Monetary Regimes, Money Supply and the US Business Cycle since 1959: Implications for Monetary Policy Today

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Abstract

To justify the operational procedures of central banks since the 2008 global financial crisis, reputable academics and practitioners proclaim the independence of interest rate policy from all things monetary. This policy debate can be traced as far back as Thornton (1802), Pigou (1917), and Tinbergen (1939, 1951) to which Hetzel (1986) points out the especially great confusion "over the effect of the choice by the monetary authority between reserves and interest rate manipulation." Using U.S. data spanning 58 years, we estimate a dynamic general equilibrium model and show that the type of monetary policy regime has significant implications for the role of monetary aggregates and interest rate policy on the U.S. business cycle. The interaction between money supply and demand and the type of monetary regime in our model does remarkably well to capture the dynamics of the U.S. business cycle. The results suggest that the evolution toward a stricter interest rate targeting regime renders central bank balance sheet expansions superfluous. In the context of the 2007−09 global financial crisis, a more flexible interest rate targeting regime would have led to a significant monetary expansion and more rapid economic recovery in the U.S.

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If the structure of the economy through which policy effects are transmitted does vary with the goals of policy, and the means adopted to achieve them, the notion of a unique 'transmission mechanism' for monetary policy is chimera and it is small wonder that we have had so little success in tracking it down.

— David Laidler (*Monetarist Perspectives*, 1982, p. 150)

1 Introduction

To justify the operational procedures of central banks since the 2008 global financial crisis, reputable academics and practitioners proclaim the independence of interest rate policy from all things monetary (e.g., Woodford (2000), Borio and Disyatat (2010) and Kashyap and Stein (2012)).[1] This policy debate can be traced as far back as Thornton (1802), Pigou (1917), and Tinbergen (1939, 1951) to which Hetzel (1986, p. 13) points out the especially great confusion "over the effect of the choice by the monetary authority between reserves and interest rate manipulation." Furthermore, central banking operates in a continuously evolving system, its policy operations are difficult to define and the transmission mechanisms of instruments are nearly impossible to pinpoint. To therefore understand the effect of monetary policy we require an endogenous monetary framework consistent with *both* theory and empirical regularity.

The purpose of this paper is twofold. Firstly, we show that the type of monetary policy regime—that is, the choice of monetary policy rule for the determination of the money stock, and hence the price level—has significant implications for the role of monetary aggregates and interest rate policy in a standard New-Keynesian framework.[2] Secondly, and as a result, we show that the U.S. economy need not succumb to the low-inflation, low-interest-rate state observed since the 2008 global financial crisis. On the one hand, an interest rate targeting regime renders central bank balance sheet expansions superfluous. At the zero lower bound this regime is also ineffective. On the other hand, relaxing the effective interest rate peg—even without the assumption of monetary non-neutrality[3]—an expansion of the central banks balance sheet will be effective.

To do this, we highlight three *bona fide* arguments in favour of a traditional model of money stock determination based on the Fisher relation, price-level determination, and the behavior of money demand. Together, these three conditions form the core of the general equilibrium framework

[1] Borio and Disyatat (2010, p. 57): "...in setting the interest rate, no open market operations need be involved at all ...the interest rate is not controlled by moving up and down a well-behaved, traditional [money] demand schedule."

[2] Most central bank models rely heavily on the benchmark New-Keynesian framework for policy analysis (Lindé et al., 2016).

[3] i.e., non-separability between consumption and money in household utility.

envisaged by McCallum (1981, 1986) and Hetzel (1986), which we develop into a New-Keynesian (NK) DSGE model. The model is estimated by Bayesian methods over the period 1959Q1−2007Q3. The estimation period is chosen for two reasons. First, the long sample period serves to highlight the empirical and theoretical coherence of the model, and provide a detailed account of the U.S. business cycle. We also compare our results with an estimated version of the model over the period 1984Q1−2007Q3, corresponding to the Great Moderation period. Second, we want to simulate the counterfactual scenario of a reduction in free reserves given the estimated structure of the model economy prior to the onset of the global financial crisis and the structural break in free reserves in 2007Q4.

The main findings show that monetary aggregates are important, not only for monetary policy, but for capturing the actual behaviour of a monetary economy. The interaction between money supply and demand and the type of monetary regime does remarkably well to capture the dynamics of the U.S. business cycle over the observed 58 years. The results suggest that the evolution toward a stricter interest rate targeting regime renders central bank balance sheet expansions superfluous. In the context of the 2007−09 global financial crisis, a more flexible interest rate targeting regime would have lead to a significant monetary expansion and more rapid economic recovery in the U.S. Specifically, a counterfactual simulation at the zero lower bound—and based on the post-crisis average free reserves held at the Federal Reserve—indicate that a $3.7 billion reduction in free reserves expands the money supply by 4 percentage points and output by 3 percentage points.

The principle contribution of this paper is to demonstrate that neither an interest rate targeting regime nor a money growth rule is desirable. At the same time, a two instrument-two goal operational framework (the 'decoupling principle') is a misidentification of monetary canon. Rather, monetary authorities should stabilize nominal income (or equivalently, the product of the broad money supply and velocity) in a market economy environment using both its monopoly over the monetary base and interest rate policy. The reason being that under certain states of the world, at either the zero lower bound or highly elastic reserve demand, either interest rate policy or money base creation can be ineffective.

As mentioned, this paper falls into the context of a long and rich literature. In the current economic state of low interest rates and ineffective monetary policy, however, the 'liquidity trap' hypothesis has resurged strongly. As the hypothesis directly applies to the arguments presented here it requires some attention before continuing. One strand of literature, in particular, posits a theory of price level determination based on the interaction between fiscal policy and monetary policy. Cochrane (2014) and Leeper (2016) form the argument by identifying three basic approaches to monetary policy and price level determination: money supply and demand in the spirit of the monetarist MV≡PY tradition; interest-rate controlling New-Keynesian models; and the fiscal theory of the price level. Their important critique, as previously raised by Sargent and Wallace (1985), is that the economy is satiated with money when the return on money (or reserves) equals the return on risk-free assets (e.g., Treasury bills). That is, any amount of money will be held at this point, and exchanging treasuries for money has no effect on the economy—the price level is therefore indeterminant. In response to this state of the world, Cochrane (2014) and Leeper (2016) show that a determinant equilibrium necessitates an *active* fiscal policy.[4] Indeed, Cochrane (2014) correctly emphasizes that this holds *only* in the current international monetary system of fiat money and central banks. But if the price level is the price of goods in terms of nominal (government) liabilities (money plus bonds), the question then begs: what determines the price level in a world of free banking with un-backed, de-centralized fiat money? Is there a more fundamental theory of price-level determination that precludes fiscal debt management and present discounted government deficits and surpluses?

In contrast, the model developed in Belongia and Ireland (2014) and Ireland (2014), based on Barnett's (1978; 1980) user cost of money and monetary aggregation theory, emphasizes the role of the true aggregate of monetary (liquidity) services demanded. Their shopping time model maintains the core New-Keynesian (IS-LM) framework and ensures that the opportunity cost on this true monetary aggregate is always positive—provided the risk-free rate is not zero. With regards to the zero lower bound, it is not immediately evident that money demand has no satiation point. While the threshold appears to be currently rather high in the market for reserves, Ireland (2009) shows evidence of a finite satiation point for broader monetary aggregates (also illustrated in Figures 1 and 2). As this is likely true, then even at zero nominal interest rates the true monetary aggregate—whether currency or highly liquid, risk-free assets—commands some positive finite transactions value (Yeager, 1986). In effect, all perfectly substitutable, perfectly liquid assets will inherit this valuable attribute. In the context of macroeconomic models, the demand for fiat money depends on whether we expected it will hold its *exchange value* in the future: its discounted present value. By backward induction, money would be valueless today if we knew with certainty that money will be valueless at some given date in the future. But if money has positive value in all future periods we can proceed.[5]

[4] "The aggregate price level is a relative price: it measures how much a basket of goods is worth in terms of nominal government liabilities—money plus bonds. This relative price must be determined by the interaction of supply and demand for these government liabilities." (Leeper, 2016, p. 2)

[5] This is basically illustrated by assuming all wealth (assets) are in the households utility function, and their corresponding rate of return has some implicit transactions value, no matter the illiquidity or riskiness—someone, somewhere is willing to trade for that asset. This is effectively Say's Law: the supply of any good, including fiat money and specie, generates a demand for all other goods (see, e.g., Yeager (1986), White, 2010). Further: I am essentially proposing some measure of "moniness" attached to any item of value, to which, as it approaches perfect

The remainder of this paper is organized as follows. Section 2 discusses the three fundamental conditions in favour of a traditional model of money stock determination. Section 3 describes the model with money stock determination and a market for bank reserves. Sections 4 to 6 present the estimation results and main findings. Section 7 concludes.

2 Revisiting three pillars of the monetary exchange economy

2.1 The Fisher relation

The "original" Fisher (1896) *effect* derives from a no-arbitrage condition on the expected terms-of-trade between money and commodities (Dimand and Betancourt, 2012, p. 188).[6] The well-known Fisher *relation* or *distinction* between nominal and real interest rates is a simplification (see Laidler, 2013, p. 3). To be consistent with theory we need a model that (1) describes how price-level expectations are formed and (2) to what extent asset markets reflect inflation expectations in the difference between nominal (i) and real (r) rates of return.

Firstly, under rational expectations the expected value of fiat money (A) expressed in terms of commodities ($1/P$) equates: $E(A) = E(1/P) = 1/E(P)$ and the "original" Fisher *effect*: $i = r - a - ra$ equates with the "conventional" Fisher *relation*: $i = r + \pi + r\pi$, where a is the expected appreciation of the value of money in terms of a basket of commodities and π is expected inflation.[7] Notably, however, price level determination with respect to *both* the money stock and the interest rate is crucial to satisfy the Fisher relationship.

Secondly, given this link between the money stock, commodity prices and rates of return, the Fisher relation further implies—as shown in, e.g., Ireland (2014) and Walsh (2010, p. 457)—that the monetary authority cannot independently determine the nominal interest rate and the expected rate of inflation (or, more correctly, the expected depreciation in the value of money). Instead, given an (intermediate) interest rate target, the money supply adjusts to a growth rate commensurate with the inflation rate, and *vice versa*.[8]

substitutability with money it will approach the finite value of liquidity (transactions) services.

[6] See also Laidler (2013) for important clarifications on usage of the Fisher relation in the Great Depression and the Great Recession—especially with respect to monetary policy discussion of the times. Dimand (1999) also distinguishes the actual contributions of Fisher from that of the development of the relation attributed to his name.

[7] It is straight forward to show, using Jensen's inequality and the Cauchy-Schwarz inequality, that $1 \leq E(P)E(1/P) \leq 1 + \frac{(b-a)^2}{4ab}$ for a bounded random variable P on the interval $[a, b] > 0$ with $\text{Prob}(a < P < b) = 1$. With negligible uncertainty about the expected price level ($a \approx b$) $E(P)E(1/P) \approx 1$.

[8] Fisher also used his hypothesis to investigate the term structure of interest rates (Dimand, 1999). It then naturally leads to questioning the central bank's control over short- and long-term interest rates, and understanding the transmission of expected policy rate changes across the term structure (see, e.g., Poole et al. (2002, p. 85), Thornton (2002, 2004) and Hummel (2013)). Here, I'd like to reiterate a possible third channel describing the Fed's interest rate policy, namely, the "interest-rate-smoothing hypothesis". Thornton (2004) shows that the lack of evidence in favour of either open market operations or open mouth operations is consistent with an endogenous

2.2 Money stock and price-level determination

The key result that Hetzel (1986, p. 7) brings to light is that for nominal money to play a causal role in determining the price level, "at least some of the determinants of nominal money supply must differ from the determinants of real money demand." Specifically, nominal shocks shift both demand and supply curves for money, whereas real shocks only shift the supply of money through changes in the real interest rate.[9] And by implication of the quantity theory of money approach, the price level adjusts to equate the real quantity of money supply with the real quantity demanded. Simply put, monetary policy generates 'money illusion'.[10]

Consequently, the problem of multiple equilibria arises if the equilibrium conditions of the model can neither determine the price level nor the nominal supply of money (McCallum, 1986). In this case, alternative price level sequences will be consistent with given paths for the nominal money stock.[11] In a regime of strict interest rate targeting, however, the standard 3-equation NK model does allow for the price level and real money balances to be determined by the money demand equation and the Fisher equation. Money is irrelevant only because the NK model lacks a deterministic path for the nominal money stock—and hence the price level. In fact, Walsh (2010, p. 460) shows that "there exists a path for the nominal money supply . . . that leads to the same real equilibrium under an interest rate peg as would occur with a flexible price regime". But again, this precludes the true specification of money stock determination. An interest rate targeting regime is simply a special case in a continuum of endogenous monetary policy regimes.

2.3 The behavior of money demand is well-defined

Going as far back as 1900, Ireland (2009) and Lucas (2000) illustrate a strikingly stable money demand relationship. This relationship holds for more recent (post-1980) periods as well (Walsh, 2010). Figures 1 and 2 highlight this relationship in the U.S. data. The fundamental implication within the context of a model of money stock determination relates to the relevance of bank reserves

policy rate rather than exogenous policy actions. That is, a central bank "smooths the transition of the policy rate to the new equilibrium required by economic shocks" (*ibid*, p. 475). More recently, Coibion and Gorodnichenko (2012) find strong evidence in favour of interest rate smoothing over persistent monetary policy shocks.

[9]Specifically, in Hetzel's model, the IS curve shifts with unanticipated price changes as reflected in the output gap (p. 2).

[10]Fisher (1928, p. 4) defines money illusion as "the failure to perceive that the dollar, or any other unit of money, expands or shrinks in value". That is, money illusion follows from an incorrectly measured (or perceived) change in the depreciation of money (Rhodes, 2008). More recently, Miao and Xie (2013, p. 84) define money illusion as "the phenomenon where people confuse nominal with real magnitudes...to account for the short-run non-neutrality of money". Patinkin (1965) summarizes it as "any deviation from decision making in purely real terms."

[11]In fact, Carlstrom and Fuerst (2001) show that, because of multiple pricing equations for the nominal interest rate, "seemingly minor modifications in the trading environment result in dramatic differences in the policy restrictions needed to ensure real determinacy." They go on to caution that policymakers should be aware that a lot depends on basic assumptions about the modelling environment in monetary models.

(Borio and Disyatat, 2010, p.73–80). The critique against the relevance of monetary aggregates is usually based on the empirical regularity that no clear and stable link exists between liquidity (reserves) and interest rates. Specifically, country's that do not employ a reserve regime, can implement the so-called 'decoupling principle' (Borio and Disyatat, 2010, p. 55–57). As a result, various levels of reserves can exist for a given interest rate. This empirical regularity, however, is somewhat different from the 'decoupling' hypothesis which allows for a 'two instrument-two goal' operational framework. Ireland (2014, p. 1301) sums it up as follows:

> Thus, although the extra degree of freedom does allow the central bank to target the short-term nominal interest rate and the real quantity of reserves simultaneously, the model shows that monetary policy actions intended to bring about long-run changes in the aggregate price level must still be accompanied by proportional changes in the nominal supply of reserves.

This means that any monetary policy operation which fixes the price of short-term debt (by, e.g., paying interest on reserves (IOR)) can remove the liquidity effect altogether *in the market for reserves* (see, e.g., Figure 3). In fact, Ireland (2014, p. 1301) finds that to bring about a 25 bp increase in the short-term interest rate both the *size* and *sign* of reserves differ from the liquidity effect that would arise under a 'traditional' reserve regime. A large *increase* in the balance sheet arises in the short-run because of the simultaneous effect of the market rate on households' demand for deposits and banks' demand for reserves . And as Fama (2013, p. 180) points out, "there is no conclusive evidence (here or elsewhere) on the role of the Fed versus market forces in the long-term path of interest rates." Ireland (2014) goes on to the show that, although IOR dramatically alters the endogenous response of reserves, in the long-run "a monetary policy action that decreases [increases] the price level always requires a proportional reduction [expansion] in the nominal supply of reserves ..." (p. 1303). That is, the short-run versus long-run dichotomy in the literature raises some concern over the long-run efficacy of a 'decoupling' policy framework. In short, the Fisher effect matters (see also, Cochrane, 2014; Hummel, 2013).[12]

Figures 3 and 4 show the relationship between reserves and the short-term nominal interest rate for quarterly U.S. data from 1959Q1 to 2007Q3. Non-borrowed reserves show no indication of a relationship with the interest rate. This is an unsurprising result given that for much of this

[12] As an interesting side note, Cochrane (2014, p. 78) emphasises the fiscal theory of the price level: "In this way, the Treasury and the Fed acting together do, in fact, institute a system in which the government as a whole sets the interest rate i_{t-1} and then sells whatever facevalue of the debt B_{t-1} that [is demanded] ... even though the Fed does not directly change the overall quantity of debt, and even though the Treasury seems to sell a fixed quantity, not at a fixed price." It is therefore quite possible that our framework implicitly assumes a "passive" fiscal policy—Leeper's (1991; 2016) "Regime M" (see also, Cochrane, 2014, p. 91).

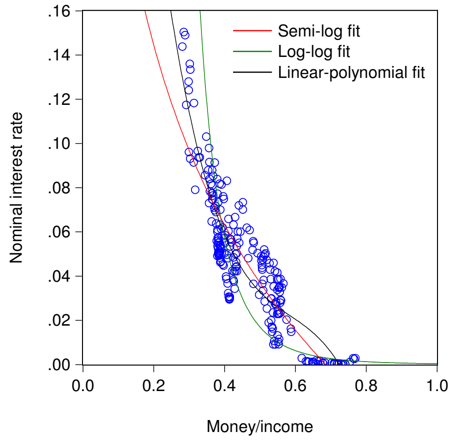

Figure 1: US Money Demand. Sample: 1959Q1−2016Q03.

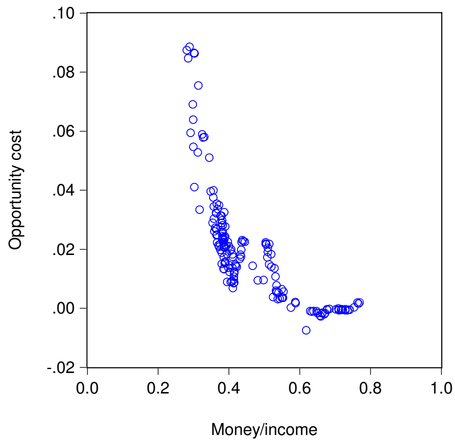

Figure 2: US Money Demand. Sample: 1975Q1−2016Q03. (Opportunity cost incl. MZM own rate)

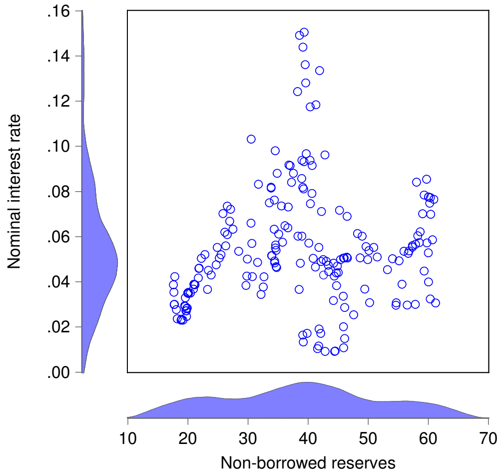

Figure 3: US Reserve Demand: Non-Borrowed Reserves (NBR). Sample: 1959Q1−2007Q03.

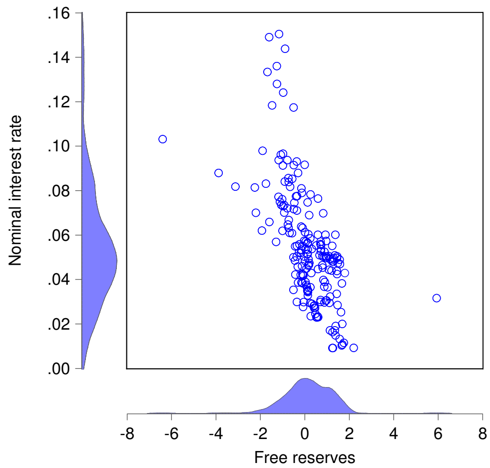

Figure 4: US Reserve Demand: Free Reserves (FR). Sample: 1959Q1−2007Q03.

period the Fed operated a *de facto* interest rate targeting regime (Hetzel, 1986; Taylor, 1993). In fact, the only extended period showing a clear negative log-linear relationship between NBR and the short-term nominal interest rate is between 1982Q3 and 1987Q1—a period in which the Federal Reserve followed a borrowed-reserves operating procedure (Walsh, 2010, p. 467).

Free reserves, on the other hand, approximate a downward sloping demand function with a Gaussian distribution for the entire period from 1959Q1 to 2007Q3 (see Figures A.1 and A.2).[13] The simple linear OLS regression gives:

$$FR = \underset{(0.15)}{1.60} - \underset{(2.40)}{27.05i} \ ,$$

with $R^2 = 0.40$ and standard errors in parentheses. In comparison, the semi-log OLS regression for money demand from Figure 1 gives:

$$ln(M/GDP) = -\underset{(0.02)}{0.54} - \underset{(0.27)}{5.54i} \ ,$$

with $R^2 = 0.69$ for the period 1959Q1−2007Q3. Of course, these results serve a descriptive purpose only; for more comprehensive analyses see, for example, Lucas (2000), Ireland (2009) and Walsh (2010).

3 The model economy

McCallum's (1981; 1986) two-equation, full employment IS-LM model with a money supply rule showed it was possible to peg the nominal interest with a money rule and obtain price determinacy. Hetzel (1986) extended McCallum (1986) to include a traditional banking sector for reserves. His model contains four key equations: a Fisher relation, a demand function for real money balances, a monetary rule, and a banking sector relationship between nominal money supply, the short-term market interest rate and bank reserves. Equations 1 to 4 represent these four equations as first-order

[13] There are only two notable outliers over the 48-and-a-half year period (-$5.2bn in 1984Q3 and $5.9bn in 2001Q3).

Taylor approximations around a deterministic steady-state:[14]

$$\text{Fisher relation} \quad : \quad i_t = E_t \pi_{t+1} + r_t \tag{1}$$

$$\text{Money demand} \quad : \quad m_t^d - p_t = \phi_y y_t - \phi_i i_t \tag{2}$$

$$\text{Monetary policy} \quad : \quad h_t = \rho_h h_{t-1} - \nu_h (i_t - \bar{i}) \tag{3}$$

$$\text{Money supply} \quad : \quad m_t^s = \frac{1}{rr}(\phi_h h_t - \phi_{fr} fr_t) \ , \tag{4}$$

where i_t, π_t and r_t are the nominal interest rate, inflation rate and real rate of interest; p_t, y_t, h_t and m_t denote the price level, output, bank reserves and the nominal money stock. ϕ_y and ϕ_i are the real income and the interest rate semi-elasticities of the demand for money, ρ_h a persistence parameter, and ν_h measures the degree to which the monetary authority smooths the nominal interest rate. rr is the reserve requirement ratio, where ϕ_h and ϕ_{fr} are the steady-state ratios of non-borrowed reserves and free reserves to the money stock.[15]

In the spirit of Benchimol and Fourçans (2012), Belongia and Ireland (2014) and Ireland (2014) we use the above approach to money stock determination to deviate from the traditional NK model with a Taylor-type monetary rule to include a monetary rule (3) and a money supply condition (4) which allows for alternative operational instruments. Specifically, ν_h captures the degree of interest rate smoothing enforced by the central bank. As $\nu_h \to \infty$ the money supply schedule becomes horizontal and we enter a monetary regime of interest rate targeting—either a 'pure' peg or a strict dynamic Taylor-rule.[16] Money and reserves become endogenous, and the reserve-money multiplier irrelevant to the determination of the money stock (Hetzel, 1986, pp. 5-6, 13, 17-18, 20). Under this type of regime the model reduces to the standard NK framework (Benchimol and Fourçans, 2012). As $\nu_h \to 0$ we enter into a 'pure' monetary aggregate targeting regime.[17]

The bank's decision problem for free reserves (fr_t) in an interest rate corridor or channel system

[14]Recently, Leeper (2016, p. 15) revisited a model of price-level determination that includes fiscal and monetary policy interaction. The model developed here could easily be extended to incorporate fiscal policy and the government budget, but under the assumption that fiscal policy is passive it is not necessary (Leeper's "Regime M:" monetary policy controls inflation and fiscal policy ensures government solvency). That said, Leeper's framework falls into the same trap identified by McCallum (1986, p. 156) in relation to Sargent and Wallace (1982), namely that the model "neglects the medium-of-exchange role of money, thereby negating the possibility of distinguishing between monetary and non-monetary assets." There is also the possibility of the *Sumner Critique*: monetary policy dominates fiscal policy if the central bank targets NGDP even in a model with sticky prices and interest rates sensitive money demand. See also posts on the *IS-LM+* model by Christensen 2012, 2013, 2016.

[15]Note: Eqs. (15) and (16) in Hetzel (1986, p. 10) imply a log-linear relationship between reserve demand schedule and reserve supply schedule.

[16]That is, by letting $\bar{i} = i_t^T$ follow some monetary policy reaction function that responds to inflation and output.

[17]Note: the reserve-money multiplier only becomes irrelevant as a result of the modelling assumption. It does not mean that the real demand for money (purchasing power) *determines* the quantity of nominal money (Tobin, 1963) (Hetzel, 1986, p. 19). Empirically, it depends on the degree that monetary aggregates (or reserve-money multipliers) become interest sensitive (elastic) (see also, Inagaki, 2009). An insensitive monetary aggregate results in a relevant and predictable reserve-money multiplier—and hence obtain price determinacy.

is based on Woodford (2001, p. 31) and Whitesell (2006) (see also, Walsh (2010, p. 544)). In this framework the net supply of settlement balances (free reserves) is zero in steady-state. As will be shown, this ensures that the effective fed funds rate hits the target policy rate in steady-state.

3.1 Households

We develop these features in the New-Keynesian tradition with an explicit role for money. The representative infinitely-lived household's utility is separable in consumption, money and leisure. The household maximizes its expected lifetime utility function given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{a}{1-\eta_c} (C_t)^{1-\eta_c} + \frac{(1-a)\xi_{m_d,t}}{1-\eta_m} (M_t^d / P_t)^{1-\eta_m} - \frac{\Psi}{1+\eta_l} (L_t)^{1+\eta_l} \right], \tag{5}$$

where β^t is the discount factor. Utility depends positively on the consumption of goods C_t, and negatively on the supply of labour hours L_t. Households' financial wealth is made up of risk-free bonds B_t and nominal money balances M_t^d. Similar to Van den Heuvel (2008), Christiano et al. (2010), Benchimol and Fourçans (2012) and Ireland (2014) we assume households derive direct utility from the liquidity services of money. This drives a positive wedge in the spread between the return on bonds and the own return on money (the opportunity cost of holding money). η_l measures the inverse of the elasticity of hours worked to the real wage. η_c captures the inverse of the intertemporal elasticity of substitution in consumption, and η_m measures the inverse of the interest rate semi-elasticity of money. $\xi_{m_d,t} = \exp(\varepsilon_t^{m_d})$ is an exogenous shock to money demand.

Eq. 6 gives the household budget constraint:

$$P_t C_t + M_t^d + Q_t B_t + P_t T_t \leq W_t L_t + B_{t-1} + M_{t-1}^d. \tag{6}$$

The household allocates periodic income from wages (W_t), risk-free bonds (B_{t-1}), and cash balances M_{t-1}^d to current consumption and new financial wealth holdings in the form of money and bonds. Q_t is the discount on one-period bond purchases such that the pay-off at maturity is the short-term nominal interest rate ($i_t = -\log Q_t$). T_t are lump-sum taxes net of transfers.

The representative household's first-order conditions for bonds, money and labour are the following:

$$U_{c,t} = \beta E_t\left[U_{c,t+1}\frac{(1+i_t)}{(1+\pi_{t+1})}\right], \tag{7}$$

$$U_{m,t} = U_{c,t} - \beta E_t\left[U_{c,t+1}\frac{1}{(1+\pi_{t+1})}\right], \tag{8}$$

$$\frac{W_t}{P_t} = \frac{U_{l,t}}{U_{c,t}}, \tag{9}$$

where π_t is the rate of inflation, $U_{c,t} = (C_t)^{-\eta_c}$ is the marginal utility of consumption, $U_{m,t} = (M_t^d/P_t)^{-\eta_m}$ is the liquidity services from holding real money balances, and $U_{l,t} = (L_t)^{\eta_l}$ is the marginal disutility of labour. Eq. 8 is the household's demand for real money balances. Eq. 9 gives the standard real wage equation: that is, the real wage equals the marginal rate of substitution between consumption and labour. Eq. 7 gives the consumption Euler equation, based on the standard asset-pricing equation for bonds.

Combining Eq. 7 and Eq. 8 illustrates the opportunity cost of holding money.

$$\frac{U_{m,t}}{U_{c,t}} = \frac{i_t}{(1+i_t)}. \tag{10}$$

Here Eq. 10 states that the marginal utility of the liquidity services, expressed in units of consumption, equals the opportunity cost of not investing money holdings in risk-free nominal bonds. If we consider measures of money other than currency in circultion, such that money earns some rate of return (for example, interest on excess reserves or the own rate of interest on M1, M2, MZM etc.) then Eq. 10 becomes:

$$\frac{U_{m,t}}{U_{c,t}} = \frac{(i_t - i_t^m)}{(1+i_t)}. \tag{11}$$

As will be seen, this will have important reserve-money multiplier effects. Furthermore, it is straightforward to impose non-separability between consumption and money in the utility function. In this case, the model can be derived as in Benchimol and Fourçans (2012, pp. 98–99).

3.2 Firms

Firms manage the goods producing sector, and are owned by households. Firm's behave optimally in a monopolistically competitive environment in which their objective is to maximize profits. In each period, only a fraction of firms $(1-\theta)$ can reset their prices. The aggregate price level then evolves as

$$(P_t) = \left[\theta(P_{t-1})^{1-\varepsilon^p} + (1-\theta)(\tilde{P}_t)^{1-\varepsilon^p}\right]^{\frac{1}{1-\varepsilon^p}}. \tag{12}$$

Firms produce goods using identical technology in the form of a standard Cobb-Douglas production function:

$$Y_t = \xi_{z,t} L_t^{\alpha}, \tag{13}$$

where L_t is the demand of labour hours, where $0 < \alpha < 1$ represents labour's decreasing returns to production. $\xi_{z,t} = \exp(\varepsilon_t^z)$ represents the exogenous technology identical to all firms.

3.3 The banking sector

3.3.1 A traditional model of the reserve market

The central bank has autonomy over the quantity of reserves supplied (3). But the money supply function is derived from the banking sectors demand for free reserves (4). Free reserves (FR) represent funds available for interbank clearing and settlement, interbank loans, as well as the portion of excess reserves (ER) less borrowed reserves (BR) allocatable to reserve requirements (RR) in the deposit (D) creation process (see also Norman et al., 2011). Assume that the central bank has a standing facility for borrowing at the discount window and that banks are required to hold reserves for a fixed reserve requirement ratio (rr). Required reserves ($RR = rrD$) and borrowed reserves (BR) are therefore not directly determined by the central bank. The central bank therefore directly determines non-borrowed reserves (H), the discount rate (i^d) and interest on (excess) reserves (i^{er}). Based on this model from Tinbergen (1939, 1951), Hetzel (1986) defines the relationship between nominal money (supply), the short-term interest rate and bank reserves:

$$\begin{aligned} FR &= H - RR = ER - BR & (14) \\ RR &= H - FR & (15) \\ RR &= rrD & (16) \\ FR &= f(i|i^{dr}, i^{er}) \ . & (17) \end{aligned}$$

Eq. 17 shows bank demand for free reserves as a function of the short-term nominal interest rate, given the spread between the discount rate on borrowed reserves and the interest on excess reserves. Deposits D equate with the nominal money supply M^s. Substituting Eqs. 16 and 17 into Eq. 15 gives the money supply function (4) for period $t = 1, 2, 3, \ldots$:

$$M_t^s = \frac{1}{rr}(H_t - FR_t) \ . \tag{18}$$

Non-borrowed reserves evolve over time t according to its supply schedule:

$$H_t = (H_{t-1})^{\rho_h} (\bar{H})^{(1-\rho_h)} \left(\frac{1+i_t}{1+\bar{i}} \right)^{-\nu_h}, \tag{19}$$

where $\bar{H}$ is the trend rate of growth of money and ρ_h determines the degree of persistence in reserve accumulation. With the elasticity of bank reserves $\nu_h \to 0$, the reserve-deposit (money) multiplier $(1/rr)$ determines the quantity of money stock created. If $\rho_h = 1$, Eq. 19 follows a random walk and independent changes to reserves are not offset. Furthermore, the market rate (i_t) equals the target rate $(\bar{i})$ in steady-state which implies that any level of reserves is independent from the interest rate.

3.3.2 The banks demand for free reserves in a corridor system

The representative bank is assumed to be risk neutral. In each period, the bank trades central bank deposit balances (free reserves) with other banks in a competitive interbank market at the market rate i. Free reserves are assumed to be subject to stochastic fluctuations ("margins of error") after the interbank market closes $(FR + \varepsilon)$.

The demand for clearing balances in the interbank market follows directly from the models of Woodford (2001) and Whitesell (2006). Given the discount rate on borrowed reserves and any interest paid on excess reserves, Eq. 20 expresses bank j's optimal (period t) demand for free reserves as a function of (1) the opportunity cost of holding a positive end-of-period reserve balance relative to lending them out in the interbank market interest rate $(i_t - i_t^{er})$, and (2) the opportunity cost holding a negative end-of-period reserve balance (overdraft) and having to borrow from the central bank rather than from the interbank market $(i_t^{dr} - i_t^{er})$:

$$F(-FR_t) = \left(\frac{i_t - i_t^{er}}{i_t^{dr} - i_t^{er}} \right), \tag{20}$$

where $F(\cdot)$ is the symmetric distribution of the reserve account shock. A symmetric distribution implies that $i^{er} = (\bar{i} - s)$ and $i^{dr} = (\bar{i} + s)$: a ceiling and floor around the target interest rate $\bar{i}$. With full information, the bank sets their desired level of period reserves $FR^* = -\varepsilon$, where $FR - E(FR) = \varepsilon$ is the end-of-day stochastic "margin of error" and where $E(\varepsilon) = 0$.[18] As a result, net settlement balances at the central bank are zero $(FR = 0)$ and $i = \bar{i}$ (Whitesell, 2006, p. 1180). Notice that this represents a strict interest rate targeting regime in Eq. 19, where $\nu_h \to \infty$—the equilibrium point where reserves become irrelevant for the determination of the money stock.

Following Woodford (2001) and Whitesell (2006), it is further assumed that $F(\cdot)$ is a cumulative

[18] The banks funding costs are therefore minimized at $-i\varepsilon$.

standard normal distribution function $N(\cdot)$ with variance σ^2.[19] Summing over all banks, indexed by j, gives the aggregate demand for reserves (depicted in Figure 1, Woodford (2001, p. 33); Figure 11.2 Walsh (2010, p. 546)):

$$FR_t = \sum_j FR_t(j) = -N^{-1}\left(\frac{i_t - i_t^{er}}{i_t^{dr} - i_t^{er}}\right)\sum_j \sigma_j = H_t - RR_t \ , \tag{21}$$

where $\sum \sigma_j$ captures the degree of uncertainty of (private) banks. Given the spread s, the function $N^{-1}(\cdot)$ can be re-written as:

$$N^{-1}\left(\frac{1}{2} + \frac{i_t - \bar{i}_t}{2s}\right)$$

Whitesell (2006, p. 1181) highlights two important characteristics of greater uncertainty in the market for reserves: (1) on the demand side, interbank uncertainty leads to interest rate smoothing (i.e., a flattening of the the demand curve for reserves); and (2) on the supply side, central bank uncertainty in reserve supply raises the volatility of interest rates. The larger the ratio of central bank uncertainty to private bank uncertainty, the fatter the tails of the resulting distribution of overnight interest rates (*ibid.*, p. 1182). We can approximate the demand for free reserves over time t according to

$$FR_t = (FR_{t-1})^{\rho_{fr}}(\bar{FR})^{(1-\rho_{fr})}\left(\frac{i_t - i_t^{er}}{i_t^{dr} - i_t^{er}}\right)^{-\nu_{fr}} \ , \tag{22}$$

where ν_{fr} determines the interest elasticity of free reserves (or the degree of interest rate smoothing in the interbank market for reserves). Under a strict interest rate peg the target rate $(\bar{i})$ is the market rate (i_t) and the central bank saturates the interbank market with reserves to narrow the width of the corridor until the elasticity of demand for reserves is infinite (ν_h and $\nu_{fr} \to \infty$). In this case, free reserves are irrelevant, along with non-borrowed reserves, for the determination of money supply.

Not only does a higher elasticity of reserve demand (flatter demand curve at equilibrium—near the target interest rate) occur with a narrower corridor, but with greater reserve balance uncertainty (Whitesell, 2006, p. 1181). And as originally indicated by Poole (1968), a higher elasticity of reserve demand essentially means a wider dispersion of reserve balances. It is important to note that ν_{fr} only captures the *sensitivity* of reserves to market rate changes. We therefore allow a degree of persistence ρ_{fr} to free reserve accumulation. $\rho_{fr} = 0$ implies that independent changes to free reserves are offset around some constant level of free reserves or constant trend growth. If $\rho_{fr} = 1$ free reserves follow a random walk. A degree of persistence $0 < \rho_{fr} < 1$ therefore captures the

[19] In fact, both free reserves and the nominal interest rate are well approximated by normal distribution (see Figures A.1 and A.2 in the Appendix). Further, the ratio of two Gaussian (normal) distributions follows a Cauchy distribution—illustrated by the Kernel Densities.

speed of mean reversion of free reserves which acts as a proxy for precautionary adjustments of free reserves to interest rate changes. This means that the demand for free reserves need not respond immediately to aggregate uncertainty implied by σ_{fr}.

As noted by Hetzel (1986, p. 12), any changes in reserve demand typically derive from credit expansion in a fractional reserve system.[20] Equating money supply with money demand we get the following expression for equilibrium in the market for reserves:[21]

$$\underbrace{H - FR}_{\substack{\text{reserve supply}\\ \text{schedule}}} = \underbrace{rr\left[\left(\frac{i}{1+i}\right)^{-\frac{1}{\eta_m}} (P_t C_t)^{\frac{\eta_c}{\eta_m}}\right] P_t^{\left(\frac{\eta_m - \eta_c}{\eta_m}\right)}}_{\substack{\text{reserve demand}\\ \text{schedule}}} \tag{23}$$

The money supply schedule is upward sloping because a higher interest rate spread between i_t and i_t^{dr} produces a lower level of free reserves and a higher level of borrowed reserves (i.e., excess reserves fall). The rise in (borrowed) reserves accommodates monetary expansion. Conversely, reserve demand is downward sloping and relates to households' demand for *real* money balances. In this sense, $\eta_m \neq \eta_c$ produces money illusion, whereas, $\eta_m = \eta_c$ implies that the determinants of the nominal quantity of reserves (and money supply) are the same is the determinants of real money demand. An excess supply of money commensurate with an excess demand for goods and services therefore leads to a one-for-one increase in the steady-state price level.

3.4 DSGE model

The usual market clearing conditions ensure $Y_t = C_t$, $M_t^s = M_t^d$ and $B_t = 0$. We now can derive the Hetzel (1986) framework Eqs. 1 to 4. For simplicity, all equations are expressed as first-order Taylor approximations around steady-state.

3.4.1 Real money demand and the velocity of money

The money demand equation (10) can be expressed in first-order Taylor approximation form as:

$$m_t^d - p_t = \frac{\eta_c}{\eta_m} c_t - \frac{1}{\eta_m} i_t \ , \tag{24}$$

where, for now, we've ignored the exogenous money demand shock $\xi_{m_d,t}$. Notice that after we impose market clearing conditions in equilibrium ($c_t = y_t$ and $m_t^d = m_t^s = m_t$), Eq.24 gives Eq.2 above, where $\phi_y = \eta_c/\eta_m$ and $\phi_i = 1/\eta_m$. Given the equation of exchange for velocity:

[20] Given Eq.17 and the accounting link between assets and liabilities.
[21] $RR = rrD = rrM^s = rrM^d$.

$v_t = p_t + y_t - m_t$, we can re-write Eq.2 as follows:

$$m_t^d - p_t = \phi_y y_t - \phi_i i_t \tag{25}$$
$$m_t^d = p_t + y_t - y_t + \phi_y y_t - \phi_i i_t \tag{26}$$
$$m_t^d + \phi_i i_t + y_t - \phi_y y_t = p_t + y_t \tag{27}$$
$$\underbrace{\phi_i i_t + (1 - \phi_y) y_t}_{\text{velocity:} v_t} = p_t + y_t - m_t \ . \tag{28}$$

We estimate the model for parameters ϕ_i and ϕ_y and determine the robustness of the estimates to the literature on interest and income semi-elasticities and velocity of money dynamics over the business cycle.

3.4.2 The Fisher relation

The first order condition for bonds (7) follows as

$$r_t = \eta_c (E_t[y_{t+1}] - y_t) \ . \tag{29}$$

Similarly, the flexible price equilibrium follows as

$$r_t^n = \eta_c (E_t[y_{t+1}^n] - y_t^n) \ , \tag{30}$$

which combines to give:

$$r_t = r_t^n + \eta_c (E_t[\tilde{y}_{t+1}] - \tilde{y}_t) \ . \tag{31}$$

where $\tilde{y}_t = y_t - y_t^n$ is the output gap. Here, y_t^n is the natural level of output commensurate with flexible prices and wages. Importantly, this version of the output gap is not the efficient level of output—markets are still imperfect (Vetlov et al., 2011, p. 10).[22]

The Fisher relationship (5) can then be re-written, using (31), as

$$i_t = E_t[\pi_{t+1}] + [r_t^n + (r_t - r_t^n)] \tag{32}$$
$$i_t = E_t[\pi_{t+1}] + [r_t^n + \eta_c(E_t[\tilde{y}_{t+1}] - \tilde{y}_t)] \tag{33}$$

In Hetzel (1986), $r_t^n = b_0 + v_t$ and $\eta_c(E_t[\tilde{y}_{t+1}] - \tilde{y}_t)] = b_1[p_t - E(p_t|I_{t-1})]$, the latter being unanticipated price realisations (analogous to output gap changes), where $b_1 < 0$. v_t represents

[22]This means that, although there is no price stickiness, the steady-state markup and markup shocks are still non-zero (see also, Hetzel, 2015).

an exogenous real shock that shifts the supply curve (i.e., both output and its flexible price (natural output) equivalent change in response to technology shocks). Furthermore, in Hetzel (1986), unanticipated price changes produce the necessary short-run trade-off between inflation and unemployment. For our rational expectations model, the short-run New-Keynesian Phillips curve derived from the firm's decision problem achieves the same end.

3.4.3 A monetary rule for money stock determination

The linearized nominal money supply Eq. 18 follows as

$$\begin{aligned} m_t^s &= \frac{1}{rr}[\frac{H}{M^s}h_t - \frac{FR}{M^s}fr_t] \\ m_t &= h_t + \frac{1}{rr}[\frac{FR}{M}(h_t - fr_t)] \ , \end{aligned} \tag{34}$$

where the monetary rule (3) is defined in terms of a reserve aggregate. Specifically, non-borrowed reserves (h_t) evolve according to its linearized supply schedule:

$$h_t = \rho_h h_{t-1} - \nu_h(i_t - \bar{i}) + \varepsilon_{h,t} \ , \tag{35}$$

where $\nu_h > 0$ determines the degree of interest rate smoothing. If i_t is used as the monetary authority's operational instrument ($\nu_h \to \infty$), then the reserve-money multiplier is irrelevant to determination of the money stock. In this case, the monetary authority can peg the policy rate at some constant rate $\bar{i}$, or follow a dynamic Taylor-type rule $\bar{i} = i_t^T = f(i_{t-1}^T, \tilde{y}_t, \pi_t, \varepsilon_{i,t})$.[23]

The demand for free reserves follows from Eq. 22 as:

$$fr_t = \rho_{fr} fr_{t-1} - \frac{\nu_{fr}}{s}(i_t - \bar{i}_t) \ . \tag{36}$$

Notice that the symmetric spread (s) serves as a "slackness" parameter in the corridor system. For example, if we assume that $\nu_h = \nu_{fr}$, a narrower (wider) spread raises (lowers) the effective elasticity of free reserves relative to non-borrowed reserves. That is, a narrower spread implies a stricter interest rate peg, a flatter demand curve for free reserves, and wider the dispersion of reserves. From 2003, $s = 0.01$ (Whitesell, 2006, p. 1179); August 17, 2007 $s = 0.005$ and March 18, 2008 $s = 0.0025$ (Walsh, 2010, p. 534).

[23] We will use this monetary rule to emulate the pre- and post-2008 GFC regimes. That is, between 1984 and 2007 the effective fed funds rate closely followed a Taylor-type rule, whereas post 2008, the effective fed funds rate was essentially pegged due to the zero lower bound.

3.4.4 System of linearized equations

Equations 28, 30, 31, 33, 34, 35, 36, plus the New-Keynesian Phillips curve: $\pi_t = \beta\pi_{t+1} + \tilde{\kappa}\tilde{y}_t$, the output gap: $\tilde{y}_t = y_t - y_t^n$, natural (flexible price equilibrium) output: $y_t^n = (1+\eta_n)/(\eta_c + \eta_n)\xi_{z,t}$, and a definition for inflation: $\pi_t = p_t - p_{t-1}$. We also assume that the policy rate target follows a Taylor-type rule, which therefore gives 12 equations and 12 endogenous variables, excluding exogenous shock processes:

$$\text{Fisher relation} : i_t = E_t[\pi_{t+1}] + [r_t^n + \eta_c(E_t[\tilde{y}_{t+1}] - \tilde{y}_t)] \tag{37}$$

$$\text{Money demand} : m_t - p_t = \frac{\eta_c}{\eta_m} y_t - \frac{1}{\eta_m} i_t + \xi_{m_d,t} \tag{38}$$

$$\text{Consumption Euler equation} : r_t = r_t^n + \eta_c(E_t[\tilde{y}_{t+1}] - \tilde{y}_t) \tag{39}$$

$$\text{Natural rate} : r_t^n = \eta_c(E_t[y_{t+1}^n] - y_t^n) \tag{40}$$

$$\text{Money supply} : m_t = h_t + \phi_{rr}(h_t - fr_t) + \xi_{m_s,t} \tag{41}$$

$$\text{Non-borrowed reserve supply} : h_t = \rho_h h_{t-1} - \nu_h(i_t - \bar{i}) \tag{42}$$

$$\text{Free reserve demand} : fr_t = \rho_{fr} fr_{t-1} - \frac{\nu_{fr}}{s}(i_t - \bar{i}_t) \tag{43}$$

$$\text{Policy target rate} : i_t^T = \rho_i i_{t-1}^T + (1-\rho_i)(\kappa_\pi \pi_t + \kappa_y \tilde{y}_t) + \epsilon_t^i \tag{44}$$

$$\text{New-Keynesian Phillips curve} : \pi_t = \beta E_t[\pi_{t+1}] + \tilde{\kappa}\tilde{y}_t \tag{45}$$

$$\text{Output gap} : \tilde{y}_t = y_t - y_t^n \tag{46}$$

$$\text{Natural output} : y_t^n = (1+\eta_n)/(\eta_c + \eta_n)\xi_{z,t} \tag{47}$$

$$\text{Inflation definition} : \pi_t = p_t - p_{t-1} \ , \tag{48}$$

where $\phi_{rr} = \frac{FR}{rrM} = \frac{FR}{RR}$. Of course, the general equilibrium can be simplified further to a system of 7 equations and 7 observables. In this case, we would have Hetzel (1986)'s 4-equation model with endogenous equations for free reserves and the target policy rate, and a short-run NK Phillips curve instead of unanticipated price changes. For the discussion of the results, we choose to expand the number of variables.

Corresponding to the 4-equation model in Hetzel (1986) (Eqs. 1−4), we capture four exogenous sources of shocks to the economy: a money demand shock $\xi_{m_d,t}$, a money supply shock $\xi_{m_s,t}$, a technology shock $\xi_{z,t}$ and an interest rate target shock ϵ_t^i. Notice that we exclude the standard price-markup shock in the New-Keynesian Phillips curve, which implies that that firm price markups are constant. The idea here is to show that the interaction between money supply and demand and the policy rate target—i.e., nominal shocks—generate sufficient shifts in aggregate demand to account for business cycle fluctuations. As will be shown, the sticky price equation is still key to generating real effects or 'money illusion' in a rational expectations framework. A non-separable utility function as in Benchimol and Fourçans (2012) would ensure monetary non-neutrality without sticky pricing. But the point of this paper is to show that the standard New-Keynesian framework—with monetary neutrality and sticky prices—assumes a special case in a continuum of monetary regimes. As such, we show that the type of monetary regime significantly alters the transmission mechanism of shocks through the economy.

4 Estimation

The model is estimated by Bayesian methods over the period 1959Q1−2007Q3. The estimation period is chosen for two reasons: first, we want to simulate the counterfactual scenario of a reduction in free reserves given the estimated structure of the model economy prior to the onset of the global financial crisis and the structural break in free reserves in 2007Q4. Second, the long sample period serves to highlight the empirical and theoretical coherence of the model. We also compare our results with an estimated version of the model over the period 1984Q1−2007Q3, corresponding to the Great Moderation. We set prior parameter values and distributions of the model to fit the U.S. economy based on Smets and Wouters (2007), Ireland (2009) and Walsh (2010). All persistence parameters are set to 0.8 with standard deviations of 0.02. We use U.S. data obtained from St. Louis FRED database over the period 1959Q1 to 2007Q3 to calibrate the relevant steady-state ratios for the banking sector and to estimate the model. The discount factor $\beta = (1+r)^{-1}$ is fixed at 0.98, corresponding to a steady-state quarterly real interest rate of 2%. The output gap, inflation, money and the nominal interest rate are treated as observables, linearly detrended following Benchimol and Fourçans (2012):

π_t yearly (quarter-on-quarter) log-difference of GDP implicit price deflator

$\tilde{y}_t$ difference between the log of real GDP per capita and real potential GDP per capita

m_t log-difference of MZM money stock per capita

i_t short term (3-month) nominal interest rate .

Table 1 reports the prior distribution, means, and standard deviations, as well as the posterior means, medians and confidence intervals of the estimated parameters.[24] The estimated structural parameters for households and firms are stable across both estimation periods. The parameters

[24] All diagnostic statistics and estimation results are available upon request.

characterising the monetary regime show that both free reserves and non-borrowed reserves are highly elastic over the Great Moderation period. Whereas, non-borrowed reserves have a greater influence over the money stock in the full-sample estimate. This finding corresponds well with the evolution of the Federal Reserves monetary operating procedures towards an interest rate targeting regime (Walsh, 2010, pp. 547-553). As will be seen, given this slant toward an (intermediate) interest rate target since 1959Q1, money demand shocks largely "determine" the price-level, which is highly persistent (e.g., Figure 10).[25] Innovations to the target policy rate can be best described as a highly smoothed AR(1) process. As such, cyclical fluctuations to the short-term nominal interest rate is largely determined endogenously through money demand and supply and technology shocks.

[See Eggertsson and Singh (2016) for estimating log-linear models at the ZLB.]

[25]By "determine" we mean that money supply accommodates demand at the given interest rate.

Table 1: Bayesian estimation of structural parameters

						1959Q01−−2007Q03				1984Q01−-2007Q03		
		Prior distribution				Posterior distribution				Posterior distribution		
	Parameter	Type	Mean	Std.dev	Mean	2.5%	Median	97.5%	Mean	2.5%	Median	97.5%
Households												
η_c	Relative risk aversion	Normal	2	0.50	4.539	4.080		5.119	3.635	3.045		4.174
η_m	Inverse elasticity of money demand	Normal	5	0.20	5.024	4.675		5.349	4.920	4.619		5.237
η_l	Inverse elasticity of labour supply	Normal	1	0.10	0.983	0.674		1.311	0.980	0.610		1.296
β	Discount factor		0.98									
Firms												
α	Labour's share in production	Beta	0.67	0.05	0.672	0.594		0.750	0.675	0.590		0.750
θ	Price stickiness	Beta	0.75	0.05	0.881	0.867		0.895	0.873	0.852		0.895
Monetary regime												
ν_h	Elasticity of non-borrowed reserves	Inv.Gamma	1	10	2.427	0.272		5.750	11.374	5.935		17.977
ν_{fr}:	Elasticity of free reserves	Inv.Gamma	10	10	55.108	35.742		76.365	12.239	8.156		17.861
ρ_h	Nonborrowed reserve persistence	Beta	0.8	0.10	0.638	0.505		0.770	0.437	0.366		0.522
ρ_{fr}	Free reserves persistence	Beta	0.8	0.10	0.175	0.111		0.234	0.201	0.130		0.269
κ_π	Weight on inflation	Gamma	1.5	0.20	1.428	1.115		1.726	1.500	1.208		1.814
κ_y	Weight on output gap	Beta	0.5	0.20	0.559	0.233		0.885	0.529	0.208		0.853
FR/RR	Ratio of free reserves to req. reserves				0.003				0.017			
AR(1) coefficients												
ρ_z	Technology	Beta	0.8	0.10	0.745	0.711		0.777	0.763	0.722		0.804
ρ_i	Interest rate target	Beta	0.8	0.10	0.998	0.996		1.000	0.996	0.992		0.999
$\rho_m s$	Money supply	Beta	0.8	0.10	0.835	0.804		0.868	0.770	0.723		0.818
$\rho_m d$	Money demand	Beta	0.8	0.10	0.999	0.998		1.000	0.992	0.986		0.999
Standard deviations												
ϵ_z	Technology	Inv.Gamma	0.02	2.00	0.066	0.056		0.075	0.038	0.031		0.044
ϵ_i	Interest rate target	Inv.Gamma	0.02	2.00	0.003	0.003		0.004	0.005	0.004		0.005
ϵ_{ms}	Money supply	Inv.Gamma	0.02	2.00	0.172	0.130		0.213	0.079	0.059		0.101
ϵ_{md}	Money demand	Inv.Gamma	0.02	2.00	0.034	0.031		0.037	0.018	0.016		0.020
Log-data density					2200.48				1243.000			
Acceptance ratio range					[24%;27%]				[22%;25%]			
Observations					195				95			

5 Empirical findings for the U.S. business cycle

5.1 Impulse response functions

Figures 5 and 6 show the impulse responses to a technology shock, an interest rate target shock, a money supply shock and a money demand shock for the full sample period (1959Q1−2007Q3) and the Great Moderation period (1984Q1−2007Q3). Two overall observations are highlighted. Firstly, for all four shocks the dynamic responses of the variables are closely consistent across both sample periods. Secondly, nominal money balances and the degree of price stickiness consistently determine the dynamic adjustment of the price-level. Therefore, to make our discussion concise, we will focus on the dynamics of the full sample.

The nominal money supply shock (top panel, Fig. 5) highlights the effect of sticky prices on real variables. An initial 1.63 pp increase in the money supply results in a 0.95 pp increase in the price-level only after 6 quarters. As a result, the monetary stimulus pushes the real interest rate down 1.3% and generates a cumulative positive output gap of 4 percentage points. A money demand shock (bottom panel, Fig. 5), on the other hand, impacts the economy negatively as households demand higher *real* money balances. To this effect, prices fall below trend as households substitute away from consumption goods to money. This negative demand shock is somewhat offset by a rise in the nominal stock of money. In the flexible equilibrium, the price level would adjust downward to immediately satisfy the increase in demand for real money balances (see Figure B.6).

A positive technology shock (top panel, Fig. 6) generates greater output, lower inflation and a negative output gap. The downward adjustment of the nominal interest rate is small and the economy converges from an initial negative output gap of 2 pp to its flexible price equilibrium after 8 quarters. The net effect on real money balances is positive. A positive shock to the target interest rate (bottom panel, Fig. 6) follows a standard New-Keynesian monetary policy shock. A 21 bp increase in the short-term nominal interest rate reduces output by 1.1 pp and inflation by 0.64%. The higher interest rate reduces real money balances (Eq. 24) and generates a persistent decline in both nominal money supply and the price-level.

5.2 Variance decomposition

Figure 7 reports the contribution of the structural shocks to the forecast error variance of money, velocity, inflation, the output gap and the nominal interest rate up to a 20-quarter horizon.

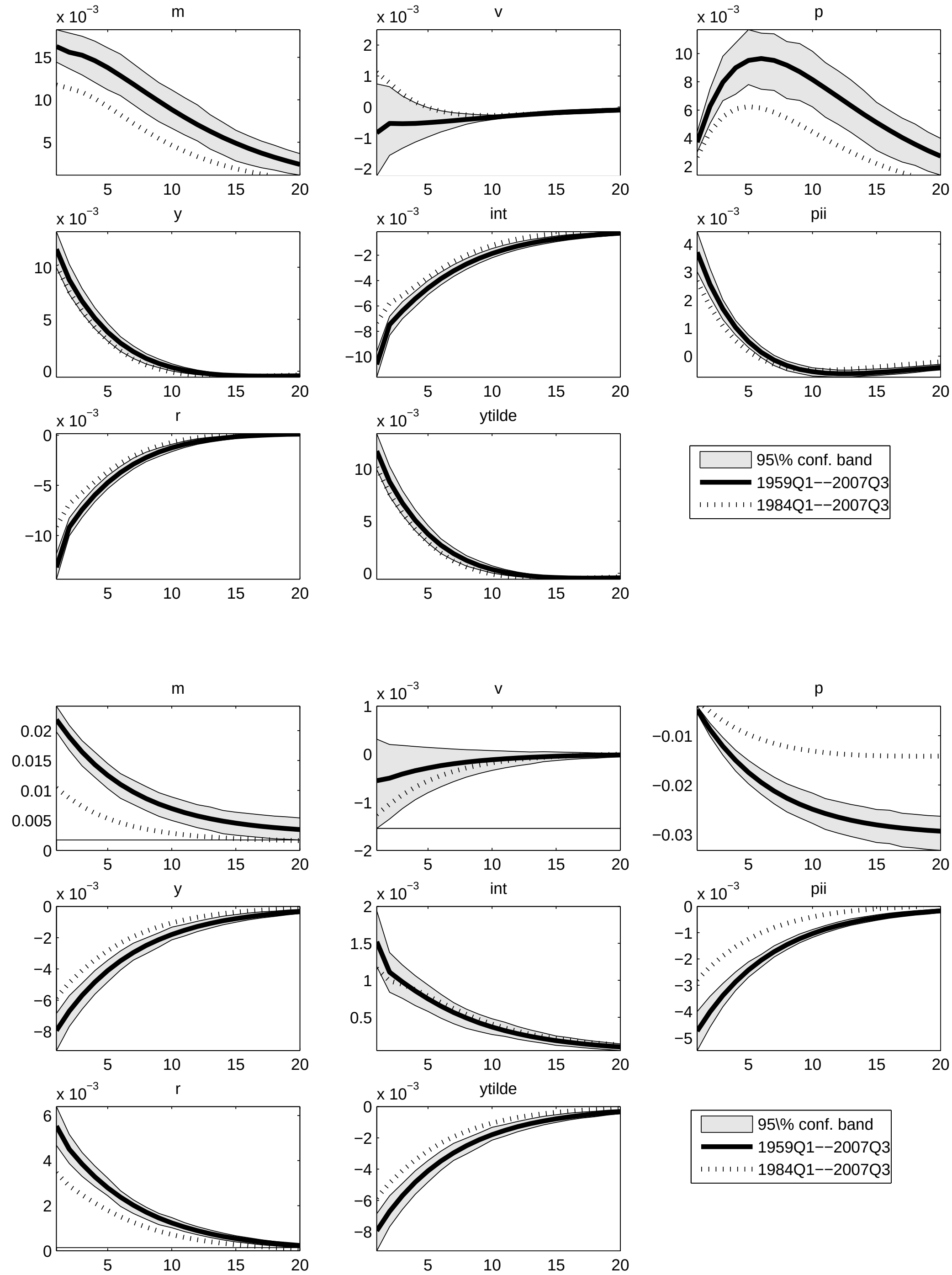

Figure 5: Impulse response to positive money supply shock (top) and positive money demand shock (bottom).

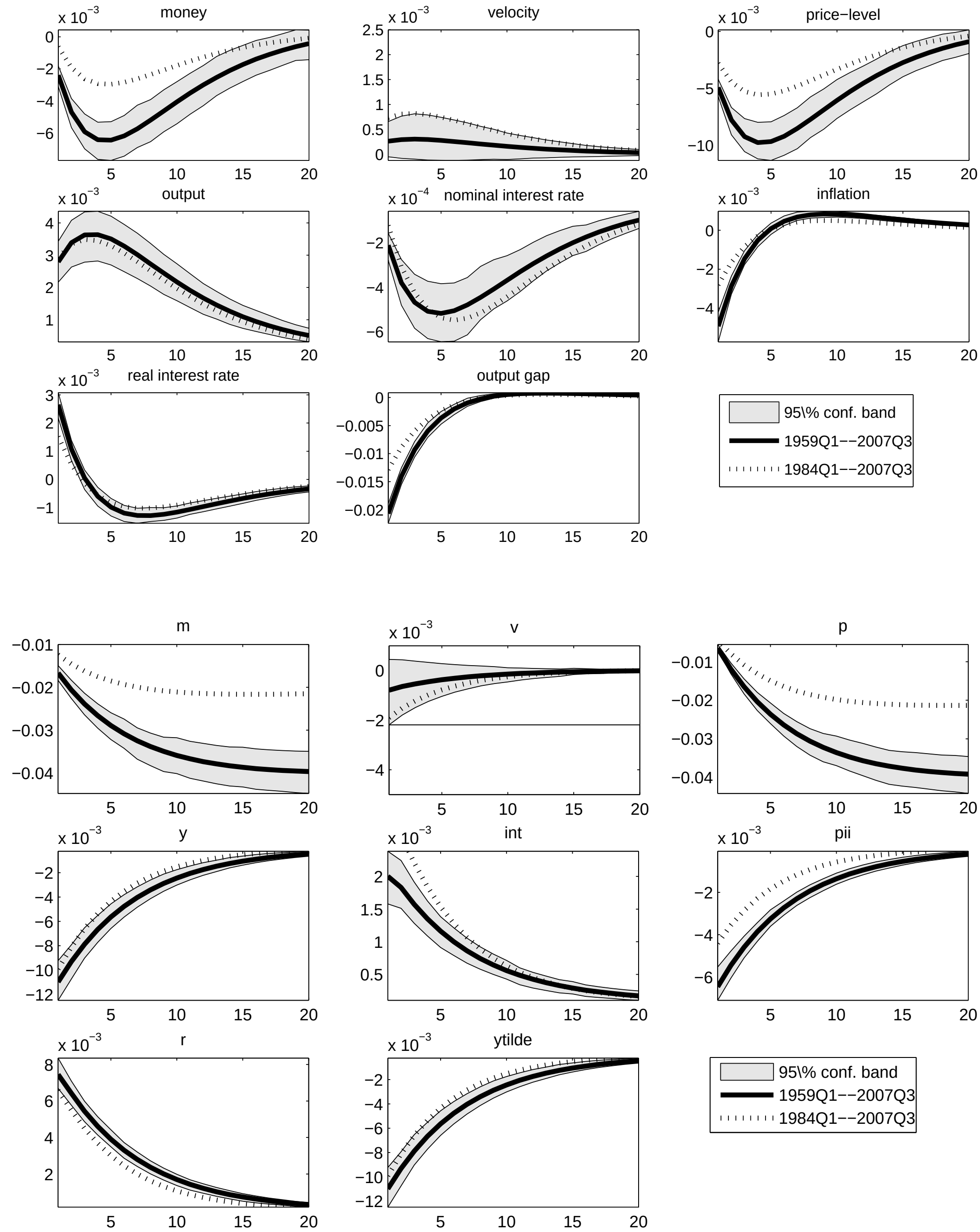

Figure 6: Impulse response to positive technology shock (top) and positive interest rate target shock (bottom).

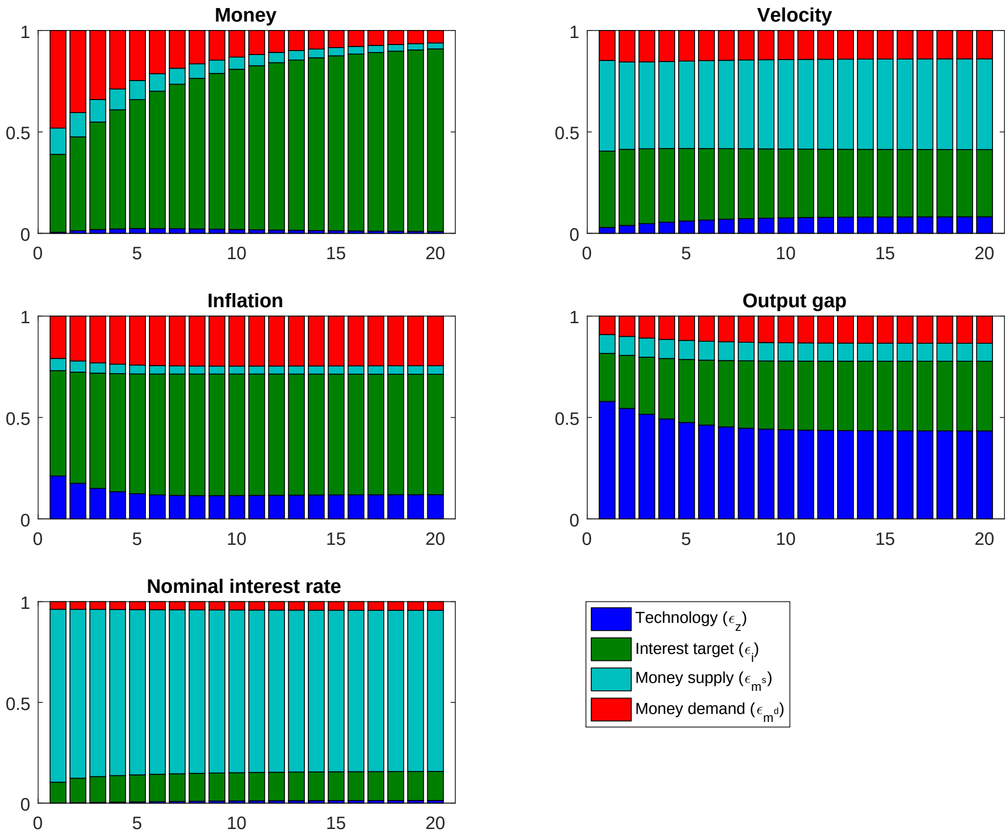

Figure 7: Variance decomposition (1959Q1−2007Q3). Money (top-left); Velocity (top-right); Inflation (middle-left); Output (middle-right); Nominal interest rate (bottom-left).

5.3 Historical decomposition

Figures 8 to 11 provide the historical shock decomposition of the main macroeconomic variables. Here, we focus on how the structural shocks predict the U.S. business cycle over the sample period 1959Q1−2007Q3.

[TBC]

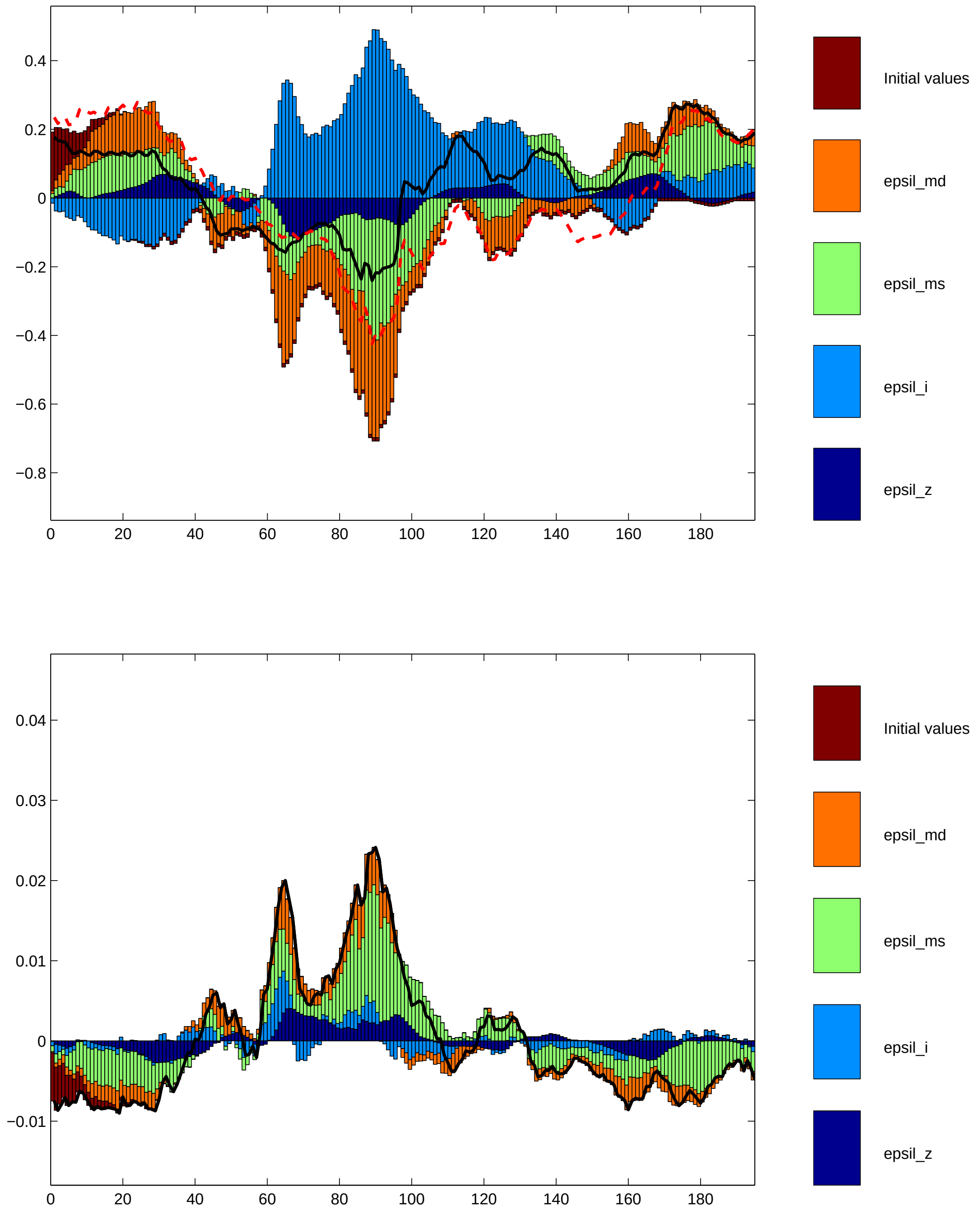

Figure 8: Historical decomposition (1959Q1−2007Q3): Money: MZM (top panel); Velocity (bottom panel). Dashed red line: actual data for the log of money-to-output ratio

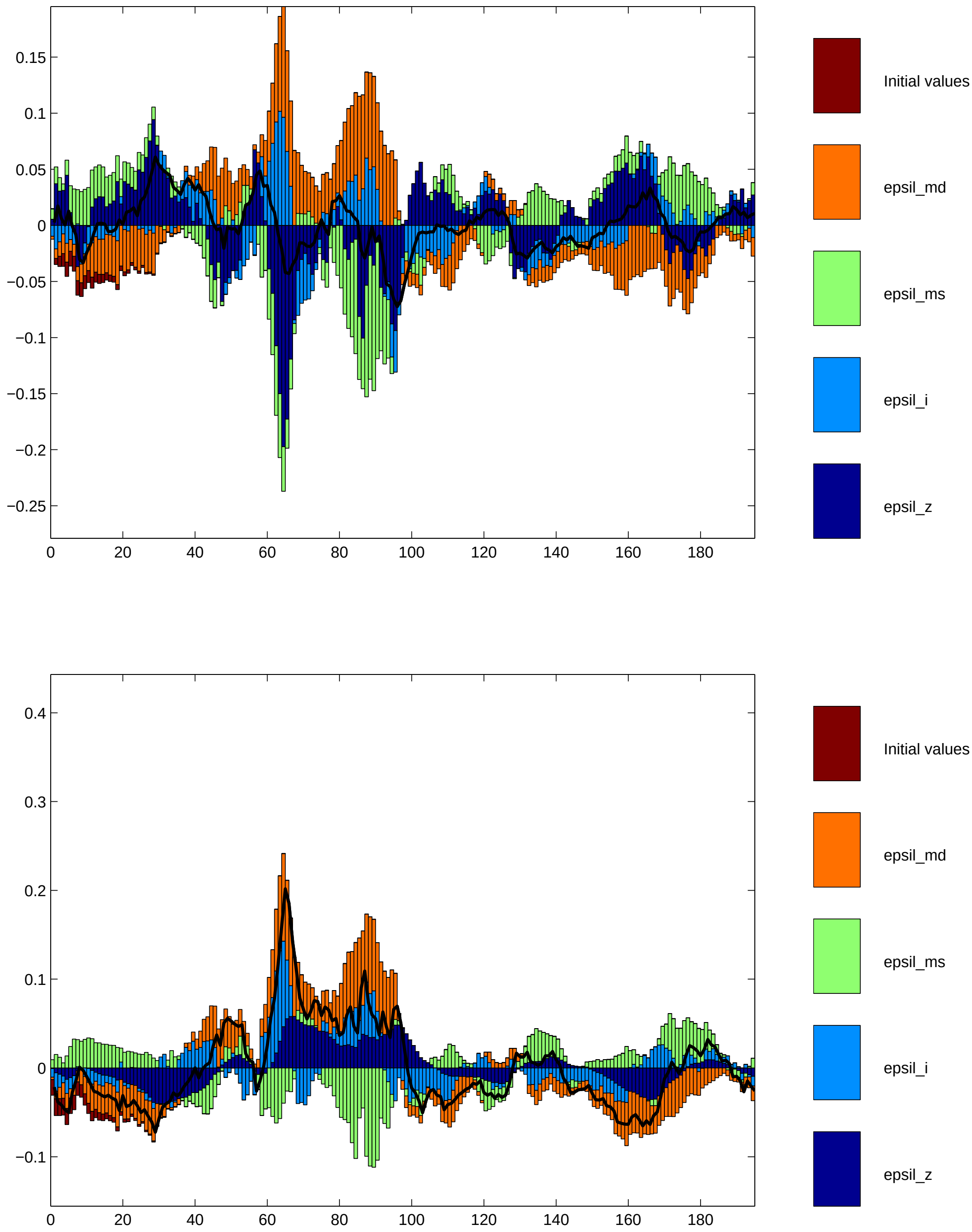

Figure 9: Historical decomposition (1959Q1−2007Q3): Output gap (top panel); Output (bottom panel).

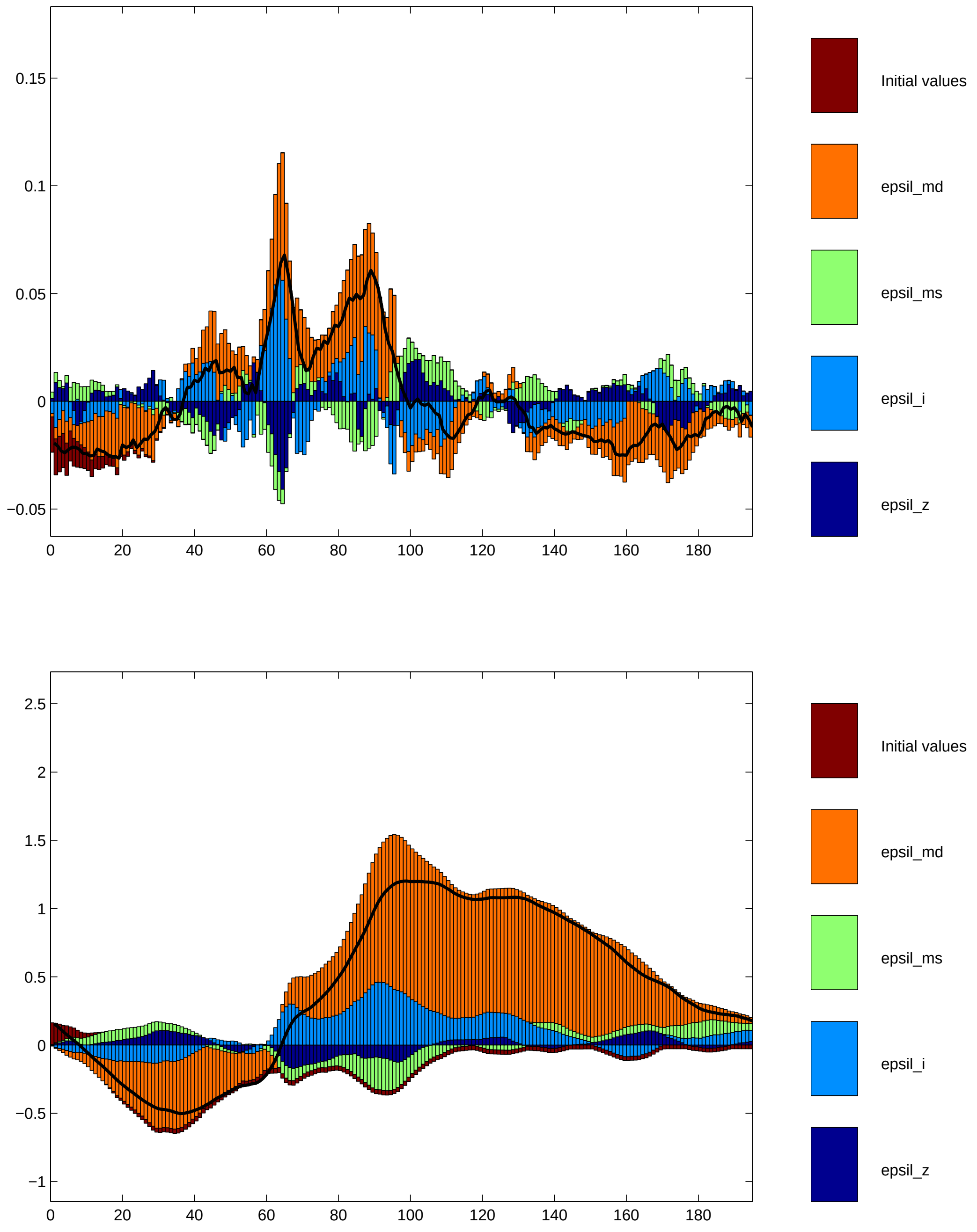

Figure 10: Historical decomposition (1959Q1−2007Q3): Inflation (top panel); Price-level (bottom panel).

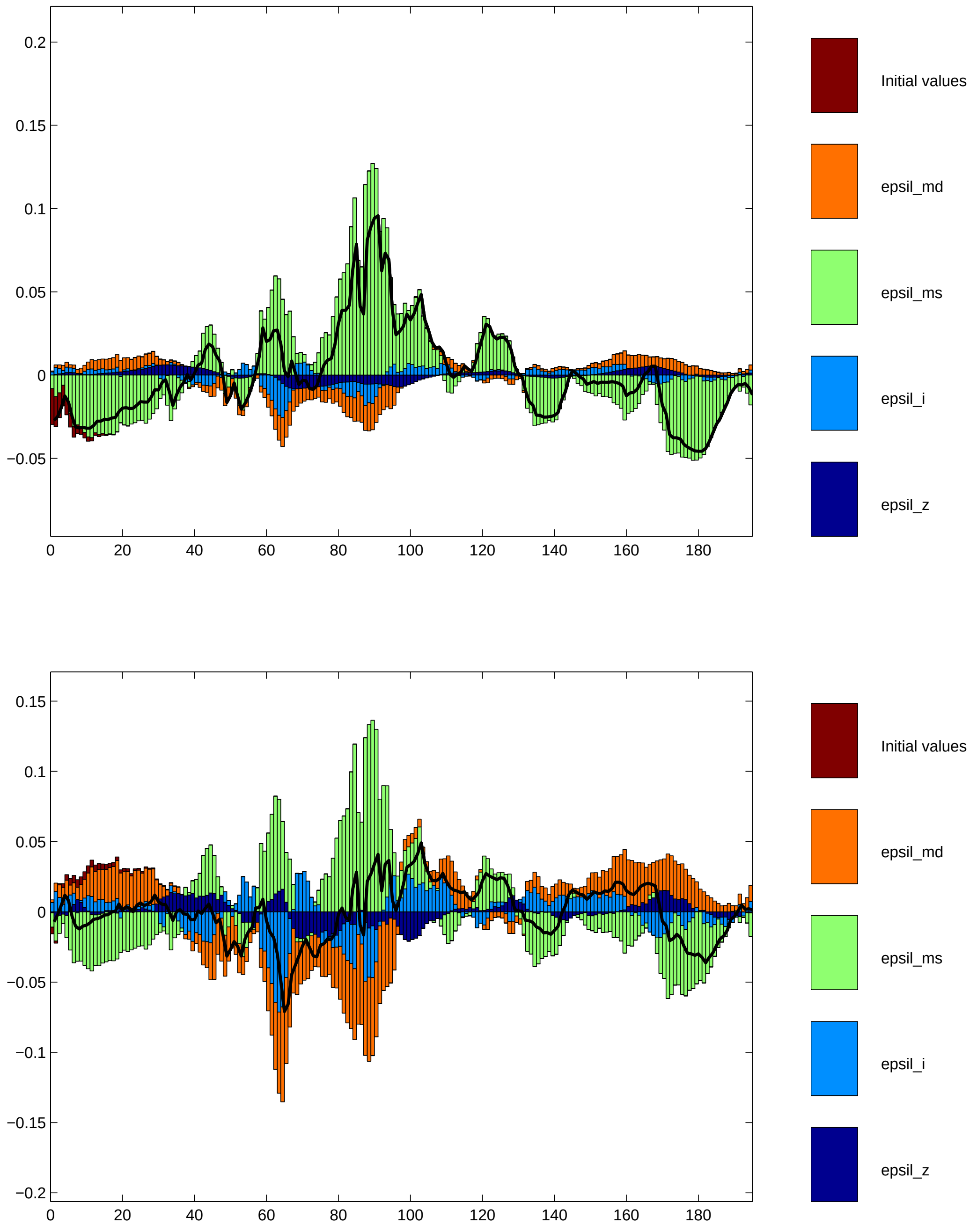

Figure 11: Historical decomposition (1959Q1−2007Q3): Nominal interest rate (top panel); Real interest rate (bottom panel).

6 A counterfactual post-crisis simulation

6.1 The behaviour of two monetary regimes

To illustrate how the choice of monetary regime changes the behaviour of the economy, Table 2 shows the variance decomposition of all the macroeconomic variables for two types of endogenous monetary regimes. The four shocks are a technology shock, an interest rate target shock, a money supply shock and a money demand shock. The structural parameters and size of the shocks are calibrated to the posterior estimates of the Great Moderation (GM) period (see Table 1). For the interest rate targeting regime, the elasticity of reserves is high: $\{\nu_h, \nu_{fr}\} = \{11, 12\}$. The monetary authorities therefore prefer to minimize (smooth) interest rate variations by allowing the money supply curve flatten. Money and reserves are therefore endogenous and weakly relevant for price determination. For the interest sensitive monetary rule $\nu_h = 1$ and $\nu_{fr} = 1$. Here, an exogenous shock to money supply directly influences the interest rate and generates 'money illusion' in the short-run. Notably, separability between consumption and money in households utility implies that money is still neutral in steady-state. The following result goes to show that the choice of monetary regime has a significant influence on the role of monetary aggregates. And although money is neutral in the long-run, sticky prices generate the necessary distortion for 'money illusion' to have a significant influence over real economic variables.

First, to illustrate price level determination, we show the variance decomposition of the nominal and real variables under a strict interest rate targeting regime, as observed over the actual GM period ($\nu_h = 11; \nu_{fr} = 12$), and a counter-factual flexible (interest sensitive) monetary regime ($\nu_h = \nu_{fr} = 1$). It is immediately clear that long-run variation in money, velocity, prices and output are mainly determined by shocks to the target interest rate under the GM monetary rule. But given that there is still a strong effect of money supply shocks on the interest rate, exogenous innovations in money supply still have a significant impact on real variables due to sticky price adjustment. The final two columns clearly show that nominal (monetary) shocks in a New-Keynesian model are neutral under flexible prices. A flexible (interest sensitive) monetary rule, in contrast, highlights the importance of exogenous innovations in money supply and demand for business cycle fluctuations. Here, money supply and demand interact to determine variation in the price-level and inflation over the entire forecast horizon. And as shown analytically by McCallum (1986), we see that the type of monetary regime can influence the transmission mechanism of monetary policy dramatically *without changing the dynamic adjustment path of the economy* (see Figures B.5 to B.7). A strict interest rate targeting regime becomes problematic, however, if the operational instrument—the policy rate—cannot lower the real interest rate enough. It is the monetary regime, not the zero lower bound, that renders monetary policy ineffective.

6.2 Effectiveness of reserves in a flexible interest rate monetary regime

Figure 12 presents simulated results for a post-global financial crisis policy response: a 25bp reduction in free reserves.[26] The results suggest that the evolution toward a stricter interest rate targeting regime renders central bank balance sheet expansions superfluous. In the context of the 2007−09 global financial crisis, a more flexible interest rate targeting regime would have lead to a significant monetary expansion and more rapid economic recovery in the U.S. Specifically, based on the post-crisis average free reserves held at the Federal Reserve, a counterfactual simulation—with the zero lower bound constraint on nominal interest rates imposed—indicates that a \$3.7 billion reduction in free reserves expands the money supply by 3.65 percentage points and output by 3.84 percentage points. Of course, this stylised example fails to capture uncertainty and regulatory constraints in the banking sector. Moreover, the Fed's 'unconventional' policy responses in the post-crisis period largely targeted long-term government securities and alternative private asset classes—i.e., quasi-debt management policies and credit policies (Borio and Disyatat, 2010, p. 62). In other words, a constant short-term nominal interest rate does not generate any intertemporal substitution of consumption in a standard New-Keynesian model. The result is therefore more indicative of the constraint on the type of monetary regime (operational framework) adopted by the Federal Reserve going into crisis—which ignores the wealth (income) effect. In fact, Brunnermeier and Sannikov (2016) develop a model in which monetary policy redistributes wealth and risk to stimulate (inside) money creation and counteract disinflationary pressures.

[26]It can be equally assumed that the Federal Reserve increases non-borrowed reserves. But given that the banking sector held on average approximately \$1570 bn in excess reserves (\$ 1483 in free reserves) for the period 2007Q4−2016Q3, we assume that (inside) money creation will likely be observed as a decline in ER and rise in RR, holding NBR constant.

Table 2: Variance decomposition of business cycle under two monetary regimes (in percent)

	Great Moderation monetary rule ($\nu_h = 11; \nu_{fr} = 12$)				Interest sensitive monetary rule ($\nu_h = \nu_{fr} = 1$)				Flexible prices ($\nu_h = \nu_{fr} = 1$)			
Shock	ϵ_z	ϵ_i	ϵ_{m^s}	ϵ_{m^d}	ϵ_z	ϵ_i	ϵ_{m^s}	ϵ_{m^d}	ϵ_z	ϵ_i	ϵ_{m^s}	ϵ_{m^d}
money (m)	0.09	98.23	1.06	0.62	0.02	3.6	96.07	0.31	0.59	2.56	96.85	0
velocity (v)	19.3	48.42	12.5	19.78	21.58	0.09	56.3	22.03	49.85	0.05	49.99	0.12
prices (p)	0.26	83.36	0.38	16	0.52	1.44	10.84	87.19	3.6	1	35.03	60.37
output (y)	12.1	43.29	30.41	14.21	3.83	0.17	87.5	8.5	100	0	0	0
real rate (r)	2.72	37.62	50.03	9.63	0.46	0.17	95.06	4.31	100	0	0	0
nominal rate (i)	1.23	18.09	77.24	3.43	0.26	0.14	97.27	2.32	35.73	0.06	64.05	0.16
nominal target rate (i^T)	0.02	99.92	0.02	0.05	0.01	99.73	0.18	0.08	0.01	99.97	0.01	0.02
inflation (π)	11.79	55.26	12.64	20.31	4.33	0.25	73.95	21.47	11.35	0.16	83.12	5.37
output gap ($\tilde{y}$)	31.66	33.65	23.64	11.04	6.19	0.17	85.35	8.29	100	0	0	0
natural output (y^n)	100	0	0	0	100	0	0	0	100	0	0	0
natural rate (r^n)	100	0	0	0	100	0	0	0	100	0	0	0
Non-Borrowed Reserves (h)	0.07	93.78	5.69	0.45	0.14	45.84	52.53	1.49	6.11	81.99	11.82	0.08
Free Reserves (fr)	0.07	93.23	6.23	0.46	0.14	42.37	55.97	1.52	6.77	80.08	13.07	0.08

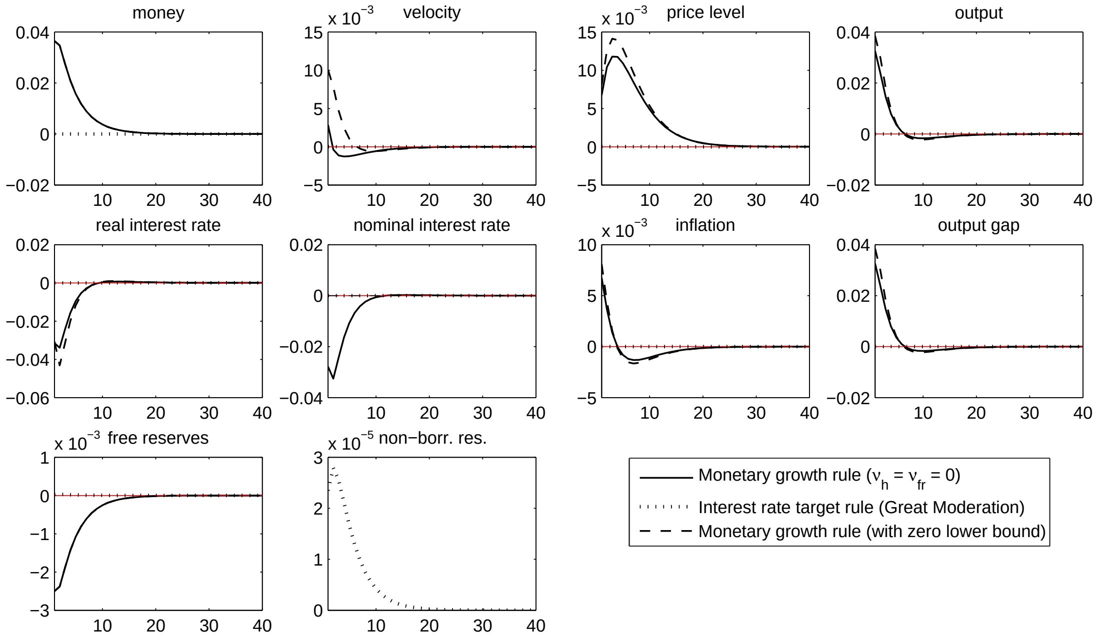

Figure 12: Negative free reserves shock: 25 basis points.

7 Concluding remarks

[TBC]

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A Distribution statistics

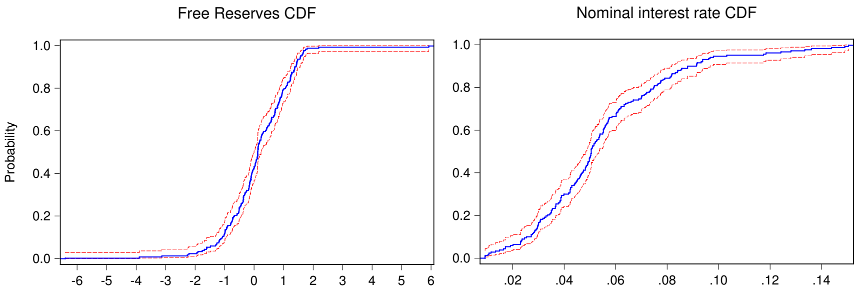

Figure A.1: Cumulative Distribution Function of Free Reserves and 3-Month T-Bill. Sample: 1959Q1−2007Q03.

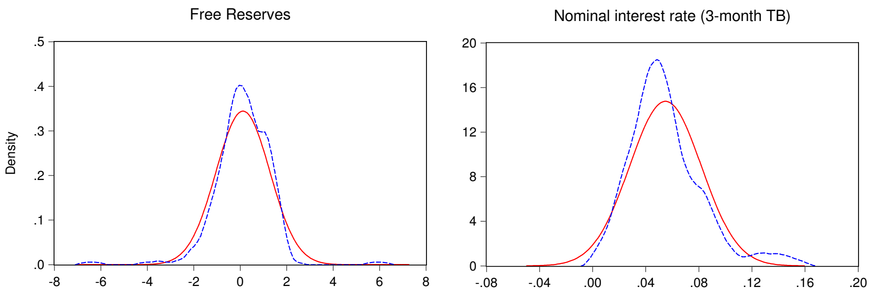

Figure A.2: Probability Density Function (dotted) and approx. Normal Distribution (solid) of Free Reserves and 3-Month T-Bill. Sample: 1959Q1−2007Q03.

B Simulation IRFs

C Estimation results

C.1 Estimation diagnostic statistics

C.2 Historical decompositions

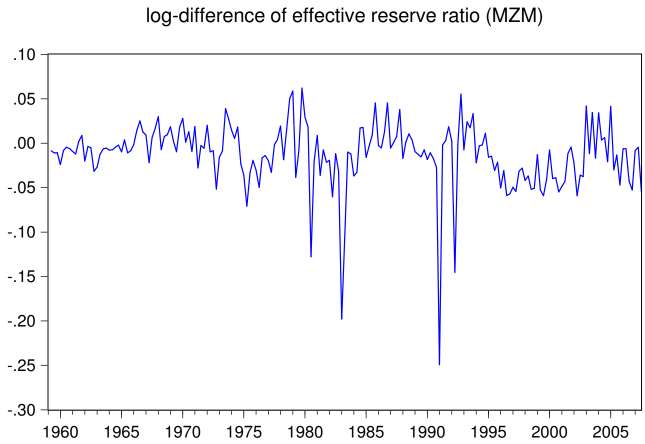

Figure A.3: Log-difference effective reserve ratio ($RR_t/MZM_t = rr_t$): 1959Q1−2007Q03.

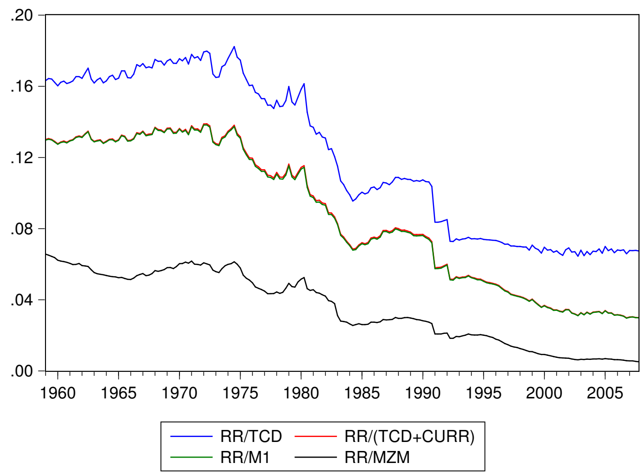

Figure A.4: Effective reserve ratio ($RR_t/M_t = rr_t$). Sample: 1959Q1−2007Q03.

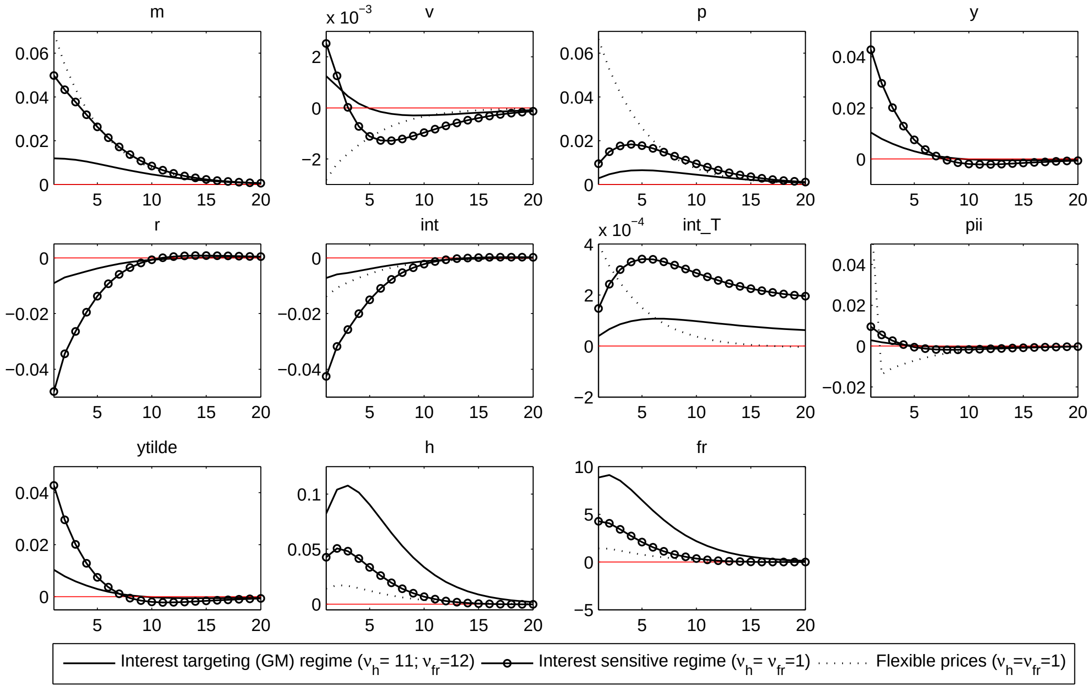

Figure B.5: Positive money supply shock.

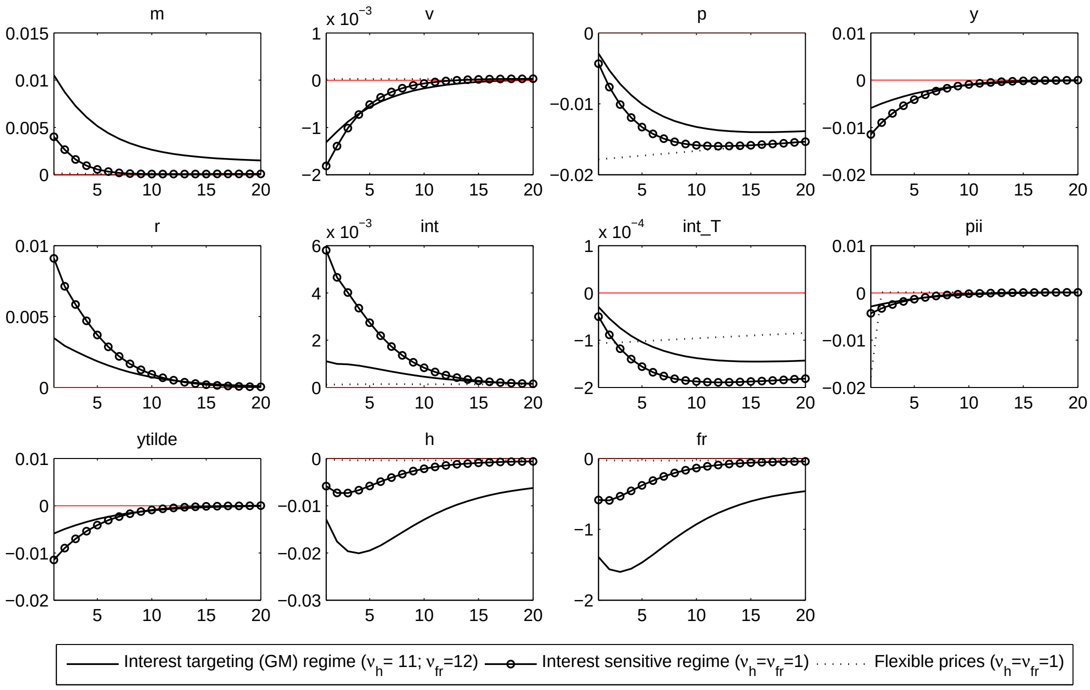

Figure B.6: Positive money demand shock.

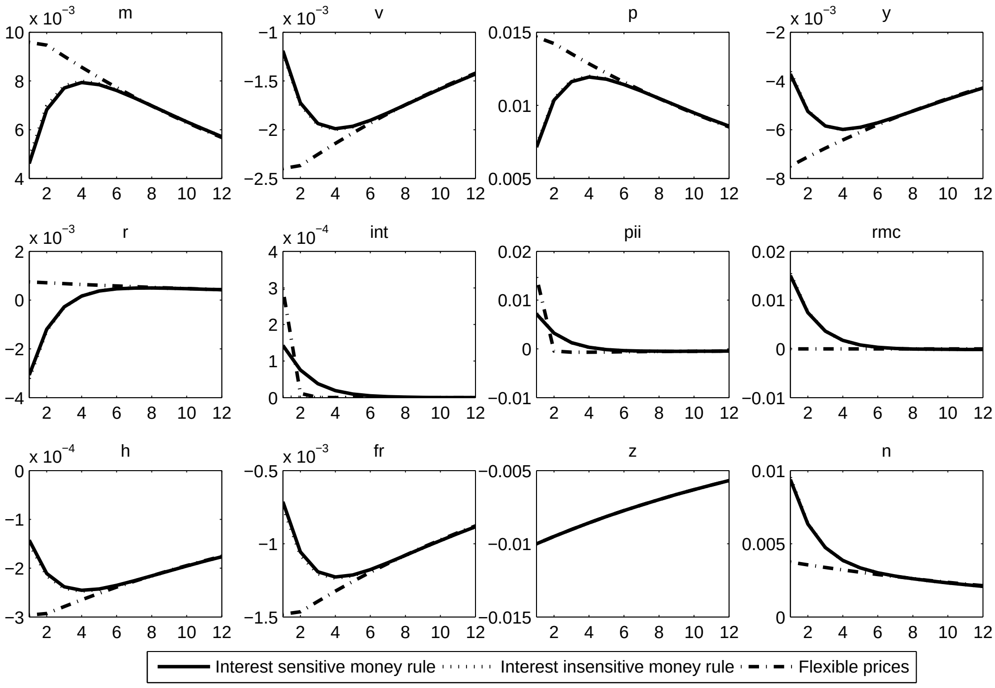

Figure B.7: Negative technology shock.

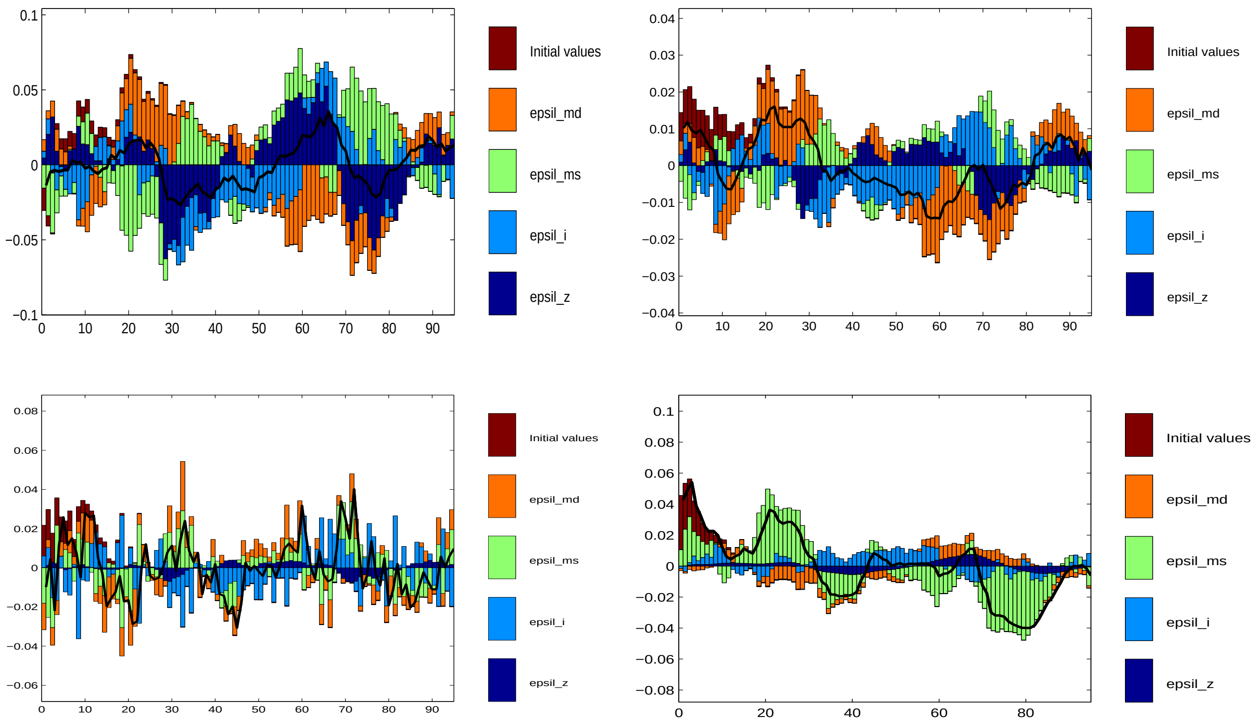

Figure C.8: Historical decomposition (1984Q1−2007Q3). Output gap (top-left); Inflation (top-right); Money (%-ch) (bottom-left); Interest rate (bottom-right).

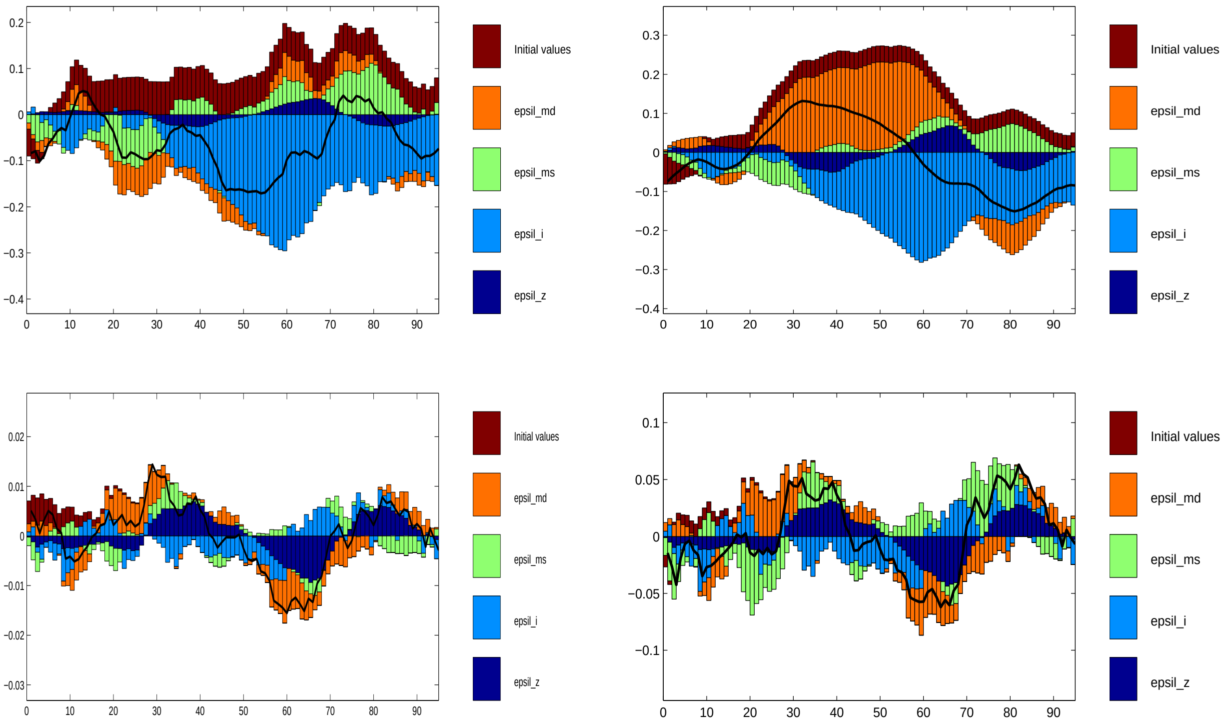

Figure C.9: Historical decomposition (1984Q1−2007Q3). Money (top-left); Velocity (top-right); Price-level (bottom-left); Output (bottom-right).