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Hedge Fund Risk Appetite:
a prospect theory perspective

By

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Declaration

I, the undersigned, hereby declare that the work contained in this assignment is my original work and that I have not previously in its entirety or in part submitted it any university for a degree.

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1. Introduction

The international hedge fund industry has witnessed tremendous growth in the past decade, and South Africa has been no exception. Managing just over a billion Rand in 2002, the industry has developed into a prominent investment class with over 31 billion Rand under management in 2011 (Novare, 2011). The industry's prominence is signified by the recent amendment of article 28 in the South African Pension Funds Act, allowing pension funds to invest up to 10% of their assets in hedge funds, a sharp increase on the previously permitted 2.5% (Riscura, 2011). These developments have come at a time when hedge fund investors internationally have grown increasingly wary of the investment class (E&Y, 2011). Forthcoming US regulation, set to come into effect in July of 2012, will further restrict the ability of banks to sponsor and invest in hedge funds, barring a large amount of the industry's source of funding and credit. The general rhetoric offered in this legislation is that hedge funds pose a systemic threat by undertaking high risk leveraged investments that fall outside the regulator's reach. Whether this view is warranted or not, hedge funds remain a prominent investment product that demands a proper understanding from the investor, creditor and regulator's perspective.

The purpose of this paper is to examine the incentives and risk attitudes prevalent in the hedge fund industry. The paper firstly considers the implications that fee structures hold for a fund manager's risk preferences under a prospect theory framework. Under prospect theory agents are thought to be loss averse in relation to a reference point and risk seeking when underperforming against their reference point. This arguably resembles the incentive structure created by hedge fund fees, as the majority of South African hedge funds rely on a performance fee as their key source of compensation (Novare, 2011). The performance fee is calculated on the return achieved in excess to a stated benchmark. The benchmark, or hurdle rate, thus serves as the pivotal reference point in fund manager's decision making and will form the starting point of this analysis.

The paper secondly proposes the inclusion of a social benchmark when evaluating a fund manager's decision making. The hedge fund industry is highly competitive and investors have the ability to withdraw funds from underperforming managers and allocate it to their rivals. This implies that hedge funds do not solely use the hurdle rate as their reference point, but also value their relative standing against other fund managers. If this hypothesis holds true, prospect theory predicts that trailing managers could increase the risk of their portfolio in fear of underperforming against their peers.

To find a useful conclusion, the paper is divided into five parts. Section one presents a discussion on prospect theory to familiarize the reader with its basic tenets. Section two discusses the key incentives in a hedge fund manager's decision making. Section three turns to the theoretical predictions of this analysis. Section four addresses the recent developments in the South African hedge fund industry. Section five concludes.

2. Prospect Theory

Recent years have revealed a rapid development in the field of behavioural finance. The discipline's insistence on viewing the investor as *real*, and not merely rational, has found a wide audience with both academics and practitioners. The increase in academic interest is clearly evident in the tremendous growth of behavioural finance literature and dedicated journals (Prast, 2004). However, the jury is still out on whether behavioural finance will supplant the mainstream approach or be able to position itself as a serious contending discipline (Stracca, 2003).

Shefrin (2008) narrows the list of differences between the conventional neoclassical approach and behavioural finance to two key features. The first is the treatment of biases and heuristics in decision making, often loosely termed as sentiment. Whereas neoclassical convention assumes that investors are largely free from bias and act rationally as a collective, behavioural finance endears itself to the so-called *irrational* aspects of investing. In rational agent models sentiment caries minor significance, if any, since rational agents will eliminate sentiment by learning and arbitrage processes (Blanchard, 1982). Conversely, behavioural finance views sentiment as a key feature in asset pricing and investor decision making, as a result of biases in investors' decision making processes.

The second departure regards preferences and utility theory. Expected utility theory has long served as the favoured workhorse in economic problem solving under uncertainty. Despite its favoured status, it is broadly recognised that it fails to explain the observed behaviour of asset prices and many other financial phenomena (Hung & Wang, 2005). Proponents of behavioural finance are critical of the neoclassical approach to preferences and often opt for theories stemming from experimental research, the most notable example being prospect theory (Kahneman and Tversky, 1979). Behavioural finance theorists point out the empirical irregularities of expected utility theory and its inability to serve as a descriptive theory of decision making under risk. Being a normative theory, it is often regarded as unrealistic and overly simplistic, both shortcomings that prospect theory tries to overcome.

2.1 Theory

Prospect theory is based on several observations repeatedly found in the economic experiments of Kahneman and Tversky (1979). In these studies they show that decision makers systematically violate the core axioms of expected utility theory. The chief underpinning of prospect theory is set forth in the value function and the decision weight function of prospect theory. The value function departs from standard expected utility theory in three distinct manners. Firstly, the value function is reference dependent; consequently gains and losses are defined relative to a reference point. Agents base their decisions on the change of wealth relative to the reference point and not on total wealth. Secondly, the value function is asymmetric in gains and losses, as agents are thought to be loss averse in relation to the reference point. Kahneman and Tversky's study found that decision makers display greater dissatisfaction with losses proportional to the satisfaction they receive from gains. Their results suggest a loss aversion coefficient of 2.25. This entails that people require R2.25 of potential winnings to justify the risk of losing one Rand. Finally, the function displays decreasing marginal returns in value, as both gains and losses increase in size. The agent is therefore risk averse when his current value equals or surpasses his reference point, but when he falls below it he becomes risk seeking. Risk aversion is generally assumed in all neoclassical analysis. Yet, there is considerable evidence that agents do not always display such behaviour. Friedman and Savage (1948) pointed out that the purchase of lottery tickets is one such example. They further stated that the simultaneous purchase of insurance and lottery tickets is a violation von Neumann-Morgenstern utility theory, as agents display both risk aversion and risk seeking behaviour in their decision making. The standard value function is defined as follows:

(1) – *if*

Where ($\lambda>1$) indicates loss aversion, and ($0 < \alpha \leq 1$), ($0 < \beta \leq 1$) indicates diminishing marginal sensitivity. The concept of diminishing marginal sensitivity infers that agents are less influenced by losses and gains as they move further away from the reference point. Empirical experiments[1] have revealed that the parameters generally adhere to these restrictions. In this representation of the value function w refers to wealth and the reference point is defined as r. While prospect theory does not explicitly specify what the reference point might be in practice. It is generally defined as the current state of order or the status quo. This definition is left wanting in a practical application, as the issue of determining and updating a reference point remains one of the major unsettled issues of the theory (Campbell, 2000). Kahneman (2000) also recognizes the possibility of multiple reference points in the decision making process, which could complicate analysis considerably.

Expected utility theory states that agents weigh possible outcomes on the probability of an outcome occurring. Correspondingly prospect theory agents select the prospect that maximizes value given the probability of it occurring, but it departs from expected utility theory by further assigning a decision weight to each probable event. Thus, the decision making process is subject to the additional probability of selecting a probability. In Kahneman and Tversky's original version of prospect theory, proposed in 1979, the decision weight method was implemented by performing a monotonic transformation to all the outcome probabilities individually. This method had two shortcomings. It did not guarantee stochastic dominance and it did not fare well in situations with a large number of outcomes, as it was developed in experimental settings which made use of binomial gambles. In 1992 Kahneman and Tversky aimed to rectify these shortcomings by offering an amended version of their original theory, called cumulative prospect theory. In this version, they transform the cumulative distribution function in its entirety, instead of transforming each probability separately. The cumulative probability weighting function also differentiates between losses and gains, by assigning separate function to each state:

$$(2) \qquad \pi_i = \delta^+(p_i + \ldots + p_n) - \delta^+(p_{i+1} + \ldots + p_n) \qquad if \quad 0 \leq i \leq n$$

$$\pi_i = \delta^-(p_{-m} + \ldots + p_i) - \delta^-(p_{-m} + \ldots + p_{i-1}) \qquad if \quad -m \leq i \leq 0$$

[1] See Kahneman and Tversky (1979), See Kahneman and Tversky (1992) & Grinblatt & Han (2004)

Where δ^+ and δ^- are the probability weighting functions for gains and losses respectively. The probability weighting functions distort the objective probability by the following function:

(3) $\delta^+ = \overline{\qquad\qquad} \quad 0 < \theta \leq 1$

(4) $\delta^- = \overline{\qquad\qquad} \quad 0 < \gamma \leq 1$

Many practical applications assume the parameters *(γ , θ)* to be equal (Han & Hsu, 2004). The decision weight function transforms objective probabilities by undervaluing very low probabilities and overvaluing probabilities near certainty. The function is thus convex near certainty and concave near impossibility; i.e. an increase in the objective probability at the 5% level is less influential in the decision making process than at the 90% mark. The shape of this function is justified from experimental studies done by Kahneman and Tversky, but it also coincides with the findings of the renowned Allais paradox (Allais, 1953). When the decision weight function and the value function are combined agents select the prospect that maximizes:

(5)

This delivers a fourfold attitude to risk. Agents are risk seeking when facing a small probability of gains and a large probability of losses. However, agents are risk averse when facing a small probability of losses and a large probability of gains. This pattern is admittedly an empirical generalization concerning choice under risk (Kahneman & Tversky, 2000), yet, it has been repeatedly substantiated in experimental research (See Cohen, et al (1987) and Wehrung (1987)). It offers a solution to the lottery ticket and insurance puzzle that Friedman and Savage (1948) were concerned about, while also offering a possible explanation for other mundane observations such as why people do not wear seat belts while driving, but fear flying (Sunstein, 2002). The probability of dying in a motor vehicle accident far surpasses the probability of dying in a plane accident, but people do not seem to differentiate between the probabilities.

Thaler (1985) explains such behaviour by introducing the concept of mental accounting. According to this hypothesis decision makers separate gambles into different accounts and solve them individually. Correspondingly, prospect theory maintains there are two stages when formulating decisions: An editing stage and an evaluation stage. At first agents edit and segregate prospects by a set of heuristics. Thereupon they evaluate the prospect independently by using the decision weight and the value function weight simultaneously (Grinblatt & Hann, 2004). Kahneman and Tversky justify the assumption of the editing stage on their experimental findings on which they postulated the framing, focusing and anchoring effect theories. The anchoring and focussing effect are similar in the fact that they both assert that individuals accentuate one aspect of a prospect and anchor expectations accordingly, leaving them unable to rationally comprehend the full consequence of their decision (Kahneman, 1992). In addition, the framing effect refers to experimental evidence where agents displayed the tendency to revise their decisions when the same problems are presented to them differently in some superficial manner (Kahneman and Tversky, 1986). This is most notable when a problem is framed in terms of losses instead of gains, an aspect that the value function incorporates by the inclusion of the loss aversion coefficient.

The notion that decision making is influenced by positive and negative framing is one of the most robust findings of prospect theory experiments. It supports the hypothesis that investors opt only to see the positive aspects of a gamble. Psychologists refer to this tendency as the positivity bias (Han & Hsu, 2004). It is also related to the psychological concept of cognitive dissonance, in which decision makers are unwilling to incorporate information that causes an element of mental discomfort. Festinger (1957) maintains that this sieving process is particularly relevant when information is in contrast with previously held beliefs. The mind's unwillingness to face regret points to an information filtering bias in the decision making process[2]. Regret can further be such a commanding negative emotion that the future anticipation of regret could also lead to suboptimal decisions (Michenaud & Solnik, 2008). This is prevalent in financial markets, where investors tend to sell high earning stocks too early and hold on to bad performing stocks too long, known as the disposition effect (Shefrin and Statman, 1985). Prospect theory offers an intuitive decision explanation for the disposition effect. When the share price drops, the investor is below his[3] reference point, the acquisition price. He therefore becomes risk seeking and willing to hold a poor stock. However, when the stock performs well, he is in the risk averse territory of the value function and opts to realize his gain earlier, even though he is holding a performing stock. Prospect theory has been successfully employed in various other economic problems, for instance tax evasion (Dhami, 2007) and the labour choice (Camerer, 1997). The paper now turns

[2] See Loomes, G. & Sugden, R. (1982) for a direct theory on regret and decision making

[3] Throughout the paper the author will refer to investors and fund managers in the masculine. This is done only to avoid clumsy alternative forms of speech and carries no other connotations.

to previous applications of prospect theory that hold specific relevance to the topic of risk management in financial markets and performance fee arrangements.

2.2 Previous Applications

The Equity Premium

Under standard neoclassical assumptions (see Lucas, 1978) the persistent difference between the returns on equity and the risk free rate implies an impossibly high level of risk aversion. The seminal work of Mehra & Prescott (1985) demonstrates that the observed equity premium in the US stock market implies a risk aversion coefficient of over thirty. That is to say, US investors are so risk averse, that they would prefer a sure $520 to an uncertain coin toss, where if they win they could receive a $1000, and even if they lose they still receive $500. In proportion to the possible gains and losses, the certainty equivalent of $520 seems highly unrealistic. Benartzi and Thaler (1995) show that a utility function which includes the loss aversion and narrow framing postulates of prospect theory is well suited to address this problem. When an investor has a short investment frame, in addition to his loss aversion, the stock market fluctuations seem very risky. He consequently requires a larger equity premium to compensate him for the risk. Their model predicts an equity premium in a two period setting, which is consistent with the observed behaviour of stock returns. Barberis et al (2001) also partially employ the value function of prospect theory with reference to changes in financial wealth, whilst keeping to the customary neoclassical definition of utility over consumption. Their model offers a similar conclusion to the 1995 Benartzi and Thaler paper, while having the added benefit of explaining the observed volatility of aggregate stock market fluctuations. Their use of a dynamic framework allows the risk attitude component to fluctuate over time, solving the excessive fluctuation puzzle. Hung & Wang (2005) define a value function that regards changes in consumption, which also delivers promising results as an asset pricing model. The key insight of these models, in relation to this study, lies in the change in risk attitudes and the risk premium. This paper will also show that the risk premium of an agent, in this case a hedge fund manager, also changes as his reference point changes.

Firm Capital Investment

While prospect theory concerns individual choices under risk, it has also been effectively utilized at the firm level. Shimizu (2007) proposes a disposition effect in firm behaviour, as managers are unwilling to divest poor performing parts of their business. It would seem they prefer to risk recovery than realize losses. Similarly, Wen (2010) identifies changing risk attitudes in firm behaviour in relation to their capital investment decision. He finds that poor performing firms which face the prospect of further losses are more prone to undergo capital investment, while firms that display positive returns are less so. This indicates that the behaviour of firm executives can be explained by prospect theory, since loss facing firms are more willing to bear the risks of new ventures. Wen (2010) finds this pattern of capital investment to be robust at different levels of financial constraints, but to recede under strong corporate governance mechanisms, such as independent directors. This is a vital insight with regards to hedge compensation and the adverse risk incentive of the performance fee. In order to beat the hurdle rate managers may be willing to take on more risk than the investor would be comfortable with, especially after repeated poor performances. Thus, the inclusion of a prudent set of governance measures in the investment mandate could curb such behaviour. The industry has made an effort to improve its governance structures. Most funds outsource administration and fund valuation to independent companies, and many make use of independent board members in its governance. However, investors' concerns have still not been put to rest, as 81% of investors believe that independent boards do not have the authority to challenge management on misconduct (E&Y, 2011).

Option Pricing

Most of the literature concerning prospect theory emphasizes the value function, but seldom do papers include the decision probability function defined by cumulative prospect theory. Shiller (2001) speculates on whether the decision probability function, equation (2), could explain the observed irregularities in option prices. Under the Black-Scholes option pricing model, option contracts that have the same expiration date should deliver the same implied volatility irrespective of the option's strike price. The variance of the underlying asset does not depend on the strike price (Hull, 2006). However, practitioners often observe that the implied volatility, derived from option prices, increases as strike price of the option increases. This is commonly known as the volatility smile. It implies that the market is overestimating the true probability of the option becoming "in the money[4]" when the probability is very small. This supports the notion laid out by cumulative prospect theory that people are risk seeking when facing a small probability of large gains. Lehenert et al (2009) derives the price of European call options for the S&P 500 index using the decision probability function laid out by cumulative prospect theory and

[4] The option is "in the money" when its payoff is positive.

find that their prices perform better at predicting the observed option prices than the reigning option pricing technique, Black-Scholes. The role of the decision weight function would also be an interesting facet to include in this study, but unfortunately falls outside its current scope As will be discussed later, the hedge fund performance fee can be regarded as a European call option, in which the manager receives a positive payoff when the fund surpasses the hurdle rate. The value of the compensation fee can thus be priced using option pricing frameworks. Many papers[5] have estimated the value of the performance fee using the Black-Scholes framework, but none, to the author's knowledge, using the decision probability distribution of cumulative prospect theory. This analysis restricts itself to the value function aspect of prospect theory, but future research may consider this as an interesting issue to tackle.

3. Hedge funds: Risks & Incentives

3.1 A social reference point?

The reference point is fundamental to prospect theory, yet there is no definite indication as to what it may be in practice. The majority of the literature employs some assumption regarding the *status quo.* The most common definition of the reference point is the prior period's level of consumption, wealth or returns. Barberis et al. (2001) offers one such example. In some cases the reference point may seem straight forward. The hedge fund hurdle rate for example is arguably a defined point which managers can compare themselves to. Yet, even in such cases the reference point is open for interpretation. The reference point could also refer to a person's future aspirations or minimum level of accepted utility (Kahneman, 2000). Another possible definition, which this paper endorses, is the notion of a social reference point, or the idea that people value their relative gains in society more than their absolute gains. The concept of '*keeping up with the Joneses'* is not a novel one in economics. Thorsten Veblen's notion of conspicuous consumption, in which people only consume to impress their peers, has made its way into many braches of economic enquiry and empirical evidence, provided by the Easterlin paradox[6], is also widely understood as evidence of relative utility.

[5] See Giuli et al (2004) &

[6] Richard Easterlin (1974) showed that the growth in real income in Western countries since the Great Depression has brought no meaningful improvement in happiness surveys.

Relativity is arguably acute in the domain of professional investing, where salaries and livelihoods are defended on the basis of outperforming competitors. Business surveys report that the majority of executive managers define success as outperforming their peers. When given the choice between reporting positive returns, but being far below the average industry return, or reporting a small loss, but being far above the average in the industry return, many managers opt for the second state (Camerer, 1997). John Maynard Keynes articulated this phenomenon when he asserted that"[w]orldy wisdom teaches us that it is better for reputation to fail conventionally than to succeed unconventionally." This is the equivalent to the loss averse facet of prospect theory, but placed in relation to considerations of social standing.

This view is reiterated in the most basic risk management courses. Students are taught that before any firm decides whether to hedge its input costs or selling price, it should be aware of the industry norm, as hedging when your competitors are not is the equivalent of speculating (Hull, 2006). It would seem fair to assume that a hedge fund manager is familiar with this concept, but the inclusion of a social reference point in the hedge fund manager's decision making process may seem strange on first inspection. The term "hedge fund" itself is almost synonymous with the idea of absolute returns and hedge funds purposefully follow investment strategies that are uncorrelated with broader markets. However, while hedge funds may not compare themselves against the market in general and to absolute standard, it is probable that they judge their performance relative to their peers, as their investors most certainly do (Agarwal et al, 2003).

The hedge fund industry is highly competitive and underperforming funds are swiftly eliminated from the market (Botha, 2007). *The Economist* (2010) remarks also that if hedge fund managers are to justify their existence and pricing structure they need to "continually outperform the market and themselves". The level of competitiveness in the industry affirms some form of a peer reference in hedge fund decision making. Apart from the incentive of merely surviving in the industry, it also does not seem unrealistic to assume that hedge fund managers set their personal aspirations on beating their peers. This paper does not posit that hedge fund managers blindly compare themselves to their peers. That would be an overly simplistic interpretation of an industry known for its dynamism and individualistic behaviour, but the inclusion of social aspects in decision making does highlight some of the acute aspects of professional investing. If such behaviour holds true, it could have significant impact on the risk preferences of hedge funds.

5.2 Incentivised compensation

The hedge fund industry is renowned for its use of incentivised compensation. Managers typically ask a management fee of 1% - 2% on total assets under management, and a performance fee of 20% on returns earned above a stated benchmark. The rationale behind this structure is evident. When managers deliver superior returns they earn superior salaries, but when they deliver dismal results they are compensated in an equally dismal fashion. For instance, George Soros' Quantum Fund earned from $393 million in fees on the 3.7 billion assets it managed in 1995. The following year, after reporting a net loss, it earned only $54 million on a balance sheet of $5.4 billion (Goetzmann et al, 2003). This compensation structure illustrates the most basic of Agency theory's premises: The higher the pay-performance nexus, the better the incentives of managers are aligned with that of the shareholder. Yet, this fee structure has been challenged on two fronts. Many market commentators have questioned whether the returns, net of the expensive fee structure, justify the risk of hedge fund investing when compared to other asset classes such as exchange traded funds (Botha, 2007). This is commonly referred to as the "catch 2&20" of hedge fund investing. Secondly commentators have asked whether the spectacular fees have not also brought about spectacular hedge fund catastrophes (The Economist, 2006). Apart from the most notable examples like Long Term Capital Management and Bernard Madoff's Ascot Partners, the hedge fund industry has routinely witnessed prominent funds declare sudden bankruptcy. For example, Amaranth Advisors, a US based hedge fund had to close its hedge fund business after losing $5 billion in one week (Financial Times, 2006). These occurrences could have their roots in the nature of the hedge fund industry's compensation structure.

The performance fee structure of the hedge fund industry is asymmetric in nature. It motivates the manager to maximize shareholder value and aligns incentives in gains, but it does not necessarily align incentives in losses (Ross, 2003). It is similar to the practice of stock option compensation. Where the shareholder or fund investor may incur losses, the manager only loses the opportunity to have gained positive compensation (Giuli et al, 2004). The use of options in executive compensation is often regarded with mixed emotions. Proponents of this arrangement argue that by "dangling the carrot" managers are incentivised to maximize shareholder value. They argue that without such compensation managers would rest on their laurels and not act in the interest of shareholders (Agarwal, 2003). Yet, the evidence on whether companies that make use of such compensation structures do deliver better returns has been mixed at best[7].

Critics claim that the asymmetric payoff of such compensation arrangements will lead to excessive risk seeking on the part of managers as they do not share in the downside risk of the concern they are managing[8]. This view is best understood from an option pricing perspective. Since any call option strictly increases in volatility, it would seem plausible that a manager receiving such an option would want to increase the risk of the underlying asset when the option is "out of the money". If the manager has not surpassed the hurdle return, or realizes there is a small probability of doing so, there is a large incentive to increase the risk of the concern he is managing, especially, as time nears the expiration date. Whether the option is "in the money" to start with will thus be critical in the determining the manager's risk preferences. The majority of stock option compensation is done with a "in the money" strike price. Agarwal et al (2009) reports that only 6% of stock option compensation is done "out of the money". This would seem prudent from a prospect theory risk perspective. If the strike price is assumed as the reference point, managers would start their decision making in gains space where they are risk averse. Yet, there is still the risk of that when the company's value falls below that of the strike price that the manager will become risk seeking. Ergo, the manager will be willing to undertake risks which are not warranted from the shareholder's perspective.

The hedge fund industry's use of the performance is arguably more severe than that of stock option compensation. The performance fee has a larger payoff and is typically set "at the money", if not "out of the money". This could easily see to a situation where the manager is forced to play catch-up to the hurdle amount. This process of risk taking would become even more acute when managers are on the brink of fund withdrawal. Since they do not share in the downside risk to the extent that the investor does the incentive to "roll the dice, just one more time" may be too tempting. There is a considerable pool of literature discussing such situations, the different payoff incentive between equity holders and bond holders in the face of bankruptcy being an apt comparison (Adler, 1991).

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[7] See Agarwal et al (2009) for a summary

[8] This statement could be regarded to be overly harsh, as managers do have a large disincentive in the form of unemployment and disgrace.

Various papers have considered what such compensation arrangements hold for risk taking behaviour under theoretical frameworks. Carpenter (2000) offers a rigorous theoretical discussion of convex compensation schemes under the standard neoclassical assumption of global risk averseness. She evaluates the optimal investment strategy of a manager that owns a call option on an asset which the manager also controls. Under various definitions of utility, she posits that the manager increases the volatility of the asset as the difference between the strike rate and the underlying becomes larger. In general, her models predict that a manager either ends with a deep "in the money" call or a deep "out the money" call, since the manager is willing to increase the volatility of the asset following the compensation scheme. However, this conclusion needs to be interpreted carefully. Her model also predicts that an increase in the performance fee ultimately enforces less risk taking when the performance fee is sufficiently large. The risk aversion over this fee, holds stronger than the asymmetrical risk seeking component implicit in the option. Ross (2003) offers a similar conclusion on stock option compensation. Both these model demonstrate that a fund manager's preferences regarding risk would be the determining factor in fund the manager's response to such forms of compensation. Under high degrees risk averseness, increasing the amount of options the manager owns could see to less volatility in the underlying asset, as the leverage implicit in the option enhances the manager's exposure to the underlying volatility, making the manager willing to decrease volatility. The pool of literature concerning this topic has witnessed significant contributions from various perspectives. Siegman and Lucas (2000) offer some insight on the matter of hedge fund investment styles from the perspective of loss averse investor, given the compensation structure. Kouwenberg & Ziemba (2005) consider a hedge fund manager displaying loss averseness toward the benchmark rate and argues that manager's increase volatility over all states. These are a few examples of the expanding literature. This paper follows in the trend by regarding the hedge fund manager's decision making from a prospect theory perspective

4. Proposal: Risk appetite under prospect theory

4.1 The Fund Manager's Wealth Function

Consider the following wealth optimization problem of a hedge fund manager; defined in a two-period time-setting[9]. The manager's initial wealth is W_t, while the value of the fund at inception is X_t. The fund X_t closes and pays out the net asset value to investors in the period *t+1*. At which point a new fund Y_{t+1} is started. The size of the new fund is a function of the relative position (percentile rank) of the previous fund in relation to the hedge industry's median fund (S_{t+1}), scaled by the previous fund's initial value. All funds in the industry at period t are assumed to be equal in size, and cannot a carry negative value[10].

$$(1) \qquad Y_{t+1} = X_t\,(X_{t+1}/S_{t+1})$$

This assumption needs some qualifying. It is based on the notion that investors will redeem funds from below average managers and allocate it to above average managers. The underperforming manager therefore has less access to funds in the second round and will manage a smaller fund than his previous fund[11]. It implies that there is a fixed pool of hedge fund assigned capital and that investors do not move between investment classes to search better yields. That is to say the only substitute investment to a bad performing hedge fund is a better one. This method insures that managers who continuously underperform see their fund size decline until they are eventually eliminated from the market in a multi period setting, as discussed in previous sections.

Assume that the manager's wealth is defined only by the wealth he earns from his occupation i.e. the manager does not own an additional private portfolio of investments. Following the fee structures discussed in the previous section, the manager receives compensation solely in the form of a management fee and a performance fee. The management fee is defined as a proportion *($\theta \geq 0$)* of the net asset value of the fund at the start of its mandate. Thus, in the second period the

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[9] Other papers concerning similar topics (See Carpenter (2000), Goetzmann et al, (2003) and Kouwenberg and Ziemba (2005)) offer a more complex framework in which managers optimize their wealth by selecting a dynamic investment strategy in a continuous time setting. This method has the added benefit of including the time value of diversification. For ease of exposition, this paper follows a simpler approach.

[10] With excessive leverage in place it is plausible that a fund can close at a negative amount, as it may owe more than its assets can pay for. This possibility is excluded in this model.

[11] Assuming that the fund manager's access capital grows or contracts in this specific proportion is arguably arbitrary, but it serves the intuition that underperforming firms are eliminated from the market. It also insures that investors do not move all their capital to the top performing hedge fund in the second round.

manager only earns the management fee of the second fund. The performance fee is defined as a percentage $(\rho \geq 0)$ of the net asset value achieved in excess of the hurdle amount H_{t+1}. The performance fee can thus be regarded as a call option on the fund value with a strike price equal to the hurdle rate. The hurdle amount is defined in the investment mandate at the start of the fund, thus the fund manager can take it as a given which is exogenously set. I further assume that the hurdle rate is the same for all hedge fund managers. This is consistent with the South African hedge fund industry, where almost 80% of hedge funds set their hurdle rate equal to the index return on one year money market funds (Novare, 2011).

For tractability, the author assumes that the median fund net asset value of the industry (S_{t+1}) is strictly greater than the hurdle fund size (H_{t+1}). The data used in this study confirms this assumption, as on average, the median for the hedge fund index outperforms liquid money market investments over the previous five years in the South African hedge fund industry (Symmetry, 2011).

In summary, the fund manager optimizes his wealth at $t+1$ by the following function:

$$(2) \qquad W_{t+1} = (\theta)Y_{t+1} + (\rho)\max\{X_{t+1} - H_{t+1}, 0\}$$

Substituting Y_{t+1} with equation (1) delivers a wealth function of:

$$(3) \qquad W_{t+1} = (\theta)(X_t)(X_{t+1})/(S_{t+1}) + (\rho)\max\{X_{t+1} - H_{t+1}, 0\}$$

Giuli et al. (2005) offers a similar definition of the performance fee, but excludes the management fee and includes the possibility of the hedge fund manager taking a share in the fund by investing his private wealth alongside the fund's investors. This is a common practice in the US hedge fund industry where some funds are even majority owned by the fund manager (Ineichen, 2002). The latest hedge fund survey regarding the South African industry (Novare, 2011) does not however indicate whether this is common in the domestic market and therefore it is excluded. Kouwenberg and Ziemba (2005) also include a management fee in their paper, but the compensation is paid in the evaluation period. To the author's knowledge, none of the work regarding this topic defines the closing of funds and opening of new funds. In addition, most define all management compensation on the fund size in the evaluation period without defining procedures regarding new capital. Consequently, all the hedge funds in these models could operate as Ponzi schemes since management could increase the fund size by luring new investors, earning them higher compensation.

4.2 Value function and Reference point

The value function proposed by prospect theory is often criticized by practitioners, as it is not continuously differentiable and difficult to implement in application. This paper uses the value function suggested by Hung and Wang (2005), with which they addressed the equity premium puzzle. This specification meets the theoretical propositions of prospect theory, whilst also being continuously differentiable.

(4) − − *if*

This function displays the following properties: 1.) Fund managers evaluate gains and losses according to the reference point (R_{t+1}). 2.) Managers display loss averseness $(\lambda>1)$ in relation to the reference point. 3.) Given that $(0<\alpha<1)$ the function is concave in gains (v'(W) > 0, v''(W) < 0), indicating risk averseness when they have surpassed their target utility. 4.) Also the function is convex in losses (v'(W) > 0, v''(W) > 0), indicating risk seeking when they are underperforming against target utility. 5.) The function shows constant absolute risk aversion (CARA) in gains and constant absolute risk seeking (CARS) in losses.

Following Kouwenberg and Ziemba (2005) the reference point (R_{t+1}) is defined as the level of wealth that the hedge fund manager accrues when his performance fee is "at the money". By this definition, compensation comprising only of a management fee (received from the new fund) is viewed as a loss by the fund manager and compensation including a performance fee is viewed as a gain. In order for the fund manager to earn a performance fee the net asset value of the fund must exceed the hurdle value. Therefore, the fund manager perceives himself to be at a loss when underperforming against the hurdle value.

This can be shown mathematically as follows[12]. From the managers wealth function (equation 1), the reference point can be written as the point where $X_{t+1} = H_{t+1}$, since that is where the performance fee is "at the money". As a result, the reference point can be written as:

$$(5) \qquad R_{t+1} = (\theta)(X_t)(H_{t+1})/(S_{t+1})$$

[12] Impetus provided by Kouwenberg and Ziemba (2005)

By substituting (1) for W_{t+1} and (5) for R_{t+1}, the dividing element of the value function between losses and gains,$W_{t+1} \leq R_{t+}$1, can be written as:

$$(6) \qquad (\theta)(X_t)(H_{t+1})/(S_{t+1}) + (\rho)\max\{X_{t+1} - H_{t+1}, 0\} \leq (\theta)(X_t)(H_{t+1})/(S_{t+1})$$

Rearranging:

$$(7) \qquad (\rho)\max\{X_{t+1} - H_{t+1}, 0\} \leq 0$$

Since (ρ) is a strictly positive percentage, it follows that $W_{t+1} \leq R_{t+1}$ is equivalent to $X_{t+1} \leq H_{t+1}$. Given this equivalence, I substitute the wealth function (1) for W_{t+1} and (5) for R_{t+1} in the value function (4). After rearranging, the value function can thus be stated as:

(8) *if*

4.3 Risk Implications

Since the value function is defined as a CARA in gains and a CARS in losses there are no 'fund wealth effects' in relation to the risk attitude in the respective states. I.e. there are two unique hedge fund manager risk premiums, one that holds globally in gains and one that holds globally in losses. The risk premium, under risk averseness, is the fixed amount of money (r) the fund manager would be willing to receive to make him indifferent between a positive risky return (Z_{t+1}) and a sure amount $[E(Z_{t+1}) - r]$ (Camerer, 2005). Thus, r is the monetary value that solves the indifference relationship: $((X_{t+1} - H_{t+1}) + Z_{t+1}) \sim ((X_{t+1} - H_{t+1}) + E(Z_{t+1}) - r)$. The risk premium is the price of risk from the hedge fund manager's perspective. It indicates the amount that the risk averse manager needs to receive for taking on additional risk and the amount that a risk seeking manager is willing to pay for additional risk. Consequently, the risk seeking manager has a negative risk premium, while the risk averse manager has a positive risk premium. An increase in the risk premium indicates that the risk averse fund manager becomes even more risk averse. An increase in the risk premium for a risk seeking manager indicates that he becomes less risk seeking, as the amount he was willing to pay for risk has decreased. Under the assumption that hedge fund returns are normally distributed, the risk premium can be proved to be equal to[13]:

[13] See Chavas (2005) for the proof of a standard risk averse investor. By using this proof I am assuming that hedge fund managers do not use the decision weight function proposed by prospect theory.

$$(9) \qquad R = (A/2)\sigma^2$$

Where A is the Arrow-Pratt coefficient of absolute risk aversion $(-V''/V')$ and σ^2 is the variance of the uncertain hedge fund return of Z_{t+1}. The Arrow-Pratt coefficient can be derived for both states using the wealth function defined in (8), resulting in the respective risk premiums:

$$(10) \qquad \textit{Gains:} \quad r^{+} = \frac{\;}{\;}\,\sigma^2$$

$$(11) \qquad \textit{Losses:} \quad r^{-} = -\frac{\;}{\;}\,\sigma^2$$

Proposition 1: Since $(\sigma^2 \geq 0)$, $(0 \leq \alpha \leq 1)$ and $(X_t, S_{t+1} \geq 0)$: *The risk premium in gains territory strictly increases as both the performance fee and the management fee increases.* Since the partial derivatives, dr^{+}/dp and $dr^{+}/d\theta$, can be shown to be positive, the risk premium for the fund manager increases as compensation increases. Therefore the fund manager, starting in gains territory, becomes more risk averse when the fees are increased. This finding corresponds to notion that granting “in the money” stock options would not result in unwarranted risk taking. It may in fact result in overly risk averse managers.

$$(12) \qquad dr^{+}/dp = \frac{\;}{\;}$$

$$(13) \qquad dr^{+}/d\theta = \frac{\;}{\;}$$

Proposition 2: Since $(\sigma^2 \geq 0)$, $(\lambda > 1)$, $(X_t, S_{t+1} \geq 0)$ and $(0 \leq \alpha \leq 1)$: *The risk premium in gains territory strictly increases as the management fee increases, but at a decreasing function of the median return:* Since the cross derivative $dr^{+}/d\theta d S_{t+1}$ is negative the positive effect of the management fee on risk averseness decreases the larger the median return becomes, since the manager will earn less from the fee as his relative peer position is worsened. This can be considered as the “keeping up with Joneses” aspect of the hedge fund manager’s wealth optimization. All things held constant, an increase in the median return makes the hedge fund manager more willing to take risk to keep with the group.

The assumption of normality does not necessarily influence the manager’s risk attitude. It just represents a simple way of showing the managers preferences for risk.

$$(14) \qquad dr^{+}/d\theta d\, S_{t+1} = - \overline{\qquad}$$

Proposition 3: Since ($\sigma^2 \geq 0$), ($\lambda > 1$), (X_t, $S_{t+1} \geq 0$) and ($0 \leq \alpha \leq 1$): *The risk premium in loss territory strictly decreases as the management fee increases.* Since $dr^{-}/d\theta$ can be shown to be negative. The decrease in the negative risk premium indicates that the manager becomes more willing to pay for additional risk. Thus, the hedge fund manager, when in loss territory, becomes more risk seeking as the management fee increases. Since the model assumed that the median fund is always larger than the hurdle value. I.e. in loss territory the manager is underperforming not just against the hurdle value, but also against his peers.

$$(15) \qquad dr^{-}/d\theta = - \overline{\qquad}$$

If the fund manager's risk attitude is known to the investor, the investor could select an optimal negative management fee to make the fund manager risk averse in all states. This could be regarded as the price the fund manager has to pay to gain the opportunity of earning the performance fee. This, in a sense, would be the price of the performance fee call option. If the investor is to make the manager risk averse in both states he would have to solve θ where the risk premiums are strictly positive:

$$(16) \qquad r^{+} = \overline{\qquad\qquad} \sigma^2 \geq 0$$

$$(17) \qquad r^{-} = - \overline{\qquad\qquad} \sigma^2 \geq 0$$

Given ($\sigma^2 \geq 0$), ($X_t/S_{t+1} > 0$) and ($0 \leq \alpha, \rho \leq 1$), equation 15 can be reduced to:

$$(18) \qquad \theta \geq -\rho \overline{\quad}$$

And given ($\sigma^2 \geq 0$), ($\lambda > 1$), ($X_t/S_{t+1} > 0$) and ($0 \leq \alpha \leq 1$), equation 16 can be reduced to:

$$(19) \qquad \theta \leq 0$$

Proposition 5: Since ($\sigma^2 \geq 0$), ($\lambda > 1$), ($0 \leq \alpha, \rho \leq 1$) *and the management fee is allowed to be negative*: By combining (18) and (19) the manager would be universally risk averse when: $-\rho \overline{\quad} \leq \theta \leq 0$

4.4 Discussion

The model highlights some possible sources of risk altering behaviour in hedge fund manager's decision making. It only considers the impact that the parameters have on risk attitudes and does not define the manager's investment opportunities[14]. Consequently, it cannot conclusively conclude on whether managers will to increase or decrease the volatility of the fund. It does however indicate what the price of risk is from a hedge fund manager's perspective and shows how this pricing changes when the hedge fund manager's incentives change. The incentives this paper highlights, namely relative performance and compensation, are shown to influence the managers preferences towards risk. It suggests that if hedge fund managers optimize their wealth according to prospect theory's value function there may be some punitive implications for risk attitudes concerning the structure of compensation. Some broad conclusions can be taken from this analysis. The assumption that an increase in the performance fee automatically leads to excessive risk taking on the part of the manager cannot be justified from this analysis. What does prove critical is the state the manager perceives himself to be in. Thus, the selection of a benchmark return rate, the division point between losses and gains, would be instrumental in determining the manager's risk attitude. If investors require higher benchmarks they might end up paying in a substantial increase in risk.

While the hedge fund industry seems determined on the current pricing structure (76% of South African hedge funds use a 1% management fee and 20% performance fee. (Novare,2011)) The model suggests that the current practice of paying a management fee towards hedge fund managers could be adapted to constrain management's risk seeking behaviour. From the model's perspective management should pay a fee scaled by performance fee and the industries median return. This fee can be regarded as the 'price' of the option of the performance fee. While, this is not a true reflection of reality as the investor shares in the upside and is not a provider of debt, it does pose an interesting question regarding the risk sharing relationship between fund manager and investor. It also brings the norm of the "2&20" pricing structure under scrutiny. The convex payoff structure of the option seems to suit the notion of the value function's risk seeking and loss averse behaviour. If a risk averse behaviour is to counter such behaviour he may be required to adapt the fee structure to ensure that managers share in the downside risk.

[14] This would be a welcomed addition to the model that future research could consider

The model offers a conclusion on the influence of peer performance and adverse incentives these could create. While this assumption is fallible it does not seem implausible. The ability of fund managers to survive in such a competitive industry would be a function of their relative performance and without cognisance of this risk impact investors and creditors might be caught off guard.

There are admittedly many shortcomings in this approach, which the author readily admits. 1.) It assumes that managers only care about their wealth in the period *t+1* and does not allow for risk optimization over time. 2.) It does not allow managers do not invest in their own funds. 3.) There are no high-water marks defined[15]. 4.) It does not consider any corporate governance measures. These shortcomings are difficult to address, but further research in these alleys can help clarify some of the uncertainty. The purpose of this model was to introduce a basic intuition of how risk preferences are affected by the key concerns it voiced. While these incentives are difficult to address in formal model and sometimes require assumptions that seem unrealistic, it does serve as a basis for analysis on which future research, hopefully, could expand.

[15] The high water mark is the level that the hedge fund manager must surpass before he can claim a performance fee when the previous year showed negative return. This could restrain risk taking behaviour, as managers know that a very poor performance this round leaves them with little probability of a performance fee in the next round.

5. Conclusion

There is no easy way to untangle the complex set of incentives that drives a hedge fund manager's decision making. If behavioural finance and prospect theory is to teach economists anything, it would be that holistic conclusions on matters as intricate as decision making should be regarded with caution. Prospect theory has provided the paper with the impetus to address the topic from a behavioural perspective. The paper has shown that prospect theory is a rich framework which has been widely applied in financial problems, and has hopefully convinced the reader of its application in the hedge fund manager's decision making process. Prospect theory, as a positive view on decision taking under uncertainty, offers insights on incentives that would have been lost under normative analysis. It is important that economists and financial market participants understand its axioms if they are to better their performance and perception of what drives market outcomes. Moreover, it is important that investors, creditors and regulators understand the thinking of the institutions they are dealing with. Hedge fund investing, as an asset class, is growing in prominence in the South African investment arena. This trend is unlikely to abate. Thus, it is important that the domestic market does not repeat some of the mistakes witnessed in other markets. This paper is not pessimistic on the hedge fund industry. It just calls for a review of the incentives. The assumption that all hedge fund managers follow the same reasoning or are hollowly influenced by their peers is not one it blindly promotes, but neither does it accept the assumption that hedge fund managers are just in all their dealings.

6. References

Adler, B.E. (1991) Bankruptcy and risk allocation. Automated categorisation of Web resources. [Online]. Available: http://heinonline.org/HOL/Page?handle=hein.journals/clqv77& &g_sent=1&collection=journals [3 January, 2012)

Agarwal, V.; Daniel, N.D. & Naik, N.Y. (2003) Flows, Perfomance, and Managerial Incentives in Hedge Funds. CFR Working Paper No. 04-04

Botha, M. (2007) A comparison of South African hedge fund risk measures. South African Journal of Economics. Vol 75:3 September 2007

Barberis, N; Huang, M & Santos, T. (1999) Prospect theory and Asset prices. Quarterly Journal of Economics 2001

Blanchard, O.J. & Watson, M.W. (1982) Bubbles, Rational expectations & Financial Markets. NBER July edition : Working Paper No. 945.

Campbell, J. (2000). Asset pricing at the millennium. Journal of Finance, 55(4), 1515, 1567.

Carpenter, J. N. (2000) Does Option Compensation Increase Managerial Risk Appetite. The Journal of Finance. Vol LV 5. October 2000

Chavas, J.P. (2005) Risk analysis in theory and practice. Elsevier Academic Press.

Cameror, C.F. (1997) Taxi Drivers and Beauty Contests. Automated categorisation of Web resources. [Online]. Available: http://www.engineeringandscience.com/17e/two [1 May 2010]

Cohen, M.; Jaffray, Y.S. and Tanios Said. (1987). "Experimental Comparison of Individual Behavior Under Risk and Under Uncertainty for Gains and for Losses," Organizational Behavior and Human Decision Processes 39, 1-22.

Easterlin, R. A. (1974) Does Economic Growth Improve the Human Lot? Some Empirical Evidence Nations and Households: Economic Growth Essays. New York: Academic Press, 89-125.

Ernst & Young (2011) Coming of age: Global hedge fund survey. [Online]. Available: http://www.ey.com/Publication/vwLUAssets/2011_survey_highlights:_global_hedge_fund_market/$FILE/Hedge%20Fund%20Survey%202011_FINAL.pdf [16 December 2011]

Festinger, L. (1957) A theory of cognitive dissonance, Stanford University Press

Friedman, M. & Savage, L.J. (1948) The utility analysis of choies involving risk. The Journal of Political Economy. Vol LVI No4. August 1948

Giuli, M.E; Maggi, M.A. & Paris, F.M. (2004) Pricing incentive fee of hedge fund manafers: a discussion of moral hazard. Automated categorisation of Web resources. [Online]. Available http://www.elsevier.com/1365 [22 November 2011]

Goetzmann, W.N.; Ingersoll, J.R. & Ross, S.A. (2003) High water marks and hedge fund management contracts. The Journal of Finance. Vol LVIII No 4. August 2003

Grinblatt, M. & Han, B. (2004) prospect Theory, Mental Accounting, and Momentum. Automated categorisation of Web resources. [Online]. Available http://www.elsevier.com/1456 [10 March 2011]

Hung, M. W. and J. Y. Wang, (2003) Asset Prices under Prospect Theory and Habit Formation The 11th Annual Conference on Pacific Basin Finance, Economics and Accounting (PBFEA 2003)

Hull, J.C. (2006) Options, futures and other derivatives. Eighth edition, New York Press

Ineichen, A.M. (2003) Absolute Returns: The risks and opportunities of hedge fund investing. Wiley Finance, New Jersey

Kahneman, D. & Tversky, A. (1979) Prospect Theory: An Analysis of Decision Under Risk. Econometrica, 47: 263-291

Kahneman, D. (1992) Reference Point, Anchors, Norms and Mixed Feelings. Organizational Behahaviour and Human decision Processes 51, 296 – 312

Keynes, J.M. (1936) The General Theory Of Employment, Interest, and Money. (1964 edition). Harcourt, Brace and World. New York

Kouwenberg, R. & Ziemba, W.T. (2005) Incentives and Risk Taking in Hedge Funds. Erasmus Working Paper Series

Loomes, G. & Sugden, R. (1982) Regret Theory: An Alternative Theory of Rational Choice Under Uncertainty. The Economic Journal , Vol. 92, No. 368 (Dec., 1982), pp. 805-824

Michenaud, S. & Solnik, B. (2008) Applying regret theory to investment choices: Currency hedging decisions Journal of International Money and finance 27 (2008) 677-694

Novare (2011) South African Hedge Fund Survey 2011. Automated categorisation of Web resources. [Online]. Available http://www.novare.co.za/uploads/files/File/Novare%20Investments%20SA%20Hedge%20Fund%20Survey%20September%202010.pdf [12 December 2011]

Prast, H. (2004) Investor Psychology: A Behavioural Explanation of Six Finance Puzzles. De Nederlandse Bank Working Paper Series

Riscura (2010) Private Equity Report: 4Th Quarter. Automated categorisation of Web resources. [Online]. Available http:// http://www.riscura.com/sa/home.htm [12 December 2011]

Ross, S. (2004) Compensation, Incentives, and The Duality of Risk Aversion and Riskiness. The Journal of Finance. Vol LIX No1 February 2004

Shefrin, H. (2005) A behavioural approach to asset pricing. First Edition. Academic Press: New York

Shiller, R. J. (2001) Human behaviour and the Efficiency and the Financial System. Cowles Foundation: Discussion Paper No. 1025.

Siegmann, A. & Lucas, A. (2002) Explaining Hedge Fund Investment Styles by Loss Aversion. Tinbergen Institute Discussion Paper 046

Stracca, L. (2003) Behavioural Finance and Aggregate Market Fluctuations: Where do we Stand?," Discussion Papers in Economics 02/10, Department of Economics, University of Leicester.

Sunstein, C.R. (2002) Probability Neglect: Emotions, Worst Cases, and Law. U Chicago Law & Economics, Olin Working Paper No. 138. Available at SSRN: http://ssrn.com/abstract=292149 or doi:10.2139/ssrn.292149

Symmetry (2011) Hedge Fund Survey Data. Provided by Symmetry to the author

Thaler, R. (1985) Mental Accounting and consumer choice. Marketing Science edition 4, 199-214

The Economist (2010) Paying the price. July 29, 2010, Print Edition

Veblen, T. (1899) The Theory of the Leisure Class. Penguin Classics. New York

Wehrung, Donald A. (1989). Risk Taking over Gains and Losses: A Study of Oil Executives. *Annals of Operations Research* 19, 115-139.

Wen, Y.F. (2010) Capital investment decision, corporate governance, and prospect theory. Procedia Journal of Social and Behavioural Finances (2010) 116-126