An overview of salient factors, relationships and values to support integrated energy-economic systems dynamic modelling

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Stellenbosch Economic Working Papers: 02/13
FEBRUARY 2013

KEYWORDS: ENERGY-ECONOMIC MODELING, SYSTEM DYNAMICS, ELASTICITIES, ECONOMIC SUBSTITUTION, TECHNICAL SUBSTITUTION, ENERGY EFFICIENCY, ENERGY COST SHARE, HEAT ENGINE EFFICIENCY
JEL: C63, Q43, Q47

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A WORKING PAPER OF THE DEPARTMENT OF ECONOMICS AND THE
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Abstract

Integrated energy-economic modeling is needed to support the development of energy and carbon policies. We propose that a systems dynamic modeling approach is needed; one that includes (a) dynamics (b) endogenous treatment of uncertainty and risks, and (c) both aggregate economic and disaggregate technical or engineering levels of analysis. To support the future development of integrated energy-economic models we review and organise the literature on energy-economy interactions into subsections covering (a) the key factors or components, (b) the relationships among these components, (c) a quantification of parameters and (d) implications for the development of an integrated energy-economic systems dynamic model. The literature is organized in discussions on economic growth and the factors of production, elasticities, macro- and technical substitutability, energy cost shares, heat engine efficiencies and energy services efficiencies. We observe non-linear relationships in production and consumption, large variations among price and income elasticity values across time frames, across countries and regions, and across energy goods, far from perfect substitution among factors of production and among energy goods on a macro level, technical/engineering limits to substitution on a micro level, as well as engineering and behavioural limits on what can be achieved with increased efficiencies. We therefore support the call to develop integrated energy-economic systems dynamic models that are able to provide new insight into the nature of energy-economic transitions.

Key words: energy-economic modeling, system dynamics, elasticities, economic substitution, technical substitution, energy efficiency, energy cost share, heat engine efficiency

1. Introduction

Academic and policy debates on how energy and carbon limits are influencing real economies world-wide are once again demanding serious attention. Tverberg (2012), for example, points to an inevitable recessionary feedback effect and a reinforcement of the financial crises from rising fossil fuel, notably oil, prices, while Dolan (2011) points towards the ability of the market mechanism, price incentives, and the protection of property as superior mechanisms to circumvent fossil fuel depletion and rising prices.

With rising public policy stakes on an increasingly complex and risky topic, it is becoming clear that partial analysis will not be sufficient. Several recent papers from influential institutions such as the IMF have begun pointing toward the need for combining both geological and economic/technological views in one integrated model. (See Benes et al. (2012) for an application to oil supply.) Kumhof and Muir (2012), in another IMF modeling paper, indicate how physical scarcity of oil could lead to very large variations in simulated outcomes if certain modeling assumptions, notably elasticities, substitutability, and production functions, are changed. The authors recommend that future research focus on "…a multidisciplinary approach to modeling, which better represents the dependence of production technologies on physical processes…" (Kumhof & Muir, 2012:4)

In the academic literature, much recent research is also now focused on the review of and development of integrated energy-economy models (Stern, 2011a; Kümmel et al. 2010; Bashmakov, 2007). Brandt (2010:3958) in a review of forty-five mathematical models of oil depletion concluded that they "have fared poorly in predicting global oil production, [and] the greatest promise … lies in simulation models that combine both physical and economic aspects of oil production". Clearly, research and modeling work is needed in the field of integrated economic-energy modeling. This paper aims to make a contribution to this emerging field.

We propose that a systems dynamic modeling approach is needed; one that includes (a) dynamics (b) endogenous treatment of uncertainty and risks, and (c) both aggregate economic and disaggregate technical or engineering levels of analysis. In this paper we follow the nomenclature of systems thinking in terms of function, components, and the relationships among these components (Meadows, 2008) to indicate a framework by which future modeling efforts can be informed. Out of the interactions among system components, a dynamic process unfolds, often with feedback loops reinforcing or counteracting original changes in the system (Deaton and Winebrake, 2000). Because complex system patterns can emerge over time from very simple interactions, it is crucial that models are built on the basis of best available science on cause-effect relationships. In this paper we organise the literature on energy-economy interactions into subsections covering (a) the key factors or components, (b) the relationships among these components, (c) a quantification of parameters and (d) implications for the development of an integrated energy-economic systems dynamic model.

The paper is structured as follows. In section 2 we review factors of production and their interrelationships cognizant of various economic growth theories. In section 3 we look into the sensitivity of one economic variable to another (or elasticity) regarding the demand and supply of energy goods to price and to income. In section 4, we review the concept of macro-substitutability, or the possibility to substitute one input to production for another, while in section 5 we review the idea of technical or engineering substitutability. In section 6 the central idea of energy cost share is reviewed. In sections 7 and 8 heat engine efficiencies and energy services efficiencies, respectively, are discussed. Finally, section 9 concludes.

2. Economic Growth and the Factors of Production

Factors and Components

In economic growth theory, changes in output (*Q*) are explained by a set of input factors: capital (*k*), labour (*l*), land (*n*), energy (*e*), materials (*m*), and knowledge (*h*). The selection of input factors is guided by economic growth theory. There are two primary types of growth theories: exogenous and endogenous.

Exogenous growth models do not include human capital, innovation, or knowledge as endogenous explanations of the growth process, and all economic growth that cannot be empirically explained by production factors such as capital (*k*), labour (*l*), land (*n*), and energy (*e*) is attributed to technological productivity (*A*). In exogenous growth theory the rate of growth is determined exogenously through the savings rate in Keynesian theories (also referred to as the Harrod-Domar model after Harrod, 1939 and Domar, 1946) or through technological progress in Neo-classical theories (also referred to as the Solow-Swan model after Solow, 1956 and Swan, 1956). Most exogenous growth models explain growth in output (*Q*), through a combination of productivity (*A*), capital (*k*), and labour (*l*), but some models include non-renewable resources (Solow 1974; Stiglitz 1974; Dasgupta & Heal, 1979; Hartwick 1977; Dixit et al., 1980) and energy (*e*) as separate production factors (Stern 2000; Kummel 1982; Berndt & Wood, 1979; Hudson & Jorgenson 1974). Environmental pollution has also been included in production functions, effectively reducing technology's (*A*) balancing contribution to growth (Xepapadeas & Vouvaki 2009; Xepapadeas 2005).

In endogenous growth models it is assumed that human capital, innovation, and knowledge are generated within the economic system itself. Several approaches exist, namely neoclassical endogenous AK models (Barro & Sala-i-Martin, 1995; Romer 1986), evolutionary Schumpeterian growth models of creative destruction (Aghion & Howitt, 1998), with resource, material, and energy constraints (Warr and Ayres, 2012; Acemoglu et al. 2009; Ayres & van den Bergh 2005; Ayres & Warr, 2005; Bretsgher, 2005), and endogenous Unified-Growth long-wave models (Jones, 2001; Galor and Weil, 2000) that focus on explaining the longer term process of economic development through a combination of factors such as technological progress and innovation, population growth and demographics, institutions, and human capital accumulation. In addition, endogenous growth models that focus on the material basis of economic growth, including the role of energy and natural resources as well as biophysical limits as governed by the laws of thermodynamics, have also been developed more recently (Smulders, 1995). Fröling (2011) specifically included energy as an input in an endogenous long-wave model.

Relationships

Not only the components of economic growth theory differ, but also the functional forms (equations) used in modelling. The functional relationships between the input factors (and in certain cases the dis-utilities of environmental pollution and waste) and output have been described as being of the form Constant Elasticity of Substitution (CES) (with special cases Linear, Cobb-Douglas, and Leontief), Translog, Quadratic, or Linear exponential (LINEX). In most cases of mainstream neo-classical economic growth modeling, CES or the special form Cobb-Douglas are used. Functional forms are described in detail elsewhere (Mishra, 2007; Ayres & Warr, 2005).

Implicit in the selection of functional form is substitutability among inputs, a deeply contentious issue among economists. One's view of the role of thermodynamics in economic growth (see Daly, 1997) informs assumptions about substitutability. We discuss substitutability from both macro-economic and technical or engineering substitutability points of view later.

Quantification

Quantification of economic growth models is accomplished by fitting unknown parameters in the growth equation (elasticities of substitution, for example) to achieve the best possible agreement with historical economic data. Quantified values therefore depend largely upon the economic growth theory employed and the functional form of the growth equation.

Implications

Earlier debates in economics concentrated on the theoretical validity of production functions, mainly focusing on how aggregate capital is measured (Robinson 1953), a debate that has not been resolved as yet (Cohen & Harcourt 2003). Shaikh (2005:462) further argued that even if aggregate production functions "appear to work on an empirical level, they provide no support for the neo-classical theory of aggregate production and distribution." The important point for an integrated systems dynamic modeling approach is that the levels and rates of change in the factors of production and limitations on output in the economy are both dependent on the functional form selected. It would be wrong to assume that macro-level aggregates are rooted in micro-foundations – specific micro-level, engineering limitations need to be specified and included in the modeling effort. We return to this issue in section 5.

3. Elasticities

Factors and Components

Elasticity refers to the sensitivity of one economic variable to another variable expressed as the ratio of percentage rates of change. Important economic variables for calculating elasticities for energy goods are the *demand* for an energy good, the *price* of the energy good, the *income* of the consumers demanding the good, the *price* of complements to and substitutes for the energy good, the *supply* of the energy good, the *output* of an energy good and the *inputs* used in the production of the energy good (Varian, 1992).

Relationships

The relationships among these variables, namely price elasticity of demand, price elasticity of supply, income elasticity of demand, cross-price elasticity of demand and output elasticity, are interpreted as follows:

- E_d, price elasticity of demand, measures the responsiveness of the quantity demanded of a good (Q_d) relative to a change in price (P) of that same good. $E_d = \frac{P\left(dQ_d / dt\right)}{Q_d\left(dP/dt\right)}$. Outcomes are inelastic demand ($-1<E_d<0$), elastic demand ($E_d<-1$) perfectly inelastic demand ($E_d=0$) or unitary elastic demand ($E_d=-1$).
- E_s, price elasticity of supply measures the responsiveness of the quantity supplied of a good (Q_s) relative to a change in price of that same good. $E_s = \frac{P\left(dQ_s / dt\right)}{Q_s\left(dP/dt\right)}$. Outcomes are inelastic supply ($E_s<1$), elastic supply ($E_s>1$), no response or "fixed supply" ($E_s=0$) or unitary elastic supply ($E_s=1$).
- E_y, income elasticity of demand measures the responsiveness of demand for a good (Q_d) relative to a change in the income (y) of those demanding the good. $E_y = \frac{y\left(dQ_d / dt\right)}{Q_d\left(dy/dt\right)}$. Outcomes are inferior goods ($E_y<0$), necessity goods ($0<E_y<1$), luxury or superior goods ($E_y>1$), or sticky goods ($E_y=0$).
- $E_{i,j}$, cross-price elasticity of demand measures the responsiveness of the demand for one good i ($Q_{d,i}$) relative to the price of another good j (P_j). $E_{ij} = \frac{P_j\left(dQ_{d,i} / dt\right)}{Q_{d,i}\left(dP_j/dt\right)}$. Products are either complements ($E_{i,i}<0$), substitutes ($E_{i,j}>0$), or independent from each other ($E_{i,j}=0$).
- E_{Qx}, output elasticity measures the responsiveness of output (Q) relative to any one input (x, which can be any of capital (k), labour (l), land (n), energy (e), materials (m), or knowledge (h)). $E_{Qx} = \frac{x\left(dQ / dt\right)}{Q\left(dx/dt\right)}$. Outcomes are either constant returns to scale ($E_{Qx}=1$), increasing returns to scale ($E_{Qx}>1$), or decreasing returns to scale ($E_{Qx}<1$) in relation to any one input factor (x), while other input factors are kept constant. If $\frac{1}{E_{Qx}} = \frac{Q\left(dx/dt\right)}{x\left(dQ/dt\right)} = 0$, output ($Q$) is said to be "decoupled" from the input (x). These output elasticities are denoted as output elasticity with respect to capital ($E_{Qk} = \alpha$), with respect to labour ($E_{Ql}= \beta$), with respect to energy ($E_{Qe} = \varepsilon$), with respect to land ($E_{Qn} = \nu$), with respect to knowledge ($E_{Qh} = \eta$) and with respect to materials (if modelled as 'active partner' in production process) ($E_{Qm} = \mu$).

Elasticities are expressed over the short run and the long run, where "long" and "short" do not refer to particular time scales. In the "short run," the quantity of at least one input is fixed, while in the "long run," quantities of all inputs vary.

Quantification

Most of the studies dealing with energy elasticities date from the 1970s and 1980s and more work is needed in deriving elasticities for contemporary energy regimes. Output elasticities should not be reported without clarity on the production function chosen, for reasons discussed in section 2. For example, production functions that include physical work as a factor of production will have larger output elasticities for energy than those that denote energy inputs by cost shares (see Lindenberger and Kummel, 2011; Auburn 2011).

Across a selection of more recent studies, and for both the short and long run, the price elasticity of energy demand (E_d) is inelastic (-0,65), the price elasticity of energy supply (E_s) is inelastic (0,14), and income elasticity of demand (E_y) signals a luxury good (1,49), although all these numbers vary greatly from one study to the next[1]. E_d is more elastic (although still inelastic) in the long than in the short run, E_s is less inelastic in the long run than in the short run, and E_y indicates that energy is a necessity good in the short run compared to a luxury good in the long run. Cross–price elasticities for energy goods are not reported here, because no recent comprehensive review could be found in the literature.

Each energy carrier can be analyzed independently. E_d for crude oil, kerosene, and gasoline are the most inelastic compared to other forms of energy, while E_y signals luxury goods for natural gas and crude oil and necessity goods for gasoline, diesel, and petroleum. E_s is inelastic for all energy sources, with the notable exception of non-OPEC countries in the short-run (Ramcharran, 2002).

Implications
The choice of values for elasticities to be used in an integrated dynamic model is not as straightforward as it may at first seem. Large variation among elasticity values across time frames, across countries and regions, and across energy goods suggest an empirical approach specific to the research question at hand and the economic theory being employed. Ultimately, we desire a systems dynamic model from which elasticities are a *result* rather than an input. With such a model, the result elasticities can be compared to the above observations to validate that the model is correctly reproducing real-world economic behaviour.

4. Macro-Substitutability

Factors and Components
The elasticity of substitution measures how easily one input (in production) or good (in consumption) may be substituted for another. In an integrated energy-economy systems dynamic model, the elasticity of substitution could indicate the substitution possibility among the input factors of production (k, l, e, m, h) or among different energy types or energy carriers. As Stern (2011b) pointed out, there are considerable differences among different definitions for substitutions and complementarity and clarification is needed when stating modeling assumptions.

Relationships
Elasticity of substitution between two factor inputs or goods is measured as the percentage response of the relative marginal products of the two factors to the percentage change in the ratio of their quantities, $E_{xy} = \frac{d\ln(\frac{y}{x})}{d\ln(MRTS_{xy})}$, where $MRTS_{xy} = -\frac{dy}{dx} = \frac{MP_x}{MP_y}$. The marginal product ($MP$) of an input factor is the extra output that can be produced by using one more unit of the input, keeping the quantities of other inputs to production constant. The closer that the elasticity of substitution comes to unity, the higher the possibility of substitution between the two input factors. Conversely, the closer that the elasticity of substitution comes to zero, the more complementary the input factors are to one another.

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[1] Studies included are: Cooper (2003), Krichene (2005), Dahl and Duggan (1998), Ramcharran (2002), Sa'ad and Shahbaz (2012), Dahl (2012).

The Cobb-Douglas production function assumes unitary factor substitution elasticity. Constant elasticities of substitution are assumed between factors of production when working with production functions specified as CES (Arrow et al. 1961).

In the case of two factor inputs, functions for the elasticity of substitution are straightforward, such as the elasticity of substitution between capital and labour (E_{KL}), between capital and energy (E_{KE}), between energy and labour (E_{EL}) or between energy and materials (E_{EM}). In the case of three or more factor inputs, nested functions are needed, such as the elasticity of substitution of capital/labour and energy ($E_{KL,E}$), capital/energy and labour ($E_{KE,L}$), energy/labour and capital ($E_{EL,K}$), capital/labour and energy/materials ($E_{KL,EM}$), capital/labour/materials and energy ($E_{KLM,E}$) or capital/labour/energy and materials ($E_{KLE,M}$).

A very relevant variation of the elasticity of substitution is the elasticity of substitution between energy inputs that are environmentally benign and those that are not. Pelli (2011) estimated that clean and dirty inputs to the production of electricity in 21 countries are complementary. Another variation is interfuel substitutability. Stern (2012) published a meta-analysis on the topic and concluded that both the level of analysis and the type of fuels matter and that substitution among energy sources is relatively easy at the industrial level, but that substitution of gas for electricity (and vice versa) or coal for electricity (and vice versa) at the industrial level is more difficult. Stern (2011b) also found that energy substitutability is practically more difficult to achieve at a macro level.

Quantification

Elasticities of substitution differ substantially among sectors and among types of inputs and goods studied (Koesler & Schymura 2012). Empirical work so far demonstrates much lower than unitary substitution elasticities between capital and labour, capital and energy, and between combinations of capital/labour and energy as well as capital/energy and labour (Koesler and Schymura, 2012; Okagawa and Ban, 2008; Balistreri et al. 2003; Van der Werf 2008). Therefore, Koesler and Schymura (2012), whom did a study for 27 EU countries and 13 other major countries, across 35 economic sectors and industries, argue that Cobb-Douglas and Leontief production functions (which assume unitary elasticity of substitution) must be rejected for the majority of economic sectors. Indicative values (averages and standard deviations over several studies, time frames, countries, industry sectors and regions) are as follows[2]:

- E_{KL} = 0,42 (σ = 0,39)
- E_{KE} = 0,55 (σ = 0,38)
- E_{EL} = 0,55 (σ = 0,36)
- E_{EM} = 0,70 (σ = n/a)
- $E_{KL,E}$ = 0,65 (σ = 1,07)
- $E_{KE,L}$ = 0,66 (σ = 0,33)
- $E_{EL,K}$ = 0,81 (σ = 0,21)
- $E_{KL,EM}$ = 0,70 (σ = n/a)
- $E_{KLM,E}$ = 0,50 (σ = n/a)
- $E_{KLE,M}$ = 0,69 (σ = 0,32)

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[2] These numbers are the averages and standard deviations as based on the following studies: Bosetti et al. (2006), Burniaux et al. (1992), Edenhofer et al. (2005), Gerlagh and van der Zwaan (2003), Goulder and Schneider (1999), Kemfert (2002), Manne et al (1995), Paltsev et al. (2005), Popp (2004), Sue Wing (2004), Van der Werf (2007), Okagawa and Ban (2008), Balistreri et al. (2007), Kemfert (1998), Koesler and Schymura (2012)

Caution in interpreting these results are needed as they are sometimes based on very few and often assumed observations.

Implications

Because all of the above results show elasticity of substitution below unity, none of the factor inputs are perfectly substitutable and all tend toward complementarity in varying degrees. Such results suggest that transitions from one production or consumption structure to another can be disruptive and that the transitions need to be modeled dynamically to the extent possible.

5. Technical Substitutability

Factors or Components

From a technical point of view, substitution from one type of energy to another is rarely simple. Important factors include:

- which raw energy types (fossil fuels, renewables, etc.) or energy carriers (electricity, refined liquid fuels, hydrogen, etc.) are being substituted,
- whether machines (wind turbines, solar panels, oil wells and refineries, etc.) are available at low-enough cost to produce the new energy types,
- whether machines (the electric grid, transportation engines, etc.) are available to transport the new energy types, and
- whether machines (factory machines, consumer goods, etc.) are available at low-enough cost to consume the new energy types.

Fouquet (2010) noted that, historically, energy substitutions (such as wind-to-coal and coal-to-oil) take several decades from the beginning of diffusion through the economy to dominance in the economy. Previous energy substitutions were accomplished because new forms of energy were perceived by consumers to be both better and cheaper. And, historically, total energy consumption was *greater* after major energy substitutions occurred.

Recent work by Jacobson and Delucchi (Jacobson, 2009; Jacobson and Delucchi, 2009; Jacobson and Delucchi, 2011; Delucchi and Jacobson, 2011) evaluated a complete world energy substitution to wind, water, and solar (WWS) raw energy sources and a fully-electric energy carrier system. With some exceptions (such as ocean shipping, long-distance road freight transport, and air travel), the substitution to an all-electric energy system is technically achievable today, but requires massive infrastructure investments and comes with significant cost. An incomplete list of factors involved in the WWS substitution proposed by Jacobson and Delucchi includes:

- Capacity and reliability of the electrical grid when significantly higher penetration of intermittent sources (wind and solar, in particular) and increased power transmission distances (required when source locations are far from consumption locations) are present.
- Complete substitution in the transportation system from internal combustion engines using refined liquid-fuel energy carriers (gasoline, diesel, and aviation fuels) to electric motors with storage batteries.
- Availability of investment capital, especially for constructing millions of new wind turbines, billions of solar panels, millions of electric motors, millions of fuel cells, and a significantly-enhanced electrical grid required by the WWS plan.
- Availability of investment energy for manufacturing new production and consumption machines.
- Availability of raw materials such as rare earth metals for electric motors, lithium for batteries, and platinum for fuel cells.

The Global Energy Assessment (GEA) (Johansson et al., 2012) suggests the following policies to achieve energy substitutions:

- removal, or at least substantial reduction, of subsidies to fossil fuels without carbon capture and storage
- stimulation of development and market entry of new renewable options
- emphasis on energy efficiency in all end-use sectors

Relationships

From a technical point of view, key quantifiable measures and relationships for WWS energy substitutions include:

- Capacity factors of WWS energy generating machines (ratio of actual energy production to energy production that would have occurred if the machine were operating continuously at rated capacity)
- Various measures of electricity supply intermittency from WWS machines
- Marginal cost increases (or decreases) to consumers (and therefore the economy as a whole) for energy source and carrier substitutions, including both the incremental cost of WWS electricity and the cost for replacing obsoleted consumption machines (e.g., automobiles with internal combustion engines replaced by electric vehicles)
- Energy cost share in the economy (see section 6 below)

Quantification

Although energy substitutions are technically possible, they must be bought at a price. Jacobson and Delucchi (2009) estimate the total cost for emplacing a WWS energy system to be $100 trillion over 20 years, or $5 trillion/year. 2011 world GDP at PPP is estimated at $69 trillion/year (CIA, 2012). So, the WWS plan would cost an additional ~7% of world GDP for the next 20 years, just to emplace.

The GEA’s plan would require additional investment in the energy sector amounting to 2–3% of GDP per year for the next 40 years (or longer). If we accelerate the GEA plan to match the timescale of the WWS plan (20 years) and if we assume that the costs scale linearly, the GWS plan reaches 4–6% of GDP per year. Thus, the WWS and GEA plans are roughly comparable in terms of investment cost to purchase substitutability of renewable energy sources for non-renewable energy sources.

Of course, cost projections are difficult in the short term and nearly impossible over a 20–40 year timeframe. However, the WWS cost figure, in particular, is likely to be an underestimate. A partial list of factors that are underestimated or not considered by Jacobson and Delucchi includes:

- Compensation for owners of obsoleted but still-useable assets (fossil fuel power plants, gasoline and diesel vehicles, oil and gas pipelines, gas ovens, etc.) (Tverberg, 2009)
- Erosion of value for owners of stock in companies with obsoleted assets (Tverberg, 2009)
- An unspecified amount of energy storage at extremely low cost (Brook, 2011)
- Significant underestimate of costs for an enhanced electrical transmission grid (Preston, 2011).
- Operations and maintenance costs (Moriarty, 2011)
- Underestimate of future electricity consumption rates (Moriarty, 2011)

Because of the above factors, both Moriarty (2011) and Tverberg (2009) have placed the cost estimate at around $200 trillion or more over 20 years, or at least 14% of GDP over 20 years for emplacing the WWS system.

Implications

Understanding of the technical aspects of energy substitutions leads to a conclusion that a worldwide transition away from fossil fuels and other non-renewable energy sources toward renewable energy sources will be costly in terms of both money and time. Integrated energy-economy systems dynamic models must account for both the cost to the economy of such a transition and the time to execute the transition. Additionally, marginal price changes that will accompany energy transitions should be included in such a model.

6. Energy Cost Share

Factors or Components

Recently, the impact on economic growth of an economy's energy cost share has received attention in the literature. The components of energy cost share in a given time period (*CS*) are energy type (*i*), energy price for each type (p_i), energy consumption rate for each type (Q_i), and *GDP*. The energy cost share for an economy at a given time *t* is calculated by

$$CS_t = \frac{\sum_i p_{i,t} Q_{i,t}}{GDP_t}. \tag{1}$$

Relationships

Recent research (Bashmakov, 2007) is showing that developed economies can sustain high total energy cost share above a threshold for a short period of time (possibly 2–3 years) before recessionary pressures destroy energy demand, stimulate energy efficiency, reduce energy prices, and return total energy cost share to its long-term sustainable range. On the other hand, reduction of total energy cost share below a lower bound provides economic stimulus, increases energy demand, provides upward pressure on energy prices, and returns the energy cost share to its long-term sustainable range.

Bashmakov (2007:3585) speculates that "energy affordability thresholds and behavioral constants" are responsible for the stable range of energy cost share over many decades. Embarking on a modern growth path appears to reduce the energy cost share in an economy from very high values (indicating that nearly all economic activity is focused on procuring energy) to small values that remain within a stable range.

Quantification

According to Bashmakov (2007) the stable range for economy-wide energy cost share is 8–10% for the U.S. and 9–11% for the OECD. The stable and narrow range of energy cost share for final consumers in the U.S. is 4–5% and in the OECD is 4.5–5.5% (Bashmakov, 2007). The oil cost share threshold that correlates with U.S. recessions is about 5.5% (Murphy et al., 2011). Sweden's energy cost share has stabilized at 12% since 1970, although it was nearly 100% in 1800 (Stern, 2012).

The South African case study is illuminating, for it shows the effects of energy cost share threshold, and it illustrates that regional effects are important. Wakeford (2012) shows that a 1979 oil cost share spike to just below 5% did not correlate with a recession, because a simultaneous gold price spike offset the negative effect of the oil price spike. However, a 1985 South African oil cost share spike to 5% correlated with a recession. The 1990 Gulf War oil price spike was mitigated in South Africa by domestic coal-to-liquids (CTL) production capabilities. Since the end of the Apartheid in 1994, global oil price spikes have led to increased energy cost share in South Africa's economy, because (a) South Africa is now integrated with the world economy and (b) the share of CTL in total consumed petroleum is declining due to increasing reliance on imported oil. Thus, the South African economy today may be more vulnerable to global oil price spikes than in the past. An oil cost share spike to nearly 7% preceded the 2009 recession in South Africa.

The picture emerging from this research shows that it is not energy *price*, per-se, that impacts the economy. Rather, the energy *cost share* (and, perhaps more narrowly, *oil* cost share) is a likely key factor. The Swedish example shows that energy cost shares evolve over time along a development path. The South African case study shows that regional and local considerations can be significant. We expect that a successful dynamic energy–economy model will exhibit a stable range for energy cost share of around 10%.

Additional research is needed to (a) isolate the economic effects of energy cost shares for different energy types (coal vs. oil, for example), (b) assess differential energy cost share effects for regional economies, (c) understand the evolution of energy cost shares as an economy develops, and (d) understand the dynamic system interactions with other elements of the economy that lead to a stable corridor of energy cost share over time.

Implications

The dynamics of energy cost share should be a *result* of an integrated energy-economy systems dynamic model rather than an input to that model. Thus, a successful integrated energy-economy systems dynamic model should predict an energy cost share range above which recessionary pressures may limit economic growth or induce further innovation and below which economic growth is stimulated.

7. Heat Engine Efficiency

Factors or Components

Heat engines, such as electric power plants and internal combustion engines, produce most of the world's useful work for moving freight, shaping material, providing light, and delivering services. Heat engines take in heat at a given rate (Q_H) at high temperature (T_H) and reject heat at a different rate (Q_C) at low temperature (T_C) as they produce a rate of useful energy, work (W). Heat engine operators pay for high temperature heat (typically in the form of coal or liquid fuels) and receive revenue (or useful energy services) from work W. Many factors affect the profitability of energy producing firms, including revenue rate, fuel price, operations and maintenance costs, and capital loan repayment costs.

Relationships

The thermal efficiency of a heat engine is given by

$$\eta = \frac{W}{Q_H}. \tag{2}$$

The theoretical maximum efficiency of a heat engine (Carnot efficiency) is a function of its operating temperatures and is given by

$$\eta_{Carnot} = 1 - \frac{T_C}{T_H}. \tag{3}$$

The existence of an upper (Carnot) limit to heat engine efficiency indicates that increasing the efficiency of heat engines cannot, by itself, completely address the challenge of depleting non-renewable energy sources.

A finite-sized plant that operates at the maximum efficiency (η_{Carnot}) for a given T_H and T_C has no output: the rate of production of sellable energy (W) is zero. Thus, there is an efficiency–power tradeoff. For an existing plant with non-zero fuel, operations and maintenance, or capital recovery costs, there is an economic incentive to produce energy at a high rate (W), thereby obtaining revenue to cover costs and turn a profit. Thus, the efficiency–power tradeoff is made in favour of power at the expense of efficiency in real-world plants.

Curzon and Ahlborn (1975) were the first to quantify the efficiency of a heat engine operating at maximum power output, i.e. a heat engine operating at the point where it is producing sellable energy at the maximum possible rate:

$$\eta_{max\ power} = 1 - \sqrt{\frac{T_C}{T_H}}. \tag{4}$$

A power plant *maximizes* revenue when it operates at maximum power ($\eta_{max\ power}$). In contrast, a power plant that operates at maximum thermodynamic efficiency (η_{Carnot}) *has no revenue*! Real-world power plants operate near the maximum power conditions ($\eta_{max\ power}$), because there is an economic incentive to do so.

Quantification

The difference in efficiency for power plants operating at maximum power ($\eta_{max\ power}$) and maximum efficiency (η_{Carnot}) for the same T_H and T_C is significant: a hypothetical coal-fired power plant operating with T_H = 565 °C (838 K) and T_C = 25 °C (298 K) will have η_{Carnot} = 0.64 and $\eta_{max\ power}$ = 0.40. Thus, there is an economic incentive to operate heat engines with efficiency that is significantly lower than (in this case 38% lower than) the thermodynamic limit.

Implications

Looking ahead, options for increasing the efficiency of practical heat engines are constrained. Equation 4 indicates that increasing T_H, reducing T_C, or both will improve the efficiency of heat engines operating at their maximum power point. Unfortunately, the lower bound on T_C is given by the nearby water or ambient air temperature, thus offering no realistic possibility for efficiency improvement. Increasing T_H is feasible only by employing higher-temperature (and, presumably, higher-cost) materials within power plant boilers, requiring technological breakthroughs in material science. After many decades of similar power plant economics, no such important breakthroughs have been forthcoming. We suggest that any energy–economy systems dynamic model assume a fixed value of heat engine efficiency that is roughly equivalent to today's value.

8. Energy Services Efficiency

Factors or Components

The efficiency of converting raw energy carriers or intermediate energy products into energy services (such as light, motion, lifting, cutting, bending, etc.) is another important consideration for developing a dynamic energy–economy model. Improvements in energy services efficiency can have unexpected effects. Jevons (1866) was the first to suggest what has become known as the *rebound effect*, wherein an energy services efficiency intervention results in less energy savings than expected. Both direct (usually behavioural) and indirect (usually economic) feedbacks can contribute to the rebound effect. An example of a direct (behavioral) feedback is that an LED light bulb may be left “on” longer when people know it consumes energy at a lower rate. An example of indirect (economic) feedback occurs when improved energy services efficiency reduces energy costs, thereby increasing cash in hand which is spent on other products and services that require energy to produce, distribute, and consume.

Energy services efficiency (η_{ES}) can be defined as

$$\eta_{ES} = \frac{E_{useful}}{E_{total}}, \tag{5}$$

where E_{useful} is the rate of useful portion of energy services consumption and E_{total} is the total rate of energy provided to the economy.

The energy intensity of an economy (I) during period t is defined as

$$I_t = \frac{E_t}{GDP_t}, \tag{6}$$

where E_t and GDP_t are energy consumed by the economy and gross domestic product, respectively, in time period t. We note that when the first derivative of energy intensity with respect to time (dI_t/dt) is less than zero, an economy exhibits relative decoupling of economic activity from energy consumption. If the following is true

$$\frac{1}{E_{Qe}} = \frac{Q\left(de/dt\right)}{e\left(dQ/dt\right)} = 0, \tag{7}$$

the economy is said to exhibit absolute decoupling from energy consumption.

The rebound effect (RE) is defined as

$$RE = 1 - \frac{S_{actual}}{S_{\exp ected}}, \tag{8}$$

where S_{actual} is the actual energy savings and $S_{expected}$ is the expected energy savings from an energy services efficiency intervention. A 10% rebound effect indicates that only 90% of an expected energy reduction has been achieved for the same level of service provided.

Relationships

Typically, increasing energy services efficiency is thought to decrease both energy consumption and energy intensity. However, an energy services efficiency intervention coupled with a strong rebound effect can increase total energy consumption of an economy, a phenomenon known as *backfire*. Jevons (1866) argued that increasing steam engine efficiency in early-industrial England led to an *increasing* rather than decreasing rate of coal use for the economy as a whole: Jevons' Paradox. Warr et al. (2010:42) say:

> We argue that energy efficiency improvements drive economic growth through [an effect similar to the] rebound effect. *Ceteris paribus* efficiency improvements provide more useful work per unit of energy purchased and hence drive down the costs of products and services. Lower prices stimulate demand enabling economies of scale and R&D. The resultant product, process, and price improvements increase revenues and further stimulate growth.

For Warr et al. (2010), the rebound effect (with backfire!) is what drives economic growth as we know it.

Quantification

Economists are divided on both the existence and magnitude of the rebound effect at both the micro and macro levels. Sorrell (2009) admits that the rebound effect is difficult to test empirically, due to the many interacting factors at play in the feedback loops.

Estimates of the magnitude of the rebound effect range from 0% to 100% and beyond (Davis, 2009). Sorrell (2009) notes that (a) the evidence for Jevons' Paradox (backfire) is inconclusive at this time and (b) perceptions of the magnitude the rebound effect are colored by assumptions about the role of energy in economic growth. Economists who contend that energy's role in economic growth is commensurate with its small cost share typically find little evidence of the rebound effect. (See, *e.g.*, Berkhout (2000) and Schipper (2000).) In contrast, economists who believe that energy's role in economic growth far exceeds its cost share tend to find that rebound effects are significant. (See, *e.g.*, Sorrell (2007) and Sorrell (2009).)

Energy intensities vary widely among economies. The UK has the lowest energy intensity at 0.102 koe/$2005p, the U.S. is at 0.173 koe/$2005p, South Africa is at 0.303 koe/$2005p, and Uzbekistan (highest) is at 0.633 koe/$2005p (enerdata.net, 2011). (Units on the preceding numbers are kg of oil equivalent per 2005$ at power purchase parity.)

Implications

We recommend that a rebound effect be included in energy–economy systems models as an adjustable exogenous parameter that can be used for sensitivity studies. Further, we posit that energy intensity should be an *outcome of* (not an input to) an energy–economy systems model. A successful integrated energy-economy systems dynamic model will predict energy intensity that is in line with today's values.

9. Conclusions

In response to both policy literature and academic literature calling for greater integration and multi-disciplinary modeling approaches, this paper attempts to provide an overview of the main factors and interrelationships for energy-economic systems. Taken together we find:

- non-linear relationships in production and consumption,
- large variations among price and income elasticity values across time frames, across countries and regions, and across energy goods
- far from perfect substitution among factors of production and among energy goods on a macro level,
- technical/engineering limits to substitution on a micro level, and
- engineering and behavioural limits on what can be achieved with increased efficiencies

We agree with the call by leading institutions and several scholars to start developing integrated energy-economic models. Such models, however, need to be able to reproduce a complex, emergent energy-economic reality. Non-linearity, large variations and the existence of enigeering and behavioural limits all indicate a need for a specific focus on the nature of transitions as informed by a modeling approach that is able to capture complex dynamics, feedback loops and an endogenous modelling of risk.

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