreign portfolio in equations that apply to portfolios of both countries.

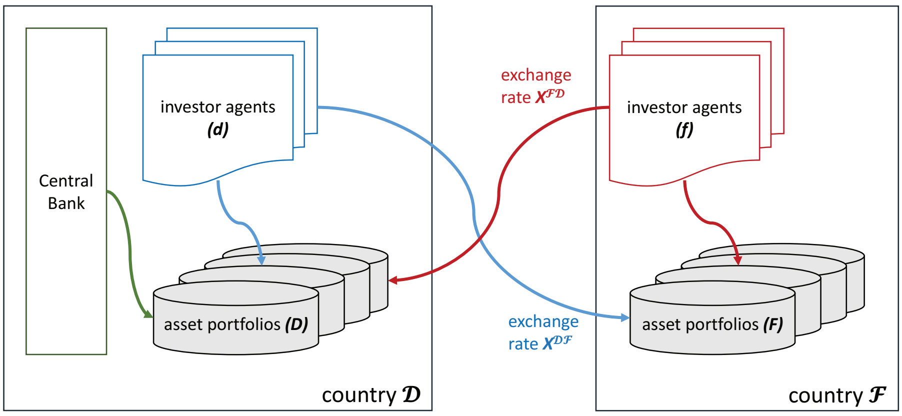


Figure 1: Model overview.

Sequence overview

To help the reader understand model dynamics before going into a detailed description, we present the algorithm in a simplified form in Figure 2.

The timing of the model includes two separate loops, an inter-day loop that governs the events from day t to $t+1$ and an intra-day loop which finds market clearing prices. At the start of the day, investors form expectations about asset returns and optimise their portfolio according to mean-variance preferences. A recursive log impact price function adjusts prices until a stopping criteria δ is satisfied, meaning that demand and supply come sufficiently close. Our model features simultaneous clearing of three markets, i.e. the domestic market, the foreign market and the exchange rate market. Subsequent to finding equilibrium prices, transaction take place and balance sheets are adjusted. Profit and maturity related factors occur overnight, before the sequence starts again. Eventually, the algorithm converges to a long-term steady state.

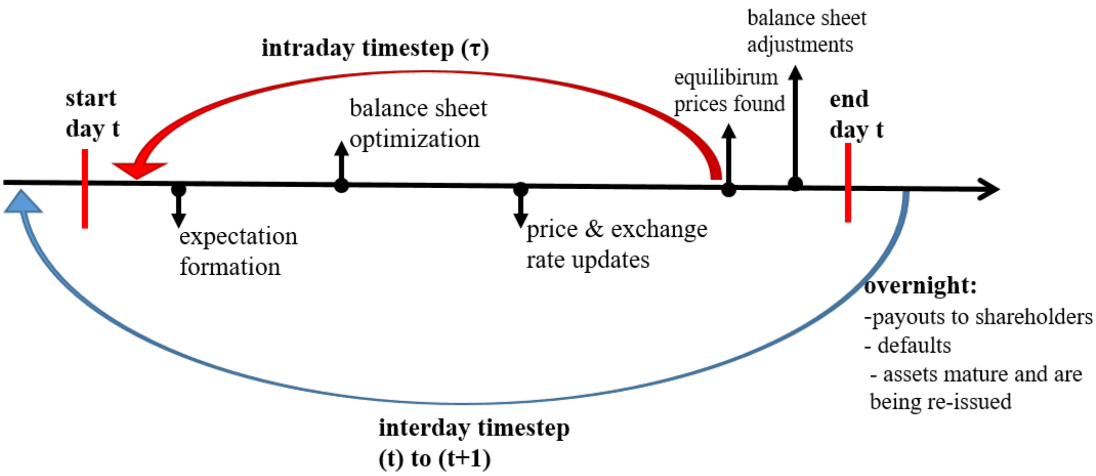


Figure 2: Model timing.

We will know proceed with the description of agents and assets.

3.1 Investor balance sheets

A domestic investor agent d has the following balance sheet structure:

Assets		Liabilities
Domestic Asset Portfolios,	$\sum_{D=1}^{n^D} Q_{d,t}^D P_t^D$	Capital, $S_{d,t}$
Foreign Asset Portfolios,	$\sum_{F=1}^{n^F} Q_{d,t}^F (P_t^F X_t^{\mathcal{DF}})$	
Domestic Cash,	$C_{d,t}^{\mathcal{D}}$	
Foreign Cash,	$C_{d,t}^{\mathcal{F}} X_t^{\mathcal{DF}}$	

with $Q_{d,t} \in \mathbb{R}^+$ denoting the quantity of shares held in a portfolio by agent d at time t and P_t being the market price of one share in the portfolio. Agents can hold cash in domestic and foreign currency ($C_{d,t}^{\mathcal{D}}$ and $C_{d,t}^{\mathcal{F}}$) and fund themselves through capital $S_{d,t}$, which they receive as an endowment in $t=0$. For the sake of simplicity, we abstract from any liability side management by agents. We thereby assume that any portfolio balancing is, on average, not due adjustments in the capital structure of agents.

3.2 Heterogeneous assets

The asset portfolios of a country can differ with regard to their maturity structure as well as the default rate of underlying assets. As agents consider expected real returns, the location of the asset portfolio determines its inflation risk.

Defaults

We assume that a fraction Ω of the asset portfolio instantly defaults overnight. Default risk is one of two exogenous risk factors that enter the model, the other one being inflation. We model defaults as stochastic process whose distribution in known by investor agents. See section 4.4 on page 20 and Equation 4.2 on page 21 for more details.

Inflation

Agents consider expected real returns, so inflation risk enters the model by deflating expected nominal returns by an expected inflation rate which is modeled as a simple normally distributed random variable $\pi_t \sim \mathcal{N}(\mathrm{E}[\pi], \sigma^\pi)$ with constant mean and variance. We describe in section 4.4 how we calibrate inflation risk to the domestic and foreign market.

Maturity

Maturity is introduced through a exogenously set repayment rate $(1-m)$ on outstanding assets in a portfolio. The parameter $m \in [0,1]$ can be interpreted as a maturity parameter, with $m = 0$ meaning that all assets inside the portfolio mature overnight, whereas $m = 1$ would entail that assets never mature.[8]

Figure 3 exemplarily shows the maturity structure of portfolios for different values of m. The higher the m, the lower the repayment rate $(1-m)$, the longer it takes for the portfolio to mature. Given $m = 0.996$ for example, the asset portfolio will have (hypothetically) matured approximately after 5 trading years from the current point in time. Note that although assets mature, the overall supply of outstanding shares is held constant by introducing an underwriter agent who instantly re-issues the maturing portfolio shares. This is a convenient way for us to deal with maturity risk without having to model maturity dates over time.

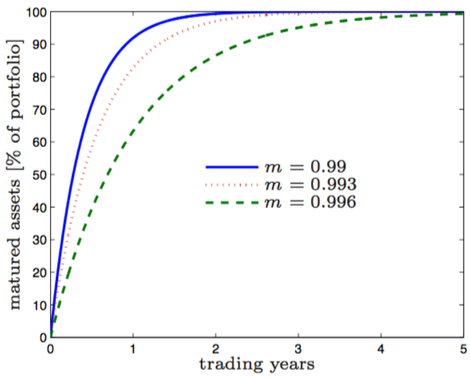


Figure 3: Exemplary maturity structures of a portfolio of assets for different values of m.

Section 7.1 in the appendix is devoted to a more detailed discussion on the appropriateness of modeling maturity through a constant repayment rate and its relation to the term structure of interest rates.

[8] Hatchondo and Martinez (2009) pioneered this one-parameter model of maturity within a standard macro model. However, rather than assuming a constant repayment rate on bonds, they assume a perpetual bond with constantly declining coupon payments. Bond duration is computed with the Macaulay definition of duration, which focuses on cash flow rather than redemption. Our modeling of maturity is identical to that in Riedler and Brückbauer (2017).

3.3 Asset performance

For an overview of performance-related variables, consider the following table:

Π_t^D	profit per portfolio share	V^D	nominal value of portfolio
$\bar{Q}^D$	portfolio shares issued	P_t^D	price of portfolio share
ρ^D	interest on portfolio share	Ω_t	default rate
m	maturity parameter	$(1-m)$	constant repayment rate
out_t	percentage outstanding	mat_t	percentage maturing

Table 1: Performance related variables for asset portfolios

There are five effects that can have an impact on an investor agent's performance. For the sake of simplicity, we assume that the events associated with these effects, i.e. interest payments, principal repayments, defaults, price movements and exchange rate movements, happen overnight (i.e. in the logical second before period t) and become public knowledge at the beginning of each period. We can compute the profit $\Pi_{d,t}$ that has accrued overnight by adding the performance effects associated with each portfolio share and unit of cash, i.e.

$$\Pi_{d,t} = \sum_{D=1}^{n^D} Q_{d,t-1}^D \Pi_t^D + \sum_{F=1}^{n^F} Q_{d,t-1}^F \Pi_t^F + C_{d,t-1}^{\mathcal{D}} \Pi_t^{C^{\mathcal{D}}} + C_{d,t-1}^{\mathcal{F}} \Pi_t^{C^{\mathcal{F}}}$$

Not all shares of a portfolio of assets are subject to the same performance effects. For the sake of a clearer presentation, we define three factors that indicate these different shares:

- The factor out_t defines the percentage of performing outstanding portfolio shares at the beginning of period t relative to the portfolio shares held at the end of period $t-1$.
- We define the factor mat_t as the percentage of performing portfolio shares that matured overnight.
- all_t defines all performing portfolio shares, i.e. those that have not defaulted overnight

So that

$$\begin{aligned} out_t &:= m(1-\Omega_t) \\ mat_t &:= (1-m)(1-\Omega_t) \\ all_t &:= out_t + mat_t = (1-\Omega_t) \end{aligned}$$

Note that the factors are time dependent because they take into account the stochastic default rate of assets in a portfolio. The profit that accrues for one share in a portfolio denominated in domestic currency is computed as follows:

$$\Pi_t^D = \underbrace{mat_t^D\left(\frac{V^D}{\bar{Q}^D} - P_{t-1}^D\right)}_{\text{repayment effect}} + \underbrace{out_t^D(P_t^D - P_{t-1}^D)}_{\text{price effect}} + \underbrace{all_t^D \frac{V^D}{\bar{Q}^D}\rho^D}_{\text{interest effect}} - \underbrace{\Omega_t^D P_{t-1}^D}_{\text{default effect}} \quad (3.1)$$

For the sake of simplicity, the face value of the assets in a portfolio V, the total number of portfolio shares $\bar{Q}$, the interest rate ρ payed per portfolio share are modeled as constants.[9] We furthermore assume that the loss given default of assets in the portfolio is 100%, i.e. shareholders incur losses of P_{t-1} for each defaulting portfolio share. The principal repayments and interest payments depend on the face value of a portfolio share, not its past valuation. When considering portfolios denominated in foreign currency, exchange rate movements between $t-1$ and t will influence the repayment and price effect. Interest rates will be valued at the current exchange rate, while losses due to default are

[9] An exogenous underwriter-agent, which instantly reissues defaulting and maturing assets is introduced in Section 3.6 in order to keep face values and quantities of portfolio shares constant.

written off at last period's exchange rate:

$$\Pi_t^F = \underbrace{mat_t^F (X_t^{\mathcal{DF}} \frac{V^F}{\bar{Q}^F} - X_{t-1}^{\mathcal{DF}} P_{t-1}^F)}_{\text{repayment effect}} + \underbrace{out_t^F (X_t^{\mathcal{DF}} P_t^F - X_{t-1}^{\mathcal{DF}} P_{t-1}^F)}_{\text{price effect}} +$$

$$+ \underbrace{all_t^F X_t^{\mathcal{DF}} \frac{V^F}{\bar{Q}^F} \rho^F}_{\text{interest effect}} - \underbrace{\Omega_t^F X_{t-1}^{\mathcal{DF}} P_{t-1}^F}_{\text{default effect}}, \tag{3.2}$$

While all cash positions pay an interest rate, the valuation of foreign cash positions are also affected by exchange rate movements:

$$\Pi_t^{C^{\mathcal{D}}} = \rho^{C^{\mathcal{D}}} \text{ and } \Pi_t^{C^{\mathcal{F}}} = X_t^{\mathcal{DF}} \rho^{C^{\mathcal{F}}} + X_t^{\mathcal{DF}} - X_{t-1}^{\mathcal{DF}}. \tag{3.3}$$

For the sake of simplicity, we model interest rates payed on cash in their respective currency as constants $\rho^{C^{\mathcal{D}}}$ and $\rho^{C^{\mathcal{F}}}$.

Given the profit $\Pi_{d,t}$ agent d accrued overnight, the first decision that agent has to make is whether to retain or to pay profits out to shareholders. In order to avoid dealing with economic growth issues within the model, we assume that all performance effects that materialize in a period (i.e. all, expect for the price effect) are payed out to shareholders. Payouts are made in both domestic and foreign currency according to the following equation:[10]

$$\begin{aligned} D_{d,t}^{\mathcal{D}} &= C_{d,t-1}^{\mathcal{D}} \rho^{C^{\mathcal{D}}} + \sum_{D=1}^{n^D} Q_{d,t-1}^D \left(mat_t^D (\frac{V^D}{\bar{Q}^D} - P_{t-1}^D) + all_t^D \frac{V^D}{\bar{Q}^D} \rho - \Omega_t^D P_{t-1}^D \right) \\ D_{d,t}^{\mathcal{F}} &= C_{d,t-1}^{\mathcal{F}} \rho^{C^{\mathcal{F}}} + \sum_{F=1}^{n^F} Q_{d,t-1}^F \left(mat_t^F (\frac{V^F}{\bar{Q}^F} - P_{t-1}^F) + all_t^F \frac{V^F}{\bar{Q}^F} \rho - \Omega_t^F P_{t-1}^F \right) \end{aligned} \tag{3.4}$$

The law of motion for the value of capital amounts to:

$$S_{d,t} = S_{d,t-1} + \Pi_{d,t} - D_{d,t}^{\mathcal{D}} - X_t^{\mathcal{DF}} D_{d,t}^{\mathcal{F}}. \tag{3.5}$$

3.4 Expectation Formation

Realized nominal returns on a portfolio share can be computed by dividing the profit per share that accrued overnight by last period's price. Taking into account an exogenous stochastic process for domestic and foreign inflation $\pi_t^{\mathcal{D}}$ and $\pi_t^{\mathcal{F}}$ respectively and defining $\mathrm{E}_{d,t}[\cdot]$ as agent d's expectation of the variable in brackets at time t, the expected real returns of domestic and foreign portfolios amount to:

$$\underset{d,t}{\mathrm{E}}[r_{t+1}^D] = \frac{1 + \mathrm{E}_{d,t}[\Pi_{t+1}^D]/P_t^D}{1 + \mathrm{E}_{d,t}[\pi_{t+1}^{\mathcal{D}}]} - 1 \quad \text{and} \quad \underset{d,t}{\mathrm{E}}[r_{t+1}^F] = \frac{1 + \mathrm{E}_{d,t}[\Pi_{t+1}^F]/(X_t^{\mathcal{DF}} P_t^F)}{1 + \mathrm{E}_{d,t}[\pi_{t+1}^{\mathcal{D}}]} - 1 \tag{3.6}$$

Analogously, real domestic and foreign cash returns amount to:

$$\underset{d,t}{\mathrm{E}}\left[r^{C^{\mathcal{D}}}{}_{t+1}\right] = \frac{1 + \mathrm{E}_{d,t}\left[\Pi_{t+1}^{C^{\mathcal{D}}}\right]}{1 + \mathrm{E}_{d,t}[\pi_{t+1}^{\mathcal{D}}]} - 1 \quad \text{and} \quad \underset{d,t}{\mathrm{E}}\left[r^{C^{\mathcal{D}}}{}_{t+1}\right] = \frac{1 + \mathrm{E}_{d,t}\left[\Pi_{t+1}^{C^{\mathcal{D}}}\right]/X_t^{\mathcal{DF}}}{1 + \mathrm{E}_{d,t}[\pi_{t+1}^{\mathcal{D}}]} - 1 \tag{3.7}$$

Since nominal profits in period $t+1$ depend on prices, exchange rates and default rates that only materialize overnight, agents need to form expectations regarding their realization. All other variables are assumed to be constant by agents. For the sake of simplicity, we assume that agents know the stochastic process behind default rates, i.e. $\mathrm{E}_{d,t}[\Omega_{t+1}] = \mu_t^{\Omega}$, with μ_t^{Ω} being the first moment of the true default rate process.[11] We furthermore assume that agents are myopic and believe in efficient

[10] When payouts are negative, agents do not receive funds, but rather retain future positive payouts until losses have been compensated.

[11] Including heterogeneous expectations about default rates is feasible within the model. We refrain from implementing them because it would require $n^d \gg 1$, which substantially increases computation time with apparent benefit for the simulations conducted.

asset markets, i.e. $\mathrm{E}_{d,t}[P_{t+1}] = P_t$. Exchange rates expectations, on the other hand, are anchored in economic fundamentals.[12] Agents believe the exchange rate will eventually revert to the purchasing power parity ($\bar{X}_t^{\mathcal{DF}}$):

$$\underset{d,t}{\mathrm{E}}[X_{t+1}^{\mathcal{DF}}] = X_t^{\mathcal{DF}} + \eta \left(\underset{d,t}{\mathrm{E}}[\bar{X}_{t+1}^{\mathcal{DF}}] - X_t^{\mathcal{DF}} \right), \quad \text{with} \quad \underset{d,t}{\mathrm{E}}[\bar{X}_{t+1}^{\mathcal{DF}}] = \bar{X}^{\mathcal{DF}} \frac{1 + \mathrm{E}_{d,t}[\pi_{t+1}^{\mathcal{F}}]}{1 + \mathrm{E}_{d,t}[\pi_{t+1}^{\mathcal{D}}]} \tag{3.8}$$

The parameter η, which e.g. depends on the responsiveness of a country's exports and imports to changes in exchange rates, defines the expected convergence speed towards the purchasing power parity. Expectations of purchasing power parity change with changes in expected inflation. In order to take into account the risk of their investments, agents compute estimates of variance and covariances of historic real returns. We generally define an agent's estimate of the covariance between variables r^x and r^y as

$$\hat{\mathrm{Cov}}_{d,t}(r^x, r^y) := \hat{\mathrm{M}}_{d,t}\left[\left(r_{t-1}^x - \hat{\mathrm{M}}_{d,t-1}[r^x, \phi]\right)\left(r_{t-1}^y - \hat{\mathrm{M}}_{d,t-1}[r^y, \phi]\right), \phi\right]. \tag{3.9}$$

with the operator

$$\hat{\mathrm{M}}_t[x, \phi] := (1 - \phi)\hat{\mathrm{M}}_{t-1}[x, \phi] + \phi x_t, \tag{3.10}$$

defining the exponentially weighted moving average of variable x. The parameter $\phi \in [0, 1]$ determines how much weight is given to the most recent observation.

3.5 Balance Sheet Optimization

Investor agents optimize their asset holdings by computing the relative weights of the portfolios on their balance sheet that optimizes a mean variance utility function:

$$\mathbf{w}_{d,t}^* = \arg\max_{\mathbf{w}} \mathbf{w}'\mathrm{E}_{d,t}[\mathbf{r}_{d,t+1}] - 0.5\mathbf{w}'\left(\boldsymbol{\lambda}_d'\boldsymbol{\Sigma}_{d,t}\boldsymbol{\lambda}_d\right)\mathbf{w} \qquad \text{s.t.} \tag{3.11}$$

$$\mathbf{w} \geq 0 \quad \text{and} \quad \mathbf{w}'\mathbf{1} = 1, \tag{3.12}$$

with $\mathbf{w}_{d,t}^* := (\mathbf{w}_{d,t}^D, \mathbf{w}_{d,t}^F, \mathbf{w}_{d,t}^C)'$ being the $N \times 1$ vector of optimal weights, $\mathrm{E}_{d,t}[\mathbf{r}_{d,t+1}] = (\mathrm{E}_{d,t}[\mathbf{r}_{d,t+1}^D], \mathrm{E}_{d,t}[\mathbf{r}_{d,t+1}^F], \mathrm{E}_{d,t}[\mathbf{r}_{d,t+1}^C])'$ being the $N \times 1$ vector of expected returns and $\boldsymbol{\Sigma}_d$ being the estimate of the $N \times N$-covariance matrix of returns. Note that $N = n^D + n^F + 2$ because $\mathbf{w}_{d,t}^D$, $\mathbf{w}_{d,t}^F$ and $\mathbf{w}_{d,t}^C$ are themselves vectors containing the optimal weights of n^D domestic asset portfolios $w_{d,t}^D$, n^F foreign asset portfolios $w_{d,t}^F$ and 2 cash positions $w_{d,t}^{C^{\mathcal{D}}}$ and $w_{d,t}^{C^{\mathcal{F}}}$. Analogously, $\mathrm{E}_{d,t}[\mathbf{r}_{d,t+1}^D], \mathrm{E}_{d,t}[\mathbf{r}_{d,t+1}^F]$ and $\mathrm{E}_{d,t+\tau}[\mathbf{r}_{d,t+1}^C]$ are themselves vectors of the corresponding expected returns.

The vector $\boldsymbol{\lambda}_d := \sqrt{(\boldsymbol{\lambda}_d^D, \boldsymbol{\lambda}_d^F, \boldsymbol{\lambda}_d^C)'}$ assigns specific risk aversions to all assets, with $\boldsymbol{\lambda}_d^D$, $\boldsymbol{\lambda}_d^F$ $\boldsymbol{\lambda}_d^C$ being $1 \times n^D$, $1 \times n^F$ and 1×2 vectors specifying the risk aversion assigned to individual domestic and foreign portfolios as well as domestic and foreign currencies, respectively. Including asset specific risk aversions is a simple way to account for idiosyncratic preferences of agents for certain assets that cannot be explained by their risk-return profile nor associated transaction costs. The substitutability of available assets crucially depends on agents' preferences for certain maturities, asset types (e.g. equities vs. bonds) and issuer nationality (home bias).

Equation (3.12) contains two constraints: a no short selling constraint[13] and a budget constraint (the sum of weights must be equal to 100%).

An agent's demand for the domestic and foreign portfolios as well as currencies are functions of the optimal weights:

[12] When agents believe that the expected exchange rate movement is zero, i.e. $\mathrm{E}_{d,t}[X_{t+1}^{\mathcal{DF}}] = X_t^{\mathcal{DF}}$, it can easily be shown that the expected return of an investment in a foreign asset is independent of the exchange rate level. This would lead to unrealistic exchange rate dynamics with excessive volatility.

[13] We follow Levich et al. (1999) with this assumption, who argue that short selling plays only a minor role in real funds' investment decisions.

$$\Delta Q^D_{d,t} = \frac{w^D_{d,t} S_{d,t}}{P^D_{d,t}} - out^D_t Q^D_{d,t-1} \tag{3.13}$$

$$\Delta Q^F_{d,t} = \frac{w^F_{d,t} S_{d,t}}{X^{\mathcal{DF}}_t P^F_{d,t}} - out^F_t Q^F_{d,t-1} \tag{3.14}$$

$$\Delta C^{\mathcal{D}}_{d,t} = w^{C^{\mathcal{D}}}_{d,t} S_{d,t} - \left(\underbrace{C^{\mathcal{D}}_{d,t-1}(1+\rho^{C^{\mathcal{D}}})}_{\text{previous cash + interest}} - \underbrace{D^{\mathcal{D}}_{d,t}}_{\text{payouts}} + \sum_{D=1}^{n^D} \underbrace{(mat^D_t + all^D_t \rho^D) Q^D_{d,t-1} \frac{V^D}{\bar{Q}^D}}_{\text{principal and interest payments}} \right) \tag{3.15}$$

$$\Delta C^{\mathcal{F}}_{d,t} = \frac{w^{C^{\mathcal{F}}}_{d,t} S_{d,t}}{X^{\mathcal{DF}}_t} - \left(C^{\mathcal{F}}_{d,t-1}(1+\rho^{C^{\mathcal{F}}}) - D^{\mathcal{F}}_{d,t} + \sum_{F=1}^{n^F} (mat^F_t + all^F_t \rho^F) Q^F_{d,t-1} \frac{V^F}{\bar{Q}^F} \right) \tag{3.16}$$

Note that demand is the difference between the desired balance sheet position (resulting from the optimal weights) and its inventory at the start of period t. For portfolios, the inventory is simply the quantity held at the end of last period reduced by shares that have defaulted or matured overnight, while for currency, the inventory takes into account overnight interest payments, principal repayments and payouts.

3.6 Price and Exchange Rate Adjustments

In equilibrium, prices and exchange rates need to take values that simultaneously clear the markets of n^D domestic portfolios, n^F foreign portfolios and two currencies. Achieving simultaneous market clearing can be challenging with standard numerical techniques. We circumvent these challenges by employing a price-setting algorithm that takes into account the economic intuition that excess demand should increase prices and excess supply should reduce them. The flow chart in Figure 4 illustrates how the algorithm, which we adopt from Riedler and Brückbauer (2017), works for any variable V_t for which economic intuition (formulated as a recursive impact function) informs the direction in which that variable is updated. In case of price adjustments we use the following impact function starting with last period's prices ($P_{t^*} = P_{t-1}$):[14]

$$\log(P_{t^*}) = \log(P_{t^*}) + \gamma \frac{\Delta Q_{t^*}}{\bar{Q}}, \tag{3.17}$$

with γ being the intensity with which prices change and

$$\Delta Q_{t^*} = \sum_{d=1}^{n^d} \Delta Q_{d,t^*} + \sum_{f=1}^{n^f} \Delta Q_{f,t^*} + \Delta Q_{U,t} + \Delta Q_{CB,t} \tag{3.18}$$

being excess demand (excess supply if $\Delta Q_{t^*} < 0$) for a portfolio of assets at the hypothetical price of P_{t^*}. Expectations of returns $\mathrm{E}_{d,t^*}[r_{t+1}]$ and prices $\mathrm{E}_{d,t^*}[P_{t+1}]$, covariance estimates $\hat{\mathrm{Cov}}_{t^*}(\cdot)$, payouts D_{d,t^*} and balance sheet size S_{d,t^*}, which are all need to be updated with the hypothetical price when computing demand. Excess demand in Eq. (3.17) is normalized by the total quantity of portfolio shares $\bar{Q}$ in circulation for a more convenient calibration of the intensity parameter γ.[15]
Monetary policy enters the model through the central bank agent's demand $\Delta Q_{CB,t}$ for assets in Eq. (3.18), while an exogenous underwriter agent reissues maturing and defaulting portfolio shares by suppling $\Delta Q_{U,t} \leq 0$. Note that neither the central bank demand nor the underwriter supply change during the iterative price setting algorithm. The demand of the central bank depends on it's inventory $Q_{CB,t-1}$ and the amount $Q^*_{CB,t}$ it desires to hold in period t, i.e.

$$\Delta Q_{CB,t} = Q^*_{CB,t} - out_t Q_{CB,t-1}. \tag{3.19}$$

[14] We add a star to the index of variables to differentiate between values within and outside of the price setting algorithm.

[15] The normalization by the total quantity of portfolio shares means that when every agent wants to sell their shares in one portfolio, then the price will fall by γ between two steps of the algorithm. The value of the intensity parameter is crucial to the efficiency of the pricing algorithm. If the value is too large, ΔQ_{t^*} may jump between positive and negative values without convergence towards the equilibrium price. If γ is too small, on the other hand, the number of iterations needed for prices to reach equilibrium can be unacceptably large. We allow for the intensity parameter to adapt within simulations. Specifically, we divide γ by three after ten jumps of ΔQ between positive and negative values. We multiply γ by 1.1 if no jump in ΔQ has occurred for 20 consecutive iterations.

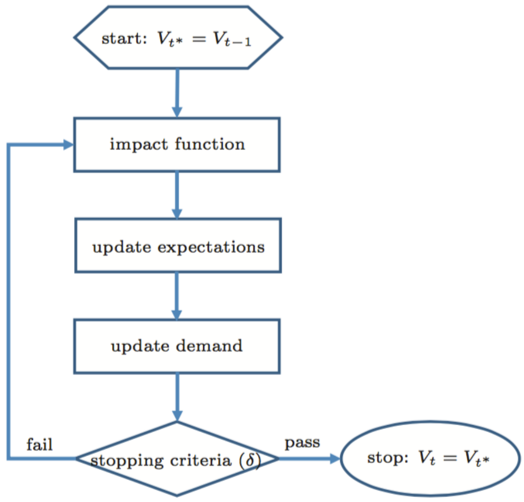


Figure 4: Pricing algorithm.

The underwriter agent, on the other hand, will supply all portfolio shares in it's inventory and all shares that have matured or defaulted overnight:

$$\Delta Q_{U,t} = -\left(out_t Q_{U,t-1} + (mat_t + \Omega_t)\bar{Q}\right) \tag{3.20}$$

The exchange rate is updated using the same iterative algorithm used to adjust price, but with the following recursive logarithmic impact function initialized at $X_{t^*}^{\mathcal{DF}} = X_{t-1}^{\mathcal{DF}}$:

$$\log(X_{t^*}^{\mathcal{DF}}) = \log(X_{t^*}^{\mathcal{DF}}) + \gamma^{EX} \frac{\Delta K_{t*}^{\mathcal{DF}}}{\sum_{d=1}^{n^d} S_{d,t^*} + \sum_{f=1}^{n^f} S_{f,t^*} X_{t^*}^{\mathcal{DF}}}, \tag{3.21}$$

with the parameter γ^{EX} defining the intensity of adjustments and

$$\Delta K_{t^*}^{\mathcal{DF}} = \sum_{d=1}^{n^d} \left(\Delta C_{d,t^*}^{\mathcal{F}} + \sum_{F=1}^{n^F} \Delta Q_{d,t^*}^{F} P_{t^*}^{F} \right) X_{t^*}^{\mathcal{DF}} - \sum_{f=1}^{n^f} \left(\Delta C_{f,t^*}^{\mathcal{D}} + \sum_{D=1}^{n^D} \Delta Q_{f,t^*}^{D} P_{t^*}^{D} \right) \tag{3.22}$$

being a measure of desired net capital flows into country $\mathcal{F}$ denominated in country $\mathcal{D}$'s currency. Analogous to normalizing excess demand in Eq. (3.17), we normalize, for convenience, desired net capital flows by the exchange rate adjusted balance sheet size of all agents.
The algorithm schematized in Figure 4 can produce prices and exchange rates that come arbitrarily close to the values that clear their respective market. Since market clearing is achieved when $\delta_{t^*}^{\{D,F\}} := \Delta Q_{t^*}^{\{D,F\}} / \bar{Q}^{\{D,F\}}$ for all portfolios and $\delta_{t^*}^{X} := \Delta K_{t*}^{\mathcal{DF}} / (\sum_{d=1}^{n^d} S_{d,t^*} + \sum_{f=1}^{n^f} S_{f,t^*} X_{t^*}^{\mathcal{DF}})$ in the foreign exchange market are zero, it is sensible to exit the algorithm when all these variables go below the bound b, i.e.

$$\delta := \begin{cases} \text{pass} & \text{if } \bigwedge_{D=1}^{n^D} (\delta_{t^*}^{D} \leq b) \wedge \bigwedge_{F=1}^{n^F} (\delta_{t^*}^{F} \leq b) \wedge (\delta_{t^*}^{X} \leq b) \\ \text{fail} & \text{else} \end{cases} \tag{3.23}$$

The higher b, the quicker the stopping criteria δ is met and the further prices and the exchange rate are from their market clearing values. Figure 3.6 illustrates this trade-off between computational complexity and achieving market clearing prices and exchange rates. We measure computational complexity as the average number of iterations needed before the stopping criteria δ is reached.

When the stopping criteria δ is satisfied, final prices and exchange rates are determined, i.e. $P_t = P_{t^*}$ and $X_t^{\mathcal{DF}} = X_{t^*}^{\mathcal{DF}} + \epsilon_t^X$. Note that we add an error term to the exchange rate in order to account for exchange rate movements that are not driven by transactions in financial assets.

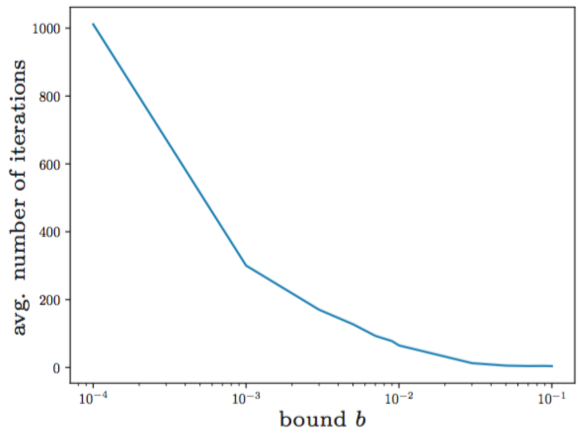


Figure 5: Trade-off between computational complexity and achieving market clearing.

3.7 Balance Sheet Adjustments

Once prices and the exchange rate have adjusted to a satisfactory extent, agents sell and buy portfolio shares and trade cash on foreign exchange markets. The consequence of using the pricing algorithm presented in the previous section is that new balance sheet positions can, in an unsystematic way, slightly deviate from the positions implied by agents' balance sheet optimization. Updating the asset side of investor agents' balance sheets thus requires taking into account excess demand and supply in shares of portfolios as well as cash payments that result from transactions in portfolio shares. The computations that lead to the new balance sheet positions are described in Appendix 7.2.

4 Calibration

We calibrate our model for the purpose of analyzing international portfolio balance effects of the ECB's expanded asset purchase programs that started in March 2015. The domestic and foreign country in the model thus represent the Eurozone (EZ) and a sample of rest-of-the-world (ROW) countries, respectively. Both regions comprise two agents, which are calibrated to match investment behavior of funds and banks. Agents can trade two asset portfolios per region representing bonds (sovereign and corporate) and equities. We use several data sources in order to get the model as close to the data as possible. Calibrated variables include balance sheet positions, interest rates, dividend yields, the maturity profile of bond portfolios, inflation risk and default risk.

4.1 Agents' asset holdings

Agents' balance sheet positions are calibrated to financial asset holdings of world-wide investment funds and banking institutions. While there is substantial heterogeneity within the group of a region's funds and banks respectively, we model EZ banks and EZ funds as well as ROW banks and ROW funds as representative agents for their respective industry, i.e. two agents per region. Our framework does not limit the number of agents included per region. However, data availability and computational complexity provide good reason to start with representative agents.

Our sample of investment funds include equity funds, bond funds, mixed funds and hedge funds. For the sake of simplicity, we exclude real estate funds, money market funds, pension funds and insurance companies. Although the investment behavior of these institutions could very well lead to a portfolio rebalancing effect in response to QE, we assume that mean-variance portfolio optimization would be a poor representation of that behavior.[16] All data points are taken from Q4 2014. Table 3 provides an overview of the balance sheet positions, while Table 13 in the appendix provides details on the

[16]The insurance and pension fund sector (ICPF) exhibits demand functions for bonds that are distinct from the rest of the investment fund sector. Due to their long-term obligations and duration gap management, the ICPF sector has an upward sloping demand function for long-term bonds, i.e. when the price of the long-term bond increases, the duration gap of any given bond portfolio increases as well and the ICPF investor reacts by purchasing additional long-term assets to match the longer duration of their liabilities Domanski et al. (2015).

data sources used. While we assume that all equity and debt securities on funds' balance sheets are held for investment purposes and therefore subject to portfolio optimization, banks may have other reasons to hold securities (e.g. to use as collateral in exchange for central bank liquidity). To account for this, we only include bank assets that are categorized as "trading assets" and "available for sale". Calibrating currency positions for funds and banks is more problematic. We are unable to distinguish cash positions held for transactional purposes and for investment purposes. For the sake of simplicity we assume that all cash positions on funds' balance sheets are the result of portfolio optimization while only excess reserves held by banks qualify as such.

4.2 Interest Rates, Dividends and Inflation

Table 2 shows the values used in simulations for EZ and ROW nominal interest rates for the respective bond portfolios and currencies, as well as nominal dividends for equity portfolios. Furthermore, since agents in our model consider real returns instead of nominal return when making investment decisions, we set agents' inflation expectations to inflation forecasts at the end of 2014 for the year 2015.[17] The nominal rates on currency are set to central bank deposit facility rates for the EZ and the ROW that were effective at the end 2014. The nominal rate for bond portfolios and dividends (as percent of an equity portfolio's face value) are calibrated in such a way, that agents' initial expected returns at $t = 0$ for the respective portfolios match empirical returns for stocks and bonds. For the calibration of expected bond returns, we use yield to maturity data from comprehensive S&P sovereign and corporate bond indices (see Table 19 in the appendix for details). We take a simple average of corporate and sovereign yields to determine the expected return of bond portfolios in the model.[18] Expected returns on equity portfolios are calibrated by using estimates of equity premia and risk-free rates for corresponding geographical regions (see Table 16 in the appendix). Note that we assume a constant dividend rate just as we assume a constant nominal interest rate on bond portfolios. While this assumption would be problematic for an individual stock, it seems reasonable for a comprehensive portfolio of stocks.

4.3 Preferences

We assume that funds' and banks' preferences are revealed by their asset holdings (Table 3) at prevailing interest rates (Table 2). In our model, these preferences come in the form of asset specific risk aversion parameters. Since changing the risk aversion parameters can have an influence on returns and their variation, we need to use the iterative algorithm schematized in Figure 6. The calibration algorithm starts by simulating the model with arbitrary parameters for bonds' nominal interest rates, for equities' nominal dividend rates and for asset specific risk aversions. The dynamics created in this simulation give agents an idea of the variance and covariances of returns associated with each asset. In a second step, asset holdings per agent are set to their target values documented in Table 3 and, for the sake of convenience, prices and exchange rates are set to 1. We then adjust nominal interest rates for bonds and nominal dividend rates for equities so that the expected returns for the respective assets is identical with the data from Table 2. The next step is the main part of the calibration algorithm.

[17]See Table 15 in the appendix for the weighting of ROW countries to obtain representative nominal rates and inflation rates

[18]Public and private debt securities have very similar market capitalizations in the Eurozone and in our sample of ROW countries.

	Nominal rate Cash	Nominal rate Bond portfolio	Nominal rate Equity portfolio	Expected Inflation
EZ	-0.20%	0.93%	5.73%	1%
ROW	0.85%	1.92%	4.33%	2.1%

Table 2: Nominal interest rates for asset portfolios. EZ cash rate is the ECB's deposit facility as of Dec 2014. The ROW cash rate is the market-cap weighted average of our sample of ROW countries. See Table 15 for details of weighting and deposit rate per individual countries. Nominal rates for bond and equity portfolios are calibrated to yield to maturity values for domestic and foreign market indices (see Table 19 for details) and equity risk premia (see Table 16) respectively. Expected inflation values are sourced from the ECB's inflation forecast for the Eurozone and the OECD's inflation forecast for the Rest-of-the-world respectively (15).

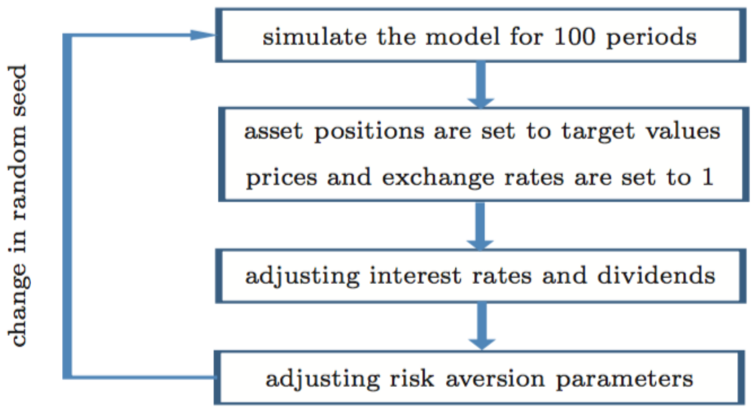


Figure 6: Calibration algorithm.

Trillion EUR		Debt Securities (DS)			Equities (Eq)			Currencies (C)			Computed Total Assets
		EZ DS	ROW DS	sum DS	EZ Eq	ROW Eq	sum Eq.	EZ C	ROW C	sum C	
Eurozone Investors	Funds	1.961	1.651	3.612	0.930	1.821	2.751	0.411	0.218	0.629	6.992
	Banks	3.204	0.656	3.860	0.571	0.075	0.646	0.08	0.016	0.096	4.602
ROW Investors	Funds	0.263	7.045	7.308	0.556	8.461	9.017	0.063	1.202	1.265	17.59
	Banks	0.568	10.009	10.577	0.078	1.692	1.770	0.136	2.4	2.536	14.883
Computed Market Size		5.996	19.361	25.357	2.135	12.049	14.184	0.69	3.836	4.526	44.067

Table 3: Balance sheet positions of funds and banks residing in the Eurozone and in the rest-of-the-world in trillion EUR. *Sources:* ECB Investment fund balance sheet statistics, CBD2, Bankscope, IMF Coordinated Portfolio Investment Survey, Morning Star Direct, Fred database and FSB Global Shadow Banking Monitoring Report.For more details for computation methods and source details see Table 18 and Table 13 in the annex.

Investors	Assets available for investment Euro area Bond Portfolio	Euro area Equity Portfolio	Rest-of-the-World Bond Portfolio	Rest-of-the World Equity Portfolio	Euro area Currency	Rest-of-the-World Currency
Euro Funds	119.35	63.22	41.09	28.21	13,233.67	126.69
Euro Banks	93.54	52.11	263.28	175.05	19860.89	142.80
ROW Funds	486.37	249.71	22.82	16.36	1646.41	80.41
ROW Banks	1317.90	719.87	54.51	40.27	3071.84	830

Table 4: Risk aversion parameters of investor agents for assets.

We adjust the asset specific risk aversion parameters for each agent so that they are willing to hold their targeted asset portfolio at the given expectations of returns. In order to adjust the asset specific risk aversion parameters we use the algorithm in Figure 4 employed to adjust prices and exchange rates within a simulation. We thereby make use of the economic intuition that an increase in asset-specific risk aversion should decrease the demand for that asset. We use the following recursive impact function to calibrate risk aversions to asset A, which could be a portfolio or a currency:

$$\log(\lambda_d^A) = \log(\lambda_d^A) + \gamma \left(\frac{w_{d,0}^A S_{d,0} - Q_{d,0}^A}{Q_{d,0}^A} \right), \tag{4.1}$$

with $w_{d,0}^A S_{d,0}$ being the demand for asset A according to agent d's balance sheet optimization, $Q_{d,0}^A$ being the targeted amount and γ being the intensity parameter that adjusts the risk aversion. Once the risk parameters for all agents have been adjusted, the model is again simulated for 100 periods and the procedure is repeated. Note that whenever the model is simulated for a 100 periods, we change the random seed, to insure that the risk aversion parameters are not only valid for a particular manifestation of the exogenous stochastic variables. We iterate the algorithm 500 times to allow for risk aversion parameters to stabilize.

Table 4 documents the calibration outcome for risk aversion parameters, while Table 5 show the standard deviation in percent of the parameters in the last 100 iterations of the calibration algorithm. Thereby it becomes clear that there is substantial uncertainty about the true preferences of agents. The uncertainty is most salient for risk aversion towards the currencies, with risk aversion parameters deviating from their means on average by 31% to 246%, while the other risk aversion parameters vary by 4% to 13%. Note that the uncertainty about the correct risk aversion parameters for currencies, reflects our uncertainty about agents' holdings of currencies for non-transactional purposes mentioned in Section 4.1. Since the risk aversion parameters can generally only be calibrated jointly, it is also likely that it is the uncertainty about currency holdings that is driving the uncertainty about risk aversion towards other assets.

Strikingly, the asset specific risk aversion parameters in Table 4 are consistently higher than the typical single digit estimates of risk aversion found in the literature. For the bond and equity portfolios the parameters range from 16.36 to 1317.90. While this seems unusual, it is more likely to indicate missing risk factors than it is to reflect a general problem with our calibration approach. Note that inflation risk and default risk are the only factors determining risk in the model. Other factors such as uncertainty about future interest rates or political uncertainty are not represented. Futhermore, the agents in the model are rather cool-headed and not inclined to overreact to changing conditions, which would generate more price volatility. The estimates of risk aversion parameters suggest that the assumed risk exposure (in the form of expected variances and covariances of returns) resulting from agents' expectation formation and trading behavior under exogenous default rate and inflation processes (see next subsection) are one order of magnitude lower than in reality. Note that this conjecture cannot easily be tested by looking at historical variations and correlations in returns, as they may differ systematically from expectations of risk exposures.

4.4 Stochastic Processes

Default risk and inflation risk are the only two risk factors that are exogenous to our model. They are modeled with random variables in order to produce realistic dynamics of endogenous variables such as prices and returns. Inflation risk is modeled as a simple normally distributed random variable $\pi_t \sim \mathcal{N}(\mathrm{E}[\pi], \sigma^\pi)$ with constant mean and variance. The mean $\mathrm{E}[\pi]$ being the region specific inflation

Assets available for investment

Investors	Euro area Bond Portfolio	Euro area Equity Portfolio	Rest-of-the-World Bond Portfolio	Rest-of-the World Equity Portfolio	Euro area Currency	Rest-of-the-World Currency
Euro Funds	7	7	4	5	31	128
Euro Banks	5	6	6	9	49	227
ROW Funds	13	11	4	5	87	246
ROW Banks	12	12	5	5	49	183

Table 5: Standard deviation in percent of risk aversion parameters for the last 100 iterations of the calibration algorithm.

	inflation process parameters		default process parameters				
	$E[\pi]$	σ^{π}	$\bar{e}$	ψ_e	σ^e	ω^{DS}	ω^{Eq}
EZ	1%	1%	15/250	0.0011	0.00176	$\frac{0.5}{250}\%$	$\frac{2}{250}\%$
ROW	2.1%	0.7%	75/250	0.0014	0.01132		

Table 6: Stochastic inflation and default processes parameters.

forecast and the standard deviation being calibrated to match that of inflation data for the respective regions from of the past 20 years, i.e. 1997 to 2017.[19] All calibration results for the stochastic processes are documented in Table 6.

For the calibration of default risk, we use data on global default events from the S&P 2017 Annual Global Corporate Default Study and Rating Transitions (see SP2017, 2018, p.5). Lacking granular data on default events for different regions, we distribute the number of events per year to the EZ and ROW region according to their relative total market capitalization. From this data basis, we estimate the parameters of the following AR(1) process:

$$\ln(E[e_t]) = \ln(E[e_{t-1}])(1-\psi_e) + \psi_e \ln(E[\bar{e}]) + \epsilon_t^e, \tag{4.2}$$

with $E[e_t]$ being the expected number of default events occurring on day t, $\bar{e}$ being the sample average number of events per day (i.e. the yearly average divided by 250), and $\epsilon_t^e \sim \mathcal{N}(0, \sigma^e)$ being a normally distributed error term with zero mean and standard deviation σ^e. Since realized default events can only be a natural number, we draw e_t from a Poisson distribution with mean $E[e_t]$. The parameters of the process in Eq. (4.2) are estimated in order to minimize the distance between the data and the stochastic process for the first order autocorrelation $A(\cdot)$ and variance $var(\cdot)$, i.e.

$$\hat{\psi}_e, \hat{\sigma}^e = \underset{\psi_e, \sigma^e}{\arg\min} \left((A(e_{\text{yearly}}) - A(\text{data}))^2 + (var(e_{\text{yearly}}) - var(\text{data}))^2 \right) \tag{4.3}$$

Since the data on default events is available on a yearly basis, while our model simulates daily data, we aggregate defaults over 250 days in order to obtain yearly default events for the numerical estimation of Eq. (4.3).

Although we assume that equity and bond portfolios of a region are subject to the same default event process, default rates Ω_t do take into account the different risk characteristics of equities and bonds. The difference in default rates for bonds and equities results from differences in the loss rate ω per default event, with $\Omega_t = e_t * \omega$. While sovereigns do not issue equity, the bond portfolio contains a 50:50 mix of corporate bonds and sovereign bonds, which for the sake of simplicity, we assume to be free of default risk. Furthermore, it is reasonable to assume a 100% loss given default for equities, whereas 50% of a defaulting corporate bond can typically be recovered (c.p. e.g. Jacobs, 2009, p. 43). With the simplifying assumption that one default event always affects $\frac{2}{250}\%$ of assets in the equity portfolios and corporate part of the bond portfolios, we arrive at our values for loss rates documented in Table 6, i.e. $\omega^{Eq} = 1 * \frac{2}{250}\%$ for equity portfolios and $\omega^{DS} = 0.5 * 0.5 * \frac{2}{250}\% = \frac{0.5}{250}\%$ for bond portfolios.[20]

[19]ROW data is taken from the IMF World Economic Outlook and weighted according to market capitalizations as documented in Table 12 of the appendix.

[20]The assumed $\frac{2}{250}\%$ of affected assets per default event means that on average 2% of corporate bond and equity issuers default each year, which is in line with the data in SP2017 (2018).

4.5 Portfolio Maturity

The average portfolio maturity parameter m is calibrated by reconstructing a portfolio that matches the maturity profile of a representative Euro area investment fund. We compile data from a sample of 15374 open-end investment funds resident in the Euro area from the Morning Star database. Table 7 contains information on the remaining maturities of fixed income securities held by the representative fund.

remaining maturity	% of total assets
0-1 year	11.5
1-3 years	22.7
3-5 years	21.7
5-7 years	15.0
7-10 years	15.1
10-15 years	4.6
15-20 years	1.9
20-30 years	4.4
30-40 years	3.1

Table 7: Maturity profile of fixed income securities of a representative open-end euro area investment fund in December 2014.

Under the assumption that the maturity dates are distributed equally within a maturity bin, we can fit the data to our model. Figure 7 shows the maturity structure of the representative fund from Table 7 and a fitted portfolio with a constant repayment rate. The maturity parameter that best fits the data is $m = 0.99936$, which corresponds to an average maturity of 6.25 years. Fitting the data to more than one portfolio with different maturity parameters only slightly increases the fit. Since stocks do not mature, we set their maturity parameter of equity portfolios to one.

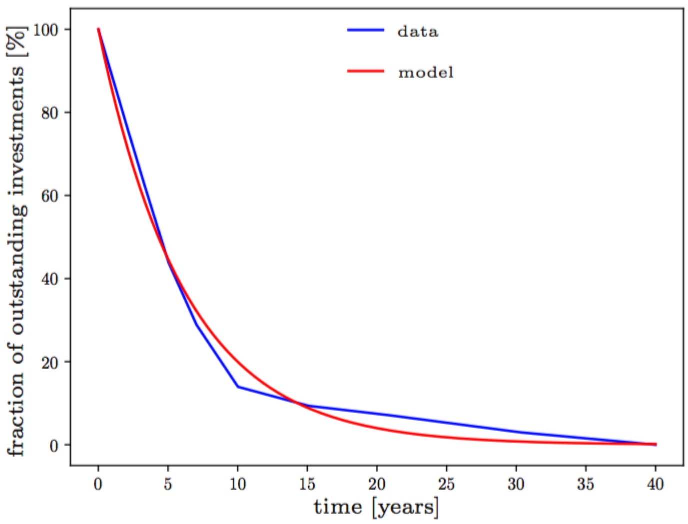


Figure 7: Maturity structure of a representative Euro area open-end investment fund compared to the maturity structure of a fitted model portfolio.

4.6 Other Parameters

Of the three parameters that remain uncalibrated the expected convergence speed (η from Eq. (3.8)) of the exchange rate towards its purchasing power parity (PPP) is arguably the most influential. Low values will lead to excess exchange rate volatility, while high values will tie the exchange rate to its PPP. We choose a convergence speed of 15% annually, which we take from Rogoff (1996).

For the memory parameter ψ in Eq. (3.9), which determines how much weight is given to the latest observation when agents update their return-covariances estimates, there is no data nor a literature that we can draw on for calibration purposes. If excessive weight (e.g. $\geq 1\%$, in our model) is given to the latest observations, agents will replace large parts of their balance sheet holdings with high frequency. This can lead to self reinforcing dynamics, where large shifts in holdings increase return volatility, which in turn increases trade volumes. The emergence of such destabilizing dynamics is unlikely to produce interesting insights as it ensues not from a realistic representation of investor behavior, but rather from an unrealistically high sensitivity of optimal portfolio weights derived from a mean-variance utility function. A memory parameter $\psi = 0.1\%$ is chosen to ensure the stability of the model.

The bound b from Eq. (3.23), which determines how close prices and the exchange rate need to come to their respective market clearing values before the stopping criteria of the pricing algorithm schematized in Figure 4 is reached. We set $b = 0.01$, which leads to reasonable simulation times (see Figure ??). This choice means that at most excess demand for an asset is $\pm 1\%$ of total quantity of that asset. Furthermore, excess demand for the respective foreign currency is at most $\pm 1\%$ of total assets.

5 Results

We conduct a series of experiments to analyze the impact of central bank asset purchases on international returns and the exchange rate. Thereby we compare simulation outcomes for a range of QE volumes to a benchmark, where the central bank does not interfere in asset markets. We simulate asset purchases from 200 bn EUR to 2.6 tn EUR, whereby the purchase volume refers to the nominal value of EZ debt securities. Each simulation run lasts for 1000 periods, i.e. 4 trading years, and is repeated 20 times with different random seeds. The repetition of simulations is important to make sure that results are valid for different manifestations of stochastic processes, in particular the default processes, which reflect different states of the economy.

5.1 Asset Returns and Prices

To obtain estimates of the portfolio balance effect of QE on asset returns, we regress the change between return expectations with and without central bank purchases on the size of these purchases. In order to avoid that serial correlation in the simulated data affects standard errors we use a cross-section of observations per random seed by averaging data points across time, which leads to 260 independently distributed observations and the following regression equation:

$$\underset{\text{seed}}{\Delta \mathrm{E}}[r|_{\mathrm{QE}}] = \alpha + \beta \frac{\mathrm{QE}}{100\text{bn EUR}} + \epsilon_{\text{seed}}, \qquad \text{with}$$
$$\underset{\text{seed}}{\Delta \mathrm{E}}[r|_{\mathrm{QE}}] = \left(\underset{\text{seed}}{\mathrm{E}}[r|_{\mathrm{QE}}] - \underset{\text{seed}}{\mathrm{E}}[r|_{\mathrm{QE}=0}] \right) * 250 * 100 * 100 \tag{5.1}$$

Note that we use annualized returns (approximated by multiplying by 250) and multiply by 10^4 in order to obtain results in basis points. We use return expectations instead of observed daily returns to isolate yield effects from price movements. Furthermore, we use EZ agents' expectations of EZ bond and equity portfolio returns and ROW agents' expectations of ROW bond and equity portfolio returns. This prevents that exchange rate expectations are reflected in return expectations. The impact of QE on exchange rate expectations is documented in ROW agents' expectations of EZ currency returns and EZ agents' expectations of ROW currency returns.

Table 8 shows the impact of QE on returns in basis points per 100 bn EUR of central bank purchases. All effects are statistically highly significant, but rather low in magnitude. The yield of the EZ bond portfolio declines on average about 0.6 basis point for each 100 bn EUR the central bank purchases of that portfolio. The low standard error on the coefficient indicates that the relationship between purchases and bond returns is well described by the linear regression. The constant is not statistically significant from zero, which is consistent with what is expected because the effect on yields by definition is zero if no QE is implemented. With a reduction of 1.96 bps per 100 bn EUR of purchases, the effect of QE is strongest on domestic equity returns. The positive impact on expected returns on the EZ currency indicates that the Euro depreciates with the central bank's asset purchases. ROW-agents assume that the Euro will eventually revert back to its PPP value and therefore expect a higher return in comparison to the scenario without central bank intervention.

We do find a statistically significant spillover into the foreign asset markets. The effects per 100 bn EUR of the central bank's purchases on the expected returns of the ROW bond portfolio and the

VARIABLES	(1) EZ bond portfolio	(2) EZ equity portfolio	(3) EZ currency
QE	-0.598***	-1.96***	0.206***
	(0.000124)	(0.000489)	(0.000119)
Constant	-0.178	0.787	0.742***
	(0.180)	(0.280)	(0.166)
Observations	260	260	260
R-squared	0.897	0.866	0.515

VARIABLES	(4) ROW bond portfolio	(5) ROW equity portfolio	(6) ROW currency
QE	-0.0271***	-0.143***	-0.205***
	(3.67e-05)	(0.000233)	(0.000118)
Constant	0.0420	0.145	-0.745***
	(0.0604)	(0.381)	(0.165)
Observations	260	260	260
R-squared	0.162	0.116	0.515

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 8: OLS regressions of QE on expected return variables with heteroskedasticity robust standard errors. *Upper half:* QE effects on domestic assets. *Lower half:* QE effects on foreign assets.

ROW equity portfolio are -0.03 bps and -0.14 bps, respectively. Thus, the impact of QE on foreign bond and equity returns, although statistically significant at the 1% level, is economically negligible.

Under our assumption that banks and funds represent the investors that are most likely to be responsible for a portfolio balance effect of QE, we can use the regression results to estimate the effects of the ECB's expanded asset purchase program (APP). Given that the ECB bought approximately 2.6 tn EUR of Eurozone debt securities between March 2015 and December 2018, our model predicts a reduction of annualized EZ bond returns of approximately 15.6 bps and a reduction of EZ equity returns of approximately 50.9 bps. While these reductions in returns are arguably too small to significantly stimulate the economy, they can have more substantial wealth effects, which are a consequence of QE induced price changes.

Table 9 shows the results of regressing percentage price differences between simulations with and without QE.[21] The price impact on the EZ equity portfolio is by far the biggest, with an increase of 0.26% per 100 bn EUR of purchased EZ bond portfolio shares. Extrapolating this to the size of the ECB's APP, predicts that the portfolio balance channel would increase EZ equity prices by approximately 6.8%. When comparing the 6.8% increase in equity prices with the annualized 15.6 bps decrease in expected bond yields, it seems fair to say that the portfolio balance effect of the ECB's APP is of greater benefit to shareholders than to issuers of debt securities.

EZ bond prices only increase by 0.035% per 100 bn EUR of purchased bonds. This is not surprising as there is a tighter relationship between the return and the price of assets with limited maturity. The relation between the exchange rate and the expected return on currency, on the other hand, is determined by the expected convergence speed (the parameter η in Eq. 3.8) towards the PPP. Hence, the predicted 0.0138% depreciation of the EUR vis-a-vis the basket of ROW currencies per 100 bn EUR of asset purchases (amounting to approx. 0.36% for the entire APP) reflects our calibration choice of $\eta = 15\%$ per year. Note that a lower (higher) expected convergence speed would lead to a stronger (weaker) depreciation without directly affecting how QE impacts expected currency returns documented in Table 8.

[21]The dependent variable is $\Delta P_{\text{seed, QE}} = \frac{P_{\text{seed, QE}} - P_{\text{seed,QE=0}}}{P_{\text{seed,QE=0}}}$, while the independent variable is the volume of EZ bond purchases.

VARIABLES	(1) EZ bond portfolio	(2) EZ equity portfolio	(3) ROW bond portfolio	(4) ROW equity portfolio
QE	0.0348***	0.260***	0.0014***	0.0152***
	(7.22e-06)	(6.39e-05)	(2.00e-06)	(2.54e-05)
Constant	0.00867	-0.180***	-0.00228	-0.0155
	(0.0104)	(0.0987)	(0.00328)	(0.0416)
Observations	260	260	260	260
R-squared	0.897	0.869	0.161	0.115

VARIABLES	(5) exchange rate
QE	0.0138***
	(7.92e-06)
Constant	0.0495***
	(0.0110)
Observations	260
R^2	0.515

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 9: OLS regressions of QE on price variables with heteroskedasticity robust standard errors.

5.2 Variation in Return and Price Effects

Although the regressions with averaged data provide estimates of the impact of QE on returns and prices with high confidence, there is a significant amount of variation in the effects when comparing individual observations. Table 10 documents the distribution of return and price effects for asset purchases worth 100 bn EUR. Note that all QE-effects apart from those on EZ bond returns and prices range from negative to positive values within the 5th and 95th percentile of the distribution. This indicates the difficulty of empirically measuring the portfolio balance effect when it is phased in slowly. It is unclear why the portfolio balance effect differs so strongly between observations. We find, however, that the state of the model economy, which we can measure by looking at stochastic default probabilities does not have a significant effect on the magnitude and direction of the portfolio balance effect.

QE-effect	Variable	Mean	Standard deviation	5% percentile	95% percentile
	exp. return EZ bonds	-0.62	0.33	-1.09	-0.22
Yields:	exp. return EZ equities	-1.88	1.56	-3.84	0.15
Effect in bps	exp. return EZ currency	0.28	0.32	-0.09	0.82
per 100 bn EUR	exp. return ROW bonds	-0.02	0.12	-0.16	0.13
assets bought	exp. return ROW equities	-0.13	0.78	-1.09	0.89
	exp. return ROW currency	-0.28	0.32	-0.81	0.09
Prices:	EZ bond price	0.036	0.019	0.012	0.063
Effect	EZ equity price	0.242	0.195	-0.018	0.488
in percentage	ROW bond price	0.001	0.006	-0.007	0.009
per 100 bn EUR	ROW equity price	0.014	0.086	-0.096	0.119
assets bought	exchange rate	0.019	0.022	-0.006	0.054

Table 10: Distribution of the QE effect on expected returns and prices following a 100 bn purchase of EZ bond assets. Sample moments are computed from 260,000 simulated observations (i.e. the average from 14 experiments with QE ranging from 200 bn EUR to 2600 bn EUR that last for 1000 days and are repeated 20 times with different random seeds).

5.3 Who Sells to the Central Bank?

Table 11 shows how agents rebalance their portfolios on the first day of asset purchases by the central bank. The upper panel documents the changes in market value, while the lower panel displays the change relative to agents' previous asset holdings. Note that sales of the EZ bond portfolio by agents do not add up to 100 bn EUR, but approximately 80 bn EUR. This is the case because the exit condition in the pricing algorithm is often satisfied before the central bank succeeds in purchasing all its desired assets. While the central bank purchases the remaining assets in the following days, we only show the results after the first day in order to avoid measuring portfolio rebalancing due to exogenously changing economic conditions (i.e. default probabilities).

We find that all agents decrease their holdings of EZ bonds and increase their holdings of EZ currency in response to QE. In relative terms, the ROW agents are more eager to sell their EZ bonds to the central bank. ROW funds sell 11.1% and ROW banks 2.3% of their EZ bonds per 100 bn EUR of EZ bonds demanded by the central bank. EZ funds and banks, who initially hold a higher share of EZ bonds, only sell 1.1% and 0.5%, respectively. The reason why ROW agents are more inclined to decrease their balance sheet share in EZ bonds, which is consistent with the data (see Koijen et al., 2016), is the increased attractiveness of the EZ currency. From the perspective of ROW agents, an expected appreciation of the Euro vis-a-vis the basket of ROW currencies increases the expected returns on EZ assets. However, since expected yields on EZ bonds and EZ equities decline at the same time, it is the investment in EZ currency that promises a higher return after QE. From the perspective of the EZ agents, holding Euros is also more attractive under QE than without it. This is the case because the expected return on holding Euros does not change with QE, while the expected returns on all other assets, domestic or foreign decline under QE. From the perspective of EZ investors, foreign expected returns decline because agents expect the ROW currency basket to depreciate in order to reach its PPP.

Change in holdings per 100bn		EZ bond	EZ equity	EZ currency	ROW bond	ROW equity	ROW currency
		EZ			Rest-of-the-World		
EZ	funds	-21.1 (8.5)	-1.3 (1.4)	22.4 (9.3)	-6.0 (11.2)	-3e-3 (0.03)	6.0 (11.3)
	banks	-16.7(9.8)	-1.1 (1.2)	16.4 (10.7)	-0.4 (1.9)	0.19 (0.3)	1.6 (5.3)
ROW	funds	-28.7 (13.9)	1.2 (1.6)	28.5 (12.5)	1.5 (6.1)	-0.1 (0.3)	-2.4 (7.1)
	banks	-13.2 (5.9)	1.2 (0.9)	12.3 (5.2)	4.4 (7.4)	-0.1 (0.2)	-4.6 (7.5)
Change in % per 100bn							
EZ	funds	-1.1 (0.4)	-0.1(0.2)	5.5 (2.3)	-0.3 (0.7)	-2e4 (0.00)	3.7 (7.0)
	banks	-0.5(0.3)	-0.2(0.2)	20.9 (13.4)	-0.1 (0.3)	0.3 (0.4)	70.0 (273.5)
ROW	funds	-11.1 (5.5)	0.2 (0.3)	42.9 (18.2)	0.02 (0.1)	-1e-3 (0.0)	-0.2 (0.6)
	banks	-2.3 (1.1)	1.5(1.1)	8.9 (3.8)	0.04 (0.07)	-0.01 (0.01)	-0.2 (0.3)

Table 11: Change in agents' balance sheet positions on the day QE is implemented. Values are means across 20 seeds, standard deviations in parentheses. *Upper half:* Difference in the market value of absolute asset holdings per 100bn EUR of asset purchases by the central bank. *Lower half:* percentage difference in asset holdings per 100bn EUR of asset purchases by the central bank.

6 Conclusions and Policy Implications

The overall topic of this paper was to investigate spillover effects from Quantitative Easing by means of a heterogenous agent model. For this purpose, a dynamic spillover model of international portfolio balance effects was developed to study the relationship between QE and financial asset prices. Our model differs from those in the literature by allowing for heterogeneity in asset characteristics and agent preferences. Including these two domains of heterogeneity is crucial in order to capture the substitutability of assets from the perspective of investors. Data clearly shows that the raw variance and covariance of returns, which are the sole determinant of asset substitutability in other portfolio models, are not sufficient to explain investor behavior. Home bias, preferences for certain asset classes and asset maturity, which we include as a characteristic of assets, all contribute in determining asset substitutability. Our two-country spillover model is complex and computationally intensive, but this is required to capture heterogeneity in assets in terms of maturity, default and inflation risk as well as in preferred habitat preferences of investors. The core of the model is about how central bank purchases impact mean-variance investors who allocate their capital across a domestic and foreign market. Asset substitutability is determined by the risk-return profile reflected in expected returns and covariance-variance structure of returns. An adaptive algorithm solves the model numerically which allows us to keep computational tractability.

The second aim of the paper was to conduct an extensive simulation study to quantify QE-induced portfolio balance effects and compare our estimates with those of the literature. The design of the simulation study was justified by incorporating theoretical and practical considerations. Section 4 calibrated the model to a two-country market with the Eurozone (EZ) representing the domestic market and a sample of rest-of-the world (ROW) countries as foreign market. It was important to capture a sample representing the global market size for portfolios of EZ and ROW bonds, equities and currencies. For this purpose, we compiled data on asset holdings of 15 374 EZ and 25 930 ROW open-end *investment funds* from the Morning Star Database, as well as data on investment portfolios of EZ and ROW *banks* from the ECB's Statistical Warehouse, Bankscope and other sources detailed in Table 13 on page 35.
The simulation study finds significant portfolio balance effects for the domestic market, albeit smaller than what could have been assumed from QE announcement studies. The average effect of EUR100bn asset purchases of the EZ central bank is a decline in the expected return of the EZ bond portfolio of 0.6 basis points, which means that the APP (EUR 2.6tn) led to an overall reduction of 15.6 basis points in domestic bond returns. This result is broadly in line with empirical work of Koijen et al. (2016) who find that the ECB's APP caused a decline of about 14 basis points in the yield of the 10-year government bond. Perhaps the most interesting result pertains to domestic equities, where the effect is 7 times stronger than for domestic bonds and the APP leads to 6.6% higher prices of the EZ equity portfolio.

Somewhat surprisingly, we don't find economically meaningful spillover effects in respect of foreign asset returns for our sample of ROW countries. While the impact of the APP program is statistically significant, price effects on ROW equities and bonds are negligible at +0.4% and +0.04% in total. When comparing these results with the empirical findings from the spillover literature, it turns out that we cannot confirm the notion that QE influenced foreign asset prices in an economically meaningful way, at least not in respect of portfolio balancing-induced price effects of the APP. In addition, domestic asset prices were impacted to a smaller degree than what was found by Motto et al. (2015); Breckenfelder et al. (2016); De Santis (2016) or what was found for US programs by Gagnon et al. (2011b); Stefania and King (2013); Neely (2015).

Do our findings contradict the conventional knowledge about the realtionship between QE and financial asset prices? There are several points to consider. First, the empirical spillover literature focuses on US-based QE (Aizenman et al. (2016); Mishra et al. (2014); Eichengreen and Gupta (2015); Chen et al. (2012b)) due to its relatively higher importance for the world economy. It is possible that portfolio balance effect from the US FED's LSAP programs were significantly higher than what we find for the APP, and thus, we need to calibrate our model to the US market to compare these findings. Second, event studies that don't specifically disentangle the underlying transmission channels may be measuring the signalling channel associated with QE programs in the US and not the portfolio balance channel. Last, but not least, there is the possibility that it is not the QE per se that is driving those empirical estimates of price effects on financial markets. Greenlaw et al. (2018) have argued that event studies determining large QE-induced effects suffer from an identification problem, i.e. it is unclear whether markets react to news on central bank asset purchases or news regarding underlying economic fundamentals. Thus, Greenlaw et al. (2018) conclude that the effect of the US programme itself on interest rates is likely smaller than what many event studies claim.

Overall, our findings seem to be in line with Greenlaw et al. (2018)'s skepticism, at least in the context of the ECB's APP. Our results lead us to believe that QE-induced asset price effects are not irrelevant, but only small in magnitude on the domestic market. Spillovers from portfolio balancing to the rest-of-the-world are negligible.

Our findings suggest that the effectiveness of QE to lower long-term interest rates by a substantial degree must be regarded with skepticism. While finding a clear relationship between asset purchases and rising prices for global bond and equity prices, the magnitude of the effect is rather small. From a domestic market perspective, it is not clear to us that policy makers should engage in massive asset purchase programs if the primary intention is to ease conditions through lowering long-term interest rates in a meaningful way. Apart from portfolio balancing, it is possible that QE works through changing expectations of market participants. However, as Eggertsson and Woodford (2003) point out, if the aim of monetary policy is to lower the expected path of short term rates, QE is not the first choice. An alternative measure is forward guidance, i.e. the credible and consistent communication about the future course of monetary policy. The effectiveness of forward guidance is not part of the analysis of this dissertation. However, it has been shown in many studies that explicit forward guidance has been effective at lowering various interest rates which is conducive to increasing economic activity and inflation (Campbell et al., 2012; Gavin et al., 2013; Swanson and Williams, 2014; Gertler and Karadi, 2015; Giannoni et al., 2015).

7 Annex

7.1 Maturity Parameter and Term Structure

Maturity of asset portfolios is modeled through a constant repayment rate $(1-m)$ on outstanding assets. In section 4.5, we describe how we calibrate the parameter m to match the maturity profile of Euro area bond portfolios. Figure 7 shows how the maturity structure we find in the data compares to our fitted portfolio with constant repayment rate. The plot shows the percentage of current portfolio still outstanding on any given date in the future. It becomes clear from Figure 7 that the portfolio within the model implicitly contains many securities with differing maturity dates. Hypothetically, a portfolio with maturity parameter $0 < m < 1$ comprises at least one security for each possible future maturity date. The expected return $r(m)$ on that portfolio is therefore a function of the yield curve $r(t)$ of underlying hypothetical securities:

$$r(m) = \sum_{t=1}^{\infty}(1-m)m^{t-1}r(t), \tag{7.1}$$

with $(1-m)m^{t-1}$ denoting the portfolio shares that mature on date t. Under certain conditions, $r(t)$ can be approximated by simulating the model with varying maturity parameters. Note, however, that the return $r(m)$ on a portfolio of average maturity T_m is generally not the same as the sport rate $r(t = T_m)$ of an individual security that matures at date T_m, with:

$$T_m = \sum_{t=1}^{\infty}(1-m)m^{t-1}t = \frac{1}{1-m}, \tag{7.2}$$

The ability to approximate $r(t)$ crucially depends on the assumption about its functional form. Specifically, $r(t)$ must lead to convergence of the infinite sum in Eq. 7.1. Conveniently, the most common parametric yield curve models used by central banks, i.e. the Nelson & Siegel model $r^{\text{N\&S}}(t)$ as well as its more flexible extension, the Svensson model $r^{\text{Sv}}(t)$, satisfy the convergence condition.

$$r^{\text{N\&S}}(t) = \beta_0 + \beta_1 \frac{1-\exp(\frac{-t}{\tau_1})}{\frac{t}{\tau_1}} + \beta_2 \left(\frac{1-\exp(\frac{-t}{\tau_1})}{\frac{t}{\tau_1}} - \exp(\frac{-t}{\tau_1}) \right) \tag{7.3}$$

$$r^{\text{Sv}}(t) = r^{\text{N\&S}}(t) + \beta_3 \left(\frac{1-\exp(\frac{-t}{\tau_2})}{\frac{t}{\tau_2}} - \exp(\frac{-t}{\tau_2}) \right) \tag{7.4}$$

With

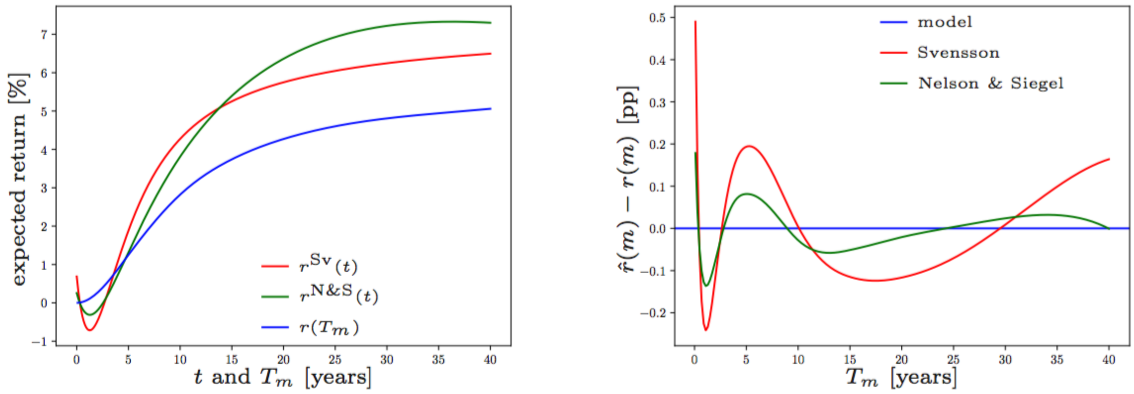
$$\begin{aligned} \hat{r}^{\text{N\&S}}(m) &= \sum_{t=1}^{\infty}(1-m)m^{t-1}r^{\text{N\&S}}(t) \\ &= \beta_0 + \frac{(\beta_1\tau_1 + \beta_2\tau_1)(m-1)(\log(1-m) - \log(1-\exp(\frac{-1}{\tau_1})m)}{m} + \frac{\beta_2(m-1)}{\exp(\frac{1}{\tau_1}) - m} \end{aligned} \tag{7.5}$$

$$\hat{r}^{\text{Sv}}(m) = \hat{r}^{\text{N\&S}}(m) + \frac{\beta_3\tau_2(m-1)(\log(1-m) - \log(1-\exp(\frac{-1}{\tau_2})m)}{m} + \frac{\beta_3(m-1)}{\exp(\frac{1}{\tau_2}) - m} \tag{7.6}$$

defining the estimate of a portfolio's return, we choose the parameters of the Nelson & Siegel and Svensson yield curve models so that they minimize the sum of squared differences to the portfolio returns obtained in simulations, i.e.

$$\{\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3, \hat{\tau}_1, \hat{\tau}_2\} = \underset{\{\beta_0,\beta_1,\beta_2,\beta_3,\tau_1,\tau_2\}}{\arg\min} \sum_m \left(r(m) - \hat{r}^{\{\text{N\&S,Sv}\}}(m) \right)^2 \tag{7.7}$$

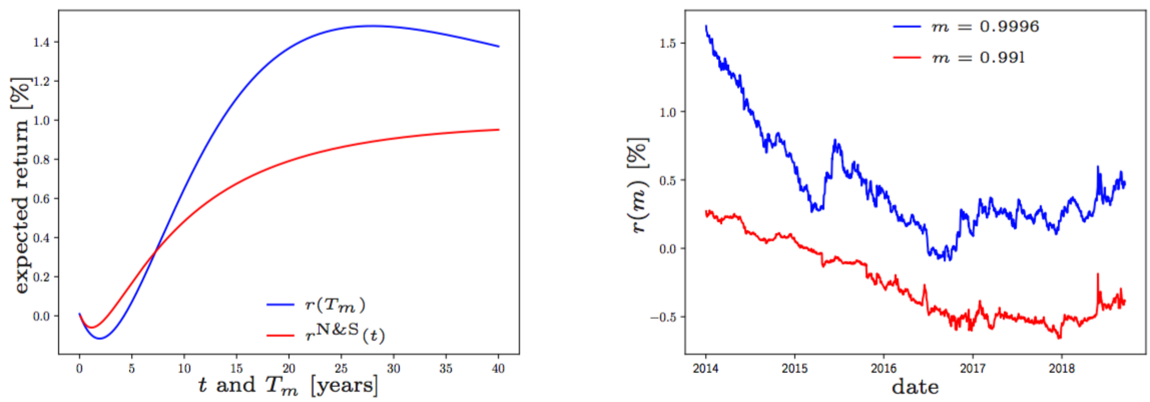
Figure 8(a) shows the yield curves $r(t)$ implied by exemplary simulated data $r(m)$ for the Nelson & Siegel and Svensson models. Unsurprisingly, the fit of the Svensson model, which has two additional parameters better fits the simulated data as displayed in Figure 8(b). Both models struggle most to fit the very short-end of $r(m)$. In particular, the inverted yield curve on the short-end seems to be an artifact of the curve fitting exercise. Such an inversion is typically interpreted as the market's expectation of an upcoming reduction in central bank interest rates, which, for the sake of simplicity, is not considered by agents in our model.



(a) portfolio vs. individual security yield curve (b) goodness of fit

Figure 8: Relation between the yield curve of securities portfolios with characterized by an maturity T_m and the yield curve of hypothetical individual securities with remaining maturity t.

Equations (7.5) and (7.6) can also be used to compute the yield curve $r(m)$ implied by yield curves fitted to real bond data. Figure 7.1 e.g. plots the Svensson model representation of the yield curve estimated with euro area sovereign bonds, which is published on a daily basis by the European Central Bank. On the basis of these transformed real data yield curves, model portfolio return dynamics can be reconstructed for calibration and validation purposes. Figure 9(b) plots the time series of two hypothetical portfolios with different maturity parameters that are reconstructed from ECB yield curve data.



(a) euro area yield curves (Dec. 2014) (b) portfolio returns

Figure 9: Transforming euro area sovereign bond yield curve into model yield curves and model portfolio return time series.

7.2 Updating Balance Sheets

Excess demand and supply of portfolio shares need to be taken into account when updating balance sheets. We define $\mathcal{Q}_+ := \{d, f, CB | \Delta Q_{\{d,f,CB\},t} \geq 0\}$ and $\mathcal{Q}_- := \{d, f, CB, U | \Delta Q_{\{d,f,CB,U\},t} < 0\}$ as the sets identifying domestic, foreign, central bank and the underwriter agents' demand and supply of portfolio shares, respectively. The factors correcting for excess demand π_t and excess supply of a portfolio of assets ν_t proportionately distribute the mismatch across agents:

$$\pi_t = 1 - \frac{\Delta Q_t}{\sum_{j \in \mathcal{Q}_+} \Delta Q_{j,t}} \quad \text{and} \quad \nu_t = 1 - \frac{\Delta Q_t}{\sum_{j \in \mathcal{Q}_-} \Delta Q_{j,t}} \tag{7.8}$$

with ΔQ_t being the aggregate excess demand for a portfolio of assets. The factors in Eq. (7.8) imply that if there is, for example, an excess demand of 10%, all agents that want to buy those portfolio shares will only be able to acquire 90% of their original demand. With the correction factors we compute the new quantities of portfolio shares agents hold on their balance sheet at the end of period t:

$$Q_{d,t} = \begin{cases} out_t Q_{d,t-1} + \Delta Q_{d,t}\pi_t & \text{if} \quad \Delta Q_{d,t} \geq 0 \text{ and } \Delta Q_t \geq 0 \\ out_t Q_{d,t-1} + \Delta Q_{d,t}\nu_t & \text{if} \quad \Delta Q_{d,t} < 0 \text{ and } \Delta Q_t < 0 \\ out_t Q_{d,t-1} + \Delta Q_{d,t} & \text{else} \end{cases} \tag{7.9}$$

The updates of central bank agent's inventory follows the logic of investor agents' new balance sheet positions, i.e.

$$Q_{CB,t} = \begin{cases} out_t Q_{CB,t-1} + \Delta Q_{CB,t}\pi_t & \text{if} \quad \Delta Q_{CB,t} \geq 0 \text{ and } \Delta Q_t \geq 0 \\ out_t Q_{CB,t-1} + \Delta Q_{CB,t}\nu_t & \text{if} \quad \Delta Q_{CB,t} < 0 \text{ and } \Delta Q_t < 0 \\ out_t Q_{CB,t-1} + \Delta Q_{CB,t} & \text{else} \end{cases} \tag{7.10}$$

while the underwriter agent, which always tries to sell it's entire inventory, does not need to consider the case of positive excess demand:

$$Q_{U,t} = \begin{cases} \Delta Q_{U,t}(\nu_t - 1) & \text{if} \quad \Delta Q_t < 0 \\ 0 & \text{else.} \end{cases} \tag{7.11}$$

Transactions in portfolio shares can lead to changes in the inventory of domestic and foreign currency holdings of investor agents, which in turn impact their demand for currency. We define

$$\Delta \tilde{C}^{\mathcal{D}}_{d,t} = \Delta C^{\mathcal{D}}_{d,t} - \sum_{D=1}^{n^D} \underbrace{(out^D_t Q^D_{d,t-1} - Q^D_{d,t})P^D_t}_{\text{asset transactions}} \quad \text{and} \tag{7.12}$$

$$\Delta \tilde{C}^{\mathcal{F}}_{d,t} = \Delta C^{\mathcal{F}}_{d,t} - \sum_{F=1}^{n^F} (out^F_t Q^F_{d,t-1} - Q^F_{d,t})P^F_t \tag{7.13}$$

as the updated respective domestic and foreign demand for cash after assets have been traded, and $\tilde{C}^{\mathcal{D}}_{d,t} = w^{C^{\mathcal{D}}}_{d,t} S_{d,t} - \Delta \tilde{C}^{\mathcal{D}}_{d,t}$ and $\tilde{C}^{\mathcal{F}}_{d,t} = w^{C^{\mathcal{F}}}_{d,t} S_{d,t}/X^{\mathcal{DF}}_t - \Delta \tilde{C}^{\mathcal{F}}_{d,t}$ as the updated respective domestic and foreign cash inventories. Cash demand needs to be updated a second time as it does not take into account the fact that an agent can only buy currency if it is able to supply an equally valued amount of a different currency. More generally, an agent will only engage in a foreign exchange transaction if its demands for the domestic and foreign currency are of opposite signs (i.e. one currency is demanded the other supplied); and the volume of the aspired cash transaction is limited by what an agent demands or supplies of the respective other currency. Taking this into account, the final demand for a currency at the end of period t amounts to:

$$\Delta \tilde{\tilde{C}}^{\mathcal{D}}_{d,t} = \begin{cases} \text{sgn}(\Delta \tilde{C}^{\mathcal{D}}_{d,t}) \cdot \min\left\{|\Delta \tilde{C}^{\mathcal{D}}_{d,t}|, |X^{\mathcal{DF}}_t \Delta \tilde{C}^{\mathcal{F}}_{d,t}|\right\} & \text{if} \quad \text{sgn}(\Delta \tilde{C}^{\mathcal{D}}_{d,t}) \neq \text{sgn}(\Delta \tilde{C}^{\mathcal{F}}_{d,t}) \\ 0 & \text{else} \end{cases} \tag{7.14}$$

$$\Delta \tilde{\tilde{C}}^{\mathcal{F}}_{d,t} = \begin{cases} \text{sgn}(\Delta \tilde{C}^{\mathcal{F}}_{d,t}) \cdot \min\left\{|\Delta \tilde{C}^{\mathcal{F}}_{d,t}|, |X^{\mathcal{FD}}_t \Delta \tilde{C}^{\mathcal{D}}_{d,t}|\right\} & \text{if} \quad \text{sgn}(\Delta \tilde{C}^{\mathcal{D}}_{d,t}) \neq \text{sgn}(\Delta \tilde{C}^{\mathcal{F}}_{d,t}) \\ 0 & \text{else} \end{cases}, \tag{7.15}$$

with sgn(·) denoting the sign or signum function, which extracts the sign of the respective updated cash demands.

Analogous to Eq. (7.8) for transactions in portfolio share, we compute correcting factors for cash transactions, i.e.

$$\pi^C_t = 1 - \frac{\Delta \tilde{\tilde{C}}_t}{\sum_{j \in \mathcal{C}_+} \Delta \tilde{\tilde{C}}_{j,t}} \quad \text{and} \quad \nu^C_t = 1 - \frac{\Delta \tilde{\tilde{C}}_t}{\sum_{j \in \mathcal{C}_-} \Delta \tilde{\tilde{C}}_{j,t}}, \tag{7.16}$$

with $\mathcal{C}_+ := \{d, f | \Delta \tilde{\tilde{C}}_{\{d,f\},t} \geq 0\}$ and $\mathcal{C}_- := \{d, f | \Delta \tilde{\tilde{C}}_{\{d,f\},t} < 0\}$ defining the sets that identify the agents demanding and supplying cash. Excess demand for domestic and foreign currency is computed as $\Delta\tilde{\tilde{C}}_t^{\mathcal{D}} = \sum_{d=1}^{n^d} \Delta\tilde{\tilde{C}}_{d,t}^{\mathcal{D}} + \sum_{f=1}^{n^f} \Delta\tilde{\tilde{C}}_{f,t}^{\mathcal{D}}$ and $\Delta\tilde{\tilde{C}}_t^{\mathcal{F}} = \sum_{d=1}^{n^d} \Delta\tilde{\tilde{C}}_{d,t}^{\mathcal{F}} + \sum_{f=1}^{n^f} \Delta\tilde{\tilde{C}}_{f,t}^{\mathcal{F}}$, respectively. Positions of domestic and foreign cash on balance sheet are computed analogously to Eq. (7.9), i.e.:

$$C_{d,t} = \begin{cases} \tilde{C}_{d,t} + \Delta\tilde{\tilde{C}}_{d,t}\pi_t^C & \text{if} \quad \Delta\tilde{\tilde{C}}_{d,t} \geq 0 \text{ and } \Delta\tilde{\tilde{C}}_t \geq 0 \\ \tilde{C}_{d,t} + \Delta\tilde{\tilde{C}}_{d,t}\nu_t^C & \text{if} \quad \Delta\tilde{\tilde{C}}_{d,t} < 0 \text{ and } \Delta\tilde{\tilde{C}}_t < 0 \\ \tilde{C}_{d,t} + \Delta\tilde{\tilde{C}}_{d,t} & \text{else} \end{cases} \tag{7.17}$$

7.3 Calibration tables

	Debt securities outstanding (BIS) Tn EUR	Stock market Capitalisation (World Bank) Tn EUR	Total Market Capitalisation Tn EUR	
Euro area	16.62	5.71	22.33	
	Tn EUR	Tn EUR	Tn EUR	Weight in ROW Sample
Total ROW	56.84	43.14	99.98	1.00
United States	29.96	21.94	51.91	0.519
United Kingdom	5.14	2.88	8.02	0.080
Australia	1.22	1.07	2.30	0.023
Brazil	1.65	0.70	2.35	0.024
Canada	1.35	1.75	3.10	0.031
Switzerland	0.27	1.25	1.52	0.015
China	4.74	5.00	9.75	0.097
Colombia	0.07	0.12	0.19	0.002
Hungary	0.06	0.01	0.07	0.001
Indonesia	0.11	0.35	0.47	0.005
India	0.56	1.30	1.86	0.019
Israel	0.16	0.17	0.33	0.003
Japan	9.12	3.65	12.77	0.128
Korea	1.20	1.01	2.21	0.022
Mexico	0.50	0.40	0.90	0.009
Norway	0.19	0.18	0.37	0.004
New Zealand	0.05	0.06	0.11	0.001
Russia	0.15	0.32	0.47	0.005
Turkey	0.16	0.18	0.35	0.003
South Africa	0.17	0.78	0.95	0.009

Table 12: Market capitalisation weights used for computation of Rest-of-the-World inflation, interest rates and expected returns

	Data Source	Data Description
A	ECB Investment fund balance sheet statistics	Eurozone funds' holdings in debt securities and equities, by counterparty region.
B	ECB Consolidated banking statistics CBD2 and Securities holding statistics SHS	Debt securities holdings of banks resident in the Eurozone area from the ECB's CBD2. Home bond bias calculated from SHS.
C	Inferred from Bankscope	Bankscope data on Rest-of-the-world banking sector securities, i.e. available-for-sale and trading securities, shows a 2.47 larger investment portfolio than for Eurozone banks.
D	FSB Global Shadow Banking Monitoring Report 2015	Global shadow banking sector without Money Market Funds, Reals estate funds and Eurozone funds
I	IMF Coordinated Portfolio Investment Survey (CPIS)	Cross-holdings of equities and debt securities in the global banking sector, i.e. 'Deposit-taking institutions except Central banks'
M^1	Morning Star Direct	Calculated by multiplying total Rest-of-the-world assets (see D above) with regional weight for Eurozone debt securities compiled from Morning Star Direct. Morning Star asset class and regional weights are based on a sample of open-end investment funds (see Table 14).
M^2	Morning Star Direct	Calculated by using weights in Eurozone equity securities in Rest-of-the-world sample compiled from Morning Star and total equity assets computed in M^4
M^3	Morning Star Direct	Computed by using debt securities weight in Rest-of-the-world sample compiled from Morning Star
M^4	Morning Star Direct	Computed by multiplying total Rest-of-the-world assets (see D) with equity securities weight compiled from Morning Star Direct
M^5	Morning Star Direct	Eurozone and Rest-of-the-World funds' cash positions are calculated by cash securities weights compiled from Morning Star Direct (see also Table 14)
R	Residual	Computed from remaining category values
RES	ECB Statistical Warehouse, Federal Reserve Bank of St. Louis FRED database	Central Bank Reserve assets (ECB, Bank of England and US Fed)
T	ECB Investment fund Statistics and Morning Star Direct	We take the sum of equities and debt securities holdings by Euro area investment funds statistics

Table 13: Data Sources for Funds and Banks' Balance Sheet Positions used in Table 18

	Sample definition	Portfolio Shares
Domestic Market Morning Star Sample	Euro area, 15374 open-end investment funds domiciled in the Euro area; Rescaling total assets to only include cash, equities and bonds Eurozone countries: Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Portugal, Slovenia, Slovakia, Spain	- Equity share: 0.44 - Bond share: 0.47 - Cash share: 0.09 - Out of all cash assets, 65.3% are domestic cash and 34.4% are foreign cash
Foreign Market Morning Star Sample	Rest-of-the-world area, 25930 open-end investment funds, Countries: Argentina, Australia, Bermuda, Botswana, Brazil, Canada, Chile, China, Colombia, Czech Republic, Denmark,Hong Kong, Hungary, India, Indonesia, Israel, Japan, Kuwait, Lebanon, Liechtenstein, Malaysia, Mexico, Monaco, Namibia, New Zealand, Norway, Oman, Pakistan, Peru, Philippines, Puerto Rico, Poland, Russia, Singapore, South Africa, South Korea, Sweden, Switzerland, Taiwan, Thailand, Turkey, United Kingdom, United States, Venezuela, Vietnam. Rescaling total assets to only include cash, equities and bonds	- Equity share: 0.51 - Bond share: 0.41 - Cash share: 0.08 - Out of all bond assets, 4% are invested in Euro area bonds - Out of all equity assets, 6% are invested in Euro area equities - Out of all cash assets, 5% are Euros

Table 14: Sample definition and portfolio shares of Euro area and Rest-of-the-world open-end investment funds collected from Morning Star. The portfolio shares are used in the calibration of the global balance sheet positions of funds.

Country	Inflation	Deposit rate	Market cap weight	Weighted deposit rate	Weighted inflation
United States	1.612	0.250	0.519	0.1298	0.837
United Kingdom	1.461	0	0.080	0	0.117
Japan	2.760	0.415	0.128	0.0530	0.352
China	1.988	0.350	0.097	0.0341	0.194
Brazil	6.329	10.024	0.024	0.2359	0.149
Canada	1.920	0.550	0.031	0.0170	0.059
Australia	2.513	2.904	0.023	0.0667	0.058
Colombia	2.905	4.089	0.002	0.0079	0.006
Hungary	-0.197	1.777	0.001	0.0012	0.000
India	5.800	5.500	0.019	0.1024	0.108
Indonesia	6.395	6.750	0.005	0.0314	0.030
Israel	0.476	0.798	0.003	0.0026	0.002
Korea	1.275	2.536	0.022	0.0560	0.028
Mexico	4.022	0.840	0.009	0.0076	0.036
New Zealand	1.220	3.250	0.001	0.0037	0.001
Norway	2.042	0.490	0.004	0.0018	0.008
Russia	7.824	6.042	0.005	0.0283	0.037
South Africa	6.090	4.801	0.009	0.0454	0.058
Switzerland	-0.012	0.020	0.015	0.0003	0.000
Turkey	8.855	7.500	0.003	0.0261	0.031
SUM			1.000	0.85	2.1

Table 15: Rest-of-the-world inflation and deposit rates. Deposit rates are sourced from the World bank development indicators, Fred database of the Federal Reserve of St. Louis, Bank of Japan, Bank of England, Norge Bank and Hungarian Central bank for December 2014. Inflation is the inflation forecast by the OECD for 2015 at the end of 2014.

	Equity premium	Equity return	Risk-free rate	Equity Market Weight	Market cap Tn USD
World	4.50	7.01	2.51	1.00	57.50
EZ	5.73	6.27	0.54	0.12	7.00
Rest-of-the-world	4.33	7.11	2.78	0.88	50.50
US			2.17		26.33
Japan			0.33		4.38
UK			1.76		3.46
Switzerland			3.66		1.50
South Africa			7.96		0.93
Mexico			6.01		0.48
Poland			2.54		0.17
Hungary			3.54		0.01
Russia			14.09		0.39
Brazil			12.43		0.84
India			7.86		1.56
Hong Kong			1.92		3.23
South Korea			2.61		1.21
China			3.66		6.00

Table 16: Equity premia and risk free rate of the Eurozone and rest-of-the-world. The Eurozone equity premium is sourced from Absolute Strategy Research, as of 31 Dec 2014. The risk-free rate is the 10-year government bond for geographical regions and provided by Datastream. Equity market capitalisation is taken from the World Bank Development indicators and World Federation of Exchanges database in 2014 US dollars.

Simulation Parameters

Agents Parameters			
Variable	*Description*	*Value*	*Source*
$\boldsymbol{\lambda}_d$	Domestic and foreign investors' risk aversion (preferred habitat) preference for domestic & foreign assets	Table 4	Calibration outcome
Asset Parameters			
Π^{C^D}	Euro area: Interest on cash,	-0.2 %	ECB Statistical Warehouse, Deposit facility Dec 2014
C^D	Domestic currency quantity	690	Table 18
Π^{C^F}	Rest-of-the-World: Interest on cash,	0.85%	Deposit interest rate, World Bank, World Development indicators; & Central banks'statistics Market-cap weighted average with weights from Table 14 and refTab:BS
C^F	Foreign currency quantity	3836	Row central banks' balance sheet (Table 18)
m_{bond}	Maturity parameter, Bond portfolio	0.99936	Morning Star Direct, calibrated to the maturity profile of open-end investment funds (Table 7)
m_{eq}	Maturity parameter, Equity portfolio	1	Equities never mature
Q^D, V^D, ρ	Euro area bond portfolio Quantity, Face Value Nominal interest	5996, 0.005%	Table 18 Calibration outcome
Q^D, V^D, ρ	Euro area equity portfolio Quantity, Face Value Nominal interet	2135, 0.03%	Table 18 Calibration outcome
Q^F, V^F, ρ	Row area bond portfolio Quantity, Face Value Nominal interest	19361 0.00943%	Table 18 Calibration outcome
Q^F, V^F, ρ	Row equity portfolio Quantity, Face Value Nominal interest	12049 0.0361%	Table 18 Calibration outcome
η	Foreign exchange reversion rate to purchasing power parity	15% annually	Rogoff (1996)
δ	Convergence parameter indicative of market clearing	1%	
ϕ	Memory parameter updating of the covariance matrix	0.001	

Table 17: Simulation parameters

Trillion EUR		Debt Securities (DS)			Equities (Eq)			Currencies (C)			Computed Total Assets
		EZ DS	ROW DS	sum DS	EZ Eq	ROW Eq	sum Eq.	EZ C	ROW C	sum C	
Eurozone Investors	Funds	1.961	1.651	3.612	0.930	1.821	2.751	0.411	0.218	0.629	6.992
	Banks	3.204	0.656	3.860	0.571	0.075	0.646	0.08	0.016	0.096	4.602
ROW Investors	Funds	0.263	7.045	7.308	0.556	8.461	9.017	0.063	1.202	1.265	17.59
	Banks	0.568	10.009	10.577	0.078	1.692	1.770	0.136	2.4	2.536	14.883
Computed Market Size		5.996	19.361	25.357	2.135	12.049	14.184	0.69	3.836	4.526	44.067

		Debt Securities (DS)			Equities (Eq)			Currencies (C)			Computed Total Assets
		EZ DS	ROW DS	sum DS	EZ Eq	ROW Eq	sum Eq.	EZ C	ROW C	sum C	
Eurozone Investors	Funds	A	R	A	A	R	A	M^5	M^5	M^5	T
	Banks	B	R	B	R	I	B	RES	RES	RES	$\sum$
ROW Investors	Funds	M^1	R	M^2	M^3	R	M^4	M^5	M^5	M^5	D
	Banks	I	R	C	I	R	C	RES	RES	RES	$\sum$
Computed Market Size		$\sum$	$\sum$	$\sum$	$\sum$	$\sum$	$\sum$	$\sum$	$\sum$	$\sum$	$\sum$

Table 18: *Upper Table*: Euro area and Rest-of-the-World balance Sheet positions in Trillion EUR. *Lower Table*: Data sources used in the compilation of balance sheet positions. Symbols are explained in Table 13.

Region	International - Ex US		Eurozone		US	
Index name	S&P International Corporate Bond Index	S&P International Sovereign Ex-U.S. Bond Index	S&P Eurozone Investment Grade Corporate Bond Index	S&P Eurozone Developed Sovereign Bond Index	S&P U.S. Treasury Bond Index	S&P 500 Investment Grade Corporate Bond Index
Description	ex-US	ex-US developed world	Investment grade bonds	Euro area sovereign bonds	US treasury bonds	US investment grade corporate bonds
Date	2014/12/29	2014/12/29	2014/12/29	2014/12/29	2014/12/29	2014/12/29
Yield to maturity	2.01	0.87	1.05	0.81	1.21	2.91
	Weighted average 50:50 1.44		Weighted average 50:50 0.93		Weighted average 50:50 2.06	
Relative Market Cap	0.59		0.23		0.41	
Weight Row	0.77					
World Yield	1.69					
Row yield	1.92					
Euro area	0.93					

Table 19: Computation of domestic and foreign market yield to maturity used in calibration of expected bond returns. Eurozone: We compute a simple average yield to maturity by using the S&P Eurozone Investment Grade Corporate Bond Index and the S&P Eurozone Developed Sovereign Bond Index as of 29 December 2014. Rest-of-the-world: We approximate the rest-of-the-world yield to maturity by combining the yield to maturity of i) a bond index excluding the US with ii) a bond index with the US to form a world bond index and iii) adjust this by the Euro area value. i) The row-ex US yield to maturity is an equally weighted average of the S&P International Corporate Bond Index and the S&P International Sovereign Ex-US Bond Index as of 29 December 2014, of 1.44%. ii) The US yield to maturity is the equally weighted average of the S&P US Treasury Bond Index and the S&P 500 Investment Grade Corporate Bond Index, of 2.06%. The combined world yield to maturity is the sum of i) and ii), weighted by their relative market size (59% for the rest-of-the-world-ex-US and 41% for the US), i.e. $0.59 * 1.44 + 0.41 * 2.06 = 1.69$. Lastly, we adjust the yield to maturity of our computed world bond index by the Eurozone value, which leads to $(1.69\% - 0.93\% * 0.23)/0.77 = 1.92\%$ for approximated yield to maturity of the foreign market in our simulation. Source: S&P Bond Indices.

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